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An opinion control rule with minimum adjustments to support the consensus reaching in bounded confidence model

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Abstract

Opinion dynamics provides a modeling tool for the public opinion management. The existing studies mainly focused on building the evolution model of opinions. However, the control of public opinions has been a key problem in practical opinion dynamics. The objective of this paper is to propose an opinion control rule to support the consensus reaching. Based on the bounded confidence model, the consensus model with the minimum adjustment is proposed. Next, based on the proposed consensus model, we propose the opinion control rule to support the consensus reaching. Furthermore, a numerical example is given to illustrate the feasibility of the proposed opinion control rule. Through simulation experiments, we investigate the effects of adjustment thresholds and bounded confidences on the opinion control rule.

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1. Introduction

Opinion dynamics can be defined as a dynamic and iterative process. In opinion dynamics, a group of agents express their initial opinions over the same issue. Based on the communication regime, their opinions are continuously updated as the time elapses. At the final stage, a consensus (or fragmentation) among the agents [2-3, 8-12, 24] is formed.

The studies on opinion dynamics went back to French. French [15] proposed the social power model to explore the patterns of interpersonal relations and agreements. Later Harary [18] provided a necessary and sufficient condition to reach a consensus in French's model. According to French's study, different types of studies on opinion

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formation have been proposed [20]: (i) opinion formation with continuous or discrete time [2,27,34], (ii) opinion formation based on different communication regimes [5-6,21,26,35], (iii) opinion formation with the multi-dimensional space of possible opinions [15,18,24], (iv) opinion formation in a specified network [16,30,33,36-37], and (v) opinion formation considering noises [22,32]. Among the studies mentioned above, bounded confidence model has become one of hot issues in opinion dynamics. Bounded confidence model assumes that each agent only communicates with the peers whose opinions are closely to its own. The earliest bounded confidence models are presented independently by Hegselmann and Krause [21] and by Deffuant and Weisbuch [6]. The two bounded confidence models are called the HK model and the DW model, respectively. In the HK model, agents synchronously update their opinions by averaging all opinions in their confidence sets; in the DW model, agents follow a pairwise-sequential updating procedure in averaging. Based on the HK model and DW model, the scholars conducted some extended studies. For example, Blondel et al. [1] discussed both the agent-based and density-based homogeneous HK models. Lorenz [28-29] reformulated the HK model as an interactive chain and analyzed the effects of heterogeneous confidence bounds. Dong et al. [13, 25] extended the HK model into the linguistic and uncertain environment.

The existing studies have made significant contributions on opinion dynamics. The existing studies mainly focused on building the evolution model of opinions. However, the control of opinions has been a key problem in practical opinion dynamics. With the rapid development of Internet, people can express their opinions conveniently. But the emergence of Internet also accelerates the spread of gossip in public opinions. If the public opinions are not controlled, it is easier to trigger the social conflicts and mass incidents. Furthermore, when controlling the public opinions, the opinion managers always hope that the opinions of all the agents reach a consensus with minimum adjustments.

The objective of this paper is to propose an opinion control rule with minimum adjustments. The reminder of this paper is arranged as follows: Section 2 introduces the HK bounded confidence model and consensus in opinion dynamics. Then, Section 3 proposes the opinion control rule. In Section 4, a numerical example is given, and several simulation experiments are designed to discuss the effects of adjustment thresholds and bounded confidences on the opinion control rule.

2. Preliminary

This section briefly introduces the basic knowledge regarding the HK model [21], and the consensus in opinion dynamics, which will provide a foundation for this study.

2.1. Bounded confidence model: the HK model

Consider a standardized opinion dynamics problem. Let $A = \{A_1, A_2, \dots, A_N\}$ be a set of agents. Let t be the discrete time, $t = 0, 1, 2, \dots$. The crisp opinion of agent $A_i \in A$ at time t is represented by $o_i(t) \in [0, 1]$. Let $O(t) = (o_1(t), o_2(t), \dots, o_N(t))^T$ be the vector of the opinions of all the agents at time t called the opinion profile. Let ε be the bounded confidence.

The process of opinion evolution in the HK model include three steps:

(1) Determination of the confidence set

In opinion dynamics problem, agent A_i only trusts the opinions which differ not more than ε from his/her opinion. Let $I(A_i, O(t))$ be the confidence set of agent A_i at time t, where

$$I(A_i, O(t)) = \{A_j \| o_i(t) - o_j(t)| \leq \varepsilon\}, \qquad i = 1, 2, \dots, N,\ t = 0, 1, 2, \dots \tag{1}$$

(2) Calculation of the weight

Agent A_i assigns the same weight to the agents in his/her confidence set $I(A_i, O(t))$. Let $w_{ij}(t)$ be the weight that agent A_i assigns to agent A_j at time t, where

$$w_{ij}(t) = \begin{cases} 1/|I(A_i, O(t))| & A_j \in I(A_i, O(t)) \\ 0, & A_j \notin I(A_i, O(t)) \end{cases}, \qquad i = 1, 2, \dots, N,\ t = 0, 1, 2, \dots \tag{2}$$

Clearly, $w_{ij}(t) \geq 0$ and $\sum_{j=1}^{N} w_{ij}(t) = 1$.

(3) Evolution of the opinions

The evolutions of the opinions in the HK model are modeled as the weighted arithmetic means of opinions in the confidence sets, i.e.,

$$o_i(t+1) = w_{i1}(t)o_1(t) + w_{i2}(t)o_2(t) + \cdots + w_{iN}(t)o_N(t), \qquad i = 1, 2, \ldots, N. \tag{3}$$

2.2. The consensus in opinion dynamics

Generally, the stabilized structures refer to the case that the opinions of all the agents are not changed after a fixed time. Consensus [19, 31], which is a typical stabilized structure, refers to the cases that all the agents hold the same opinions. Thus, the mathematical definition for the stabilized structures and consensus can be given as follows:

Definition 1 Let t_0 and t_1 ($t_1 \geq t_0$) be any two times, if $o_i(t_1) = o_i(t_0)$ ($i = 1, 2, \ldots, N$), and $o_j(t_0 - 1) \neq o_j(t_0)$, $\exists \ j \in \{1, 2, \ldots, N\}$, then we call the opinion profile converge to the stabilized structures at time t_0.

Definition 2 Let t_0 as defined before. If $o_i(t_0) = o_j(t_0)$ ($i, j = 1, 2, \ldots, N$), then a consensus among the agents is reached. Otherwise, the fragmentation among the agents are formed.

In the framework of bounded confidence, the existing studies (eg., [27]) have provided the sufficient conditions to the consensus reaching, i.e., **Theorem 1**.

Theorem 1 [27] Let $I(A_i, O(t))$ and $I(A_j, O(t))$ be as defined before. If there exist a time t_0, such that $I(A_i, O(t)) \cap I(A_j, O(t)) \neq \emptyset$, for all $i, j \in \{1, 2, \ldots, N\}$, then a consensus among the agents will be reached.

3. The proposed opinion control rule

In this section, we propose the consensus model with minimum adjustments. Then, based on the proposed consensus model, we propose the opinion control rule.

3.1. The consensus model with minimum adjustment

In the proposed opinion control problem, let $o_i(t)$ as before, and let $\bar{o}_i(t)$ be the adjusted opinion of agent $A_i \in A$ at time t. Let $x_i^{(t)}$ be the 0-1 variable which counts the number of adjusted opinions, i.e.,

$$x_i^{(t)} = \begin{cases} 1, & o_i(t) \neq \bar{o}_i(t) \\ 0, & o_i(t) = \bar{o}_i(t) \end{cases}, \qquad i = 1, 2, \ldots, N. \tag{4}$$

Then, minimizing the number of adjusted opinions can be described as:

$$Min \sum_{i=1}^{N} x_i^{(t)} \tag{5}$$

Meanwhile, it is natural that the distance between $o_i(t)$ and $\bar{o}_i(t)$ has the acceptable level, i.e.,

$$|\bar{o}_i(t) - o_i(t)| \leq \beta, \qquad i = 1, 2, \ldots, N, \tag{6}$$

where β is the established adjustment threshold. The larger β values indicate the agents can accept the bigger changes of their opinions.

Let $r_{ij}^{(t)}$ be the 0-1 variable for determining whether agent A_j belongs to the confidence set $I(A_i, O(t))$, i.e.,

$$r_{ij}^{(t)} = \begin{cases} 1, & \bar{o}_i(t) - \varepsilon \leq \bar{o}_j(t) \leq \bar{o}_i(t) + \varepsilon \\ 0, & otherwise \end{cases}, \qquad i, j = 1, 2, \ldots, N. \tag{7}$$

Let $g_{ikj}^{(t)}$ be the 0-1 variable for determining whether agent A_j belongs to both the confidence sets $I(A_i, O(t))$ and $I(A_k, O(t))$, i.e.,

$$g_{ikj}^{(t)} = \begin{cases} 1, & r_{ij}^{(t)} + r_{kj}^{(t)} = 2 \\ 0, & otherwise \end{cases}, \qquad i, k = 1, 2, \ldots, N, \tag{8}$$

where $r_{ij}^{(t)} + r_{kj}^{(t)} = 2$ denotes that agent A_j belongs to the confidence sets $I(A_i, O(t))$ and $I(A_k, O(t))$.

Let $f_{ik}^{(t)}$ be the 0-1 variable for determining whether the confidence sets $I(A_i, O(t))$ and $I(A_k, O(t))$ have the common agent, i.e.,

$$f_{ik}^{(t)} = \begin{cases} 1, & \sum\limits_{j=1}^{N} g_{ikj}^{(t)} \geq 1 \\ 0, & \sum\limits_{j=1}^{N} g_{ikj}^{(t)} = 0 \end{cases}, \qquad i,k = 1,2,\ldots,N. \tag{9}$$

Let $z^{(t)}$ be the threshold which counts the pairs of agents who have the common agents in their confidence sets. In practical opinion dynamics, the opinion manager can set the value of $z^{(t)}$ by the real need.

As a result, the consensus model can be constructed as follows:

$$Min \sum_{i=1}^{N} x_i^{(t)} \tag{10}$$

$$s.t. \quad |\bar{o}_i(t) - o_i(t)| \leq \beta, \qquad i = 1,2,\ldots,N, \tag{11}$$

$$\sum_{i=1}^{N-1} \sum_{k=i+1}^{N} f_{ik}^{(t)} \geq z^{(t)} \tag{12}$$

$$x_i^{(t)} = \begin{cases} 1, & o_i(t) \neq \bar{o}_i(t) \\ 0, & o_i(t) = \bar{o}_i(t) \end{cases}, \qquad i = 1,2,\ldots,N, \tag{13}$$

$$r_{ij}^{(t)} = \begin{cases} 1, & \bar{o}_i(t) - \varepsilon \leq \bar{o}_j(t) \leq \bar{o}_i(t) + \varepsilon \\ 0, & otherwise \end{cases}, \qquad i,j = 1,2,\ldots,N, \tag{14}$$

$$g_{ikj}^{(t)} = \begin{cases} 1, & r_{ij}^{(t)} + r_{kj}^{(t)} = 2 \\ 0, & otherwise \end{cases}, \qquad i,k = 1,2,\ldots,N, \tag{15}$$

$$f_{ik}^{(t)} = \begin{cases} 1, & \sum\limits_{j=1}^{N} g_{ikj}^{(t)} \geq 1 \\ 0, & \sum\limits_{j=1}^{N} g_{ikj}^{(t)} = 0 \end{cases}, \qquad i,k = 1,2,\ldots,N, \tag{16}$$

where $x_i^{(t)}$ $(i = 1,2,\ldots,N)$ are the decision variables in model (10)-(16).

Note: In this paper, we don't discuss the method to obtain the optimal solutions to model (10)-(16). In the future, an extended version of this conference paper will be provided to discuss this problem in detail.

3.2. The proposed algorithm

The details of the opinion control rule based on the consensus model are depicted in the following algorithm.

Input: The initial opinions $o_i(0)$ $(i = 1,2,\ldots,N)$, the bounded confidence ε, and the adjustment threshold β.

Output: The adjusted opinions $\bar{o}_i(t)$ $(i = 1,2,\ldots,N)$, and the number of the iterations t.

Step 1: Let $t = 0$ and $o_i(t) = o_i(0)$, $i = 1,2,\ldots,N$.

Step 2: Let $\bar{o}_i(t)$ be the adjusted opinion of agent A_i at time t. Then, using model (10)-(16) obtains the adjusted opinions $\bar{o}_i(t)$.

Step 3: Based on the adjusted opinions $\bar{o}_i(t)$, we use the HK model (i.e., Eqs. (1)-(3)) to calculate the opinion $o_i(t+1)$, $i = 1,2,\ldots,N$.

Step 4: If the conditions regarding the consensus (See **Definition 1**) are satisfied, then go to Step 6, Otherwise, go to Step 5.

Step 5: Let $z^{(t)}$ be as defined before, and let $t = t + 1$. If $z^{(t)} = \frac{N(N-1)}{2}$, then go to Step 3; Otherwise, go to Step 2.

Step 6: Output $\bar{o}_i^{(t)}$ $(i = 1,2,\ldots,N)$ and t.

4. Numerical example and simulation experiment

In this section, we firstly provide a numerical example. Then, through simulation experiments, we investigate the effects of adjustment thresholds and bounded confidences on the opinion control rule.

4.1. Numerical example

In this example, assume that there are 50 agents who are participated in opinion dynamics. Their initial opinions $o_i(0)$ $(i = 1, 2, \ldots, 50)$ are given as: $o_i(0) = \frac{i}{50}$.

Let $\varepsilon = 0.1$. If the evolution of opinions is not controlled, we can obtain the evolution of the original opinions, which are shown in Fig. 1.

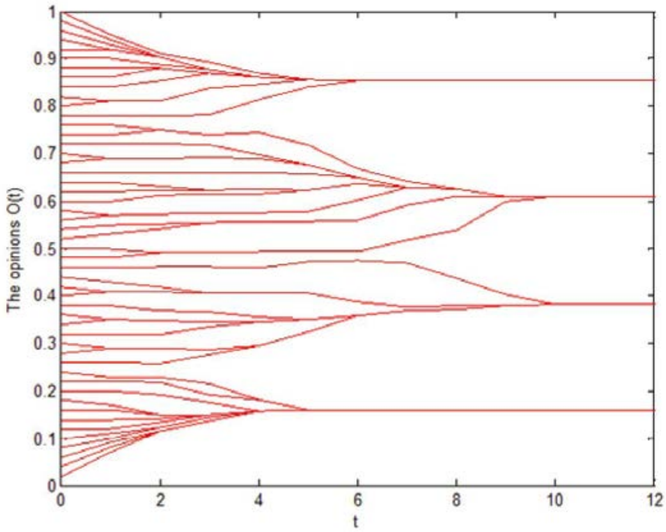

Fig. 1. The evolution of the original opinions in the HK model.

From Fig. 1, we obtain the following observations: The opinions of all the agents become stabilize at time $t = 10$. And four clusters among the opinions of all the agents are formed in the stabilized result.

Let $\beta = 0.15$. Then we use the proposed opinion control rule to determine the adjusted opinions of the agents, which are shown in Table 1.

Table 1. The adjusted opinions at each time.

t	$\bar{o}_i(t)$	$z(t)$
$t = 0$	$\bar{o}_1(0) = 0.17$, $\bar{o}_2(0) = 0.19$, $\bar{o}_{49}(0) = 0.83$, $\bar{o}_{50}(0) = 0.85$	454
$t = 1$	$\bar{o}_1(1) = 0.3325$, $\bar{o}_6(1) = 0.3247$, $\bar{o}_8(1) = 0.2817$, $\bar{o}_{41}(1) = 0.6736$, $\bar{o}_{43}(1) = 0.6518$	567
$t = 2$	$\bar{o}_{10}(2) = 0.3325$, $\bar{o}_{12}(2) = 0.3745$, $\bar{o}_{45}(2) = 0.6518$, $\bar{o}_{49}(2) = 0.6407$, $\bar{o}_{50}(2) = 0.6407$	617
$t = 3$	$\bar{o}_3(3) = 0.4385$, $\bar{o}_4(3) = 0.4385$, $\bar{o}_5(3) = 0.4385$, $\bar{o}_{18}(3) = 0.5219$, $\bar{o}_{19}(3) = 0.5619$, $\bar{o}_{22}(3) = 0.5266$	724
$t = 4$	$\bar{o}_{10}(4) = 0.4847$, $\bar{o}_{12}(4) = 0.4561$, $\bar{o}_{14}(4) = 0.4451$, $\bar{o}_{15}(4) = 0.4561$, $\bar{o}_{26}(4) = 0.5519$, $\bar{o}_{28}(4) = 0.5883$	795
$t = 5$	$\bar{o}_{11}(5) = 0.4936$, $\bar{o}_{13}(5) = 0.4916$, $\bar{o}_{17}(5) = 0.5243$, $\bar{o}_{28}(5) = 0.5270$, $\bar{o}_{30}(5) = 0.5473$, $\bar{o}_{35}(5) = 0.5641$	913
$t = 6$	$\bar{o}_5(6) = 0.4922$, $\bar{o}_9(6) = 0.5175$, $\bar{o}_{36}(6) = 0.5043$, $\bar{o}_{39}(6) = 0.5275$, $\bar{o}_{30}(6) = 0.5447$, $\bar{o}_{41}(6) = 0.5775$	1070
$t = 7$	$\bar{o}_4(7) = 0.5072$, $\bar{o}_7^{(7)} = 0.5055$, $\bar{o}_{32}(7) = 0.5235$, $\bar{o}_{42}(7) = 0.5235$, $\bar{o}_{43}(7) = 0.5229$, $\bar{o}_{45}(7) = 0.5392$	1176

Then, based on the adjusted opinion $\bar{o}_i(t)$, we obtain the evolution of adjusted opinions, which are shown in Fig. 2.

From Fig. 2, we obtain the following observations: The opinions of all the agents become stabilize at time $t = 12$. And a consensus among the agents is formed in the stabilized result.

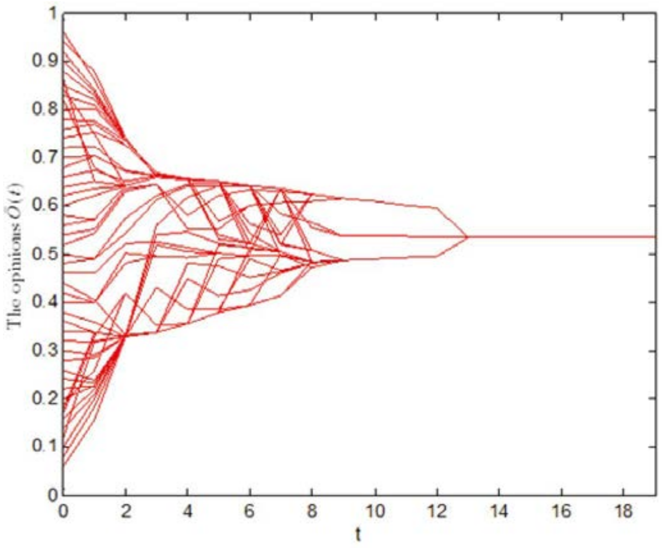

Fig. 2. The evolution of the adjusted opinions in the HK model

4.2. Discussion

In this subsection, we propose Experiments I and II based on the example presented in Subsection 4.1. Specifically, in Experiment I, we set different adjustment thresholds (i.e., β). Let $\varepsilon = 0.1$, then based on the example 4.1, we use the proposed opinion control rule to the obtain the average amounts of adjusted opinions at each iteration (i.e., $AOE = \sum_{t=0}^{l}\sum_{i=1}^{N} x_i^t \Big/ l$). The experimental results are shown in Fig. 3.

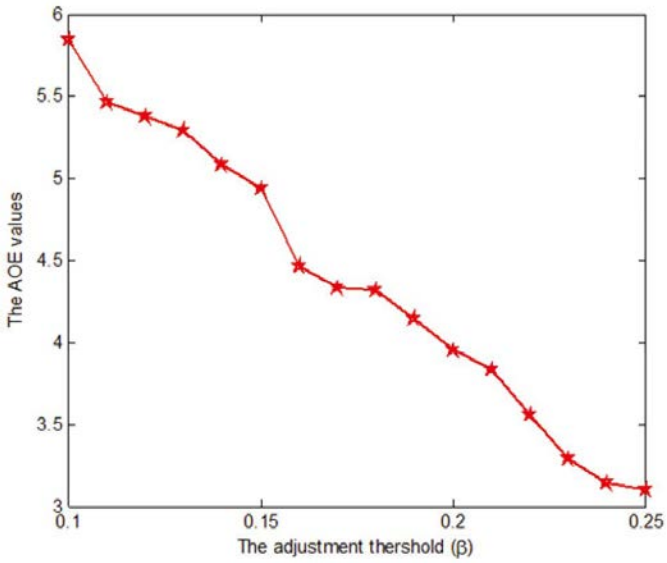

Fig. 3. The AOE values under different β values

In Experiment II, we set different bounded confidences (i.e., ε). Let $\beta = 0.15$. Then, based on the Example 4.1, we use the proposed opinion control rule to the obtain the total amounts of adjusted opinions (i.e., $AO = \sum_{t=0}^{l}\sum_{i=1}^{N} x_i^t$). The experimental results are shown in Fig. 4.

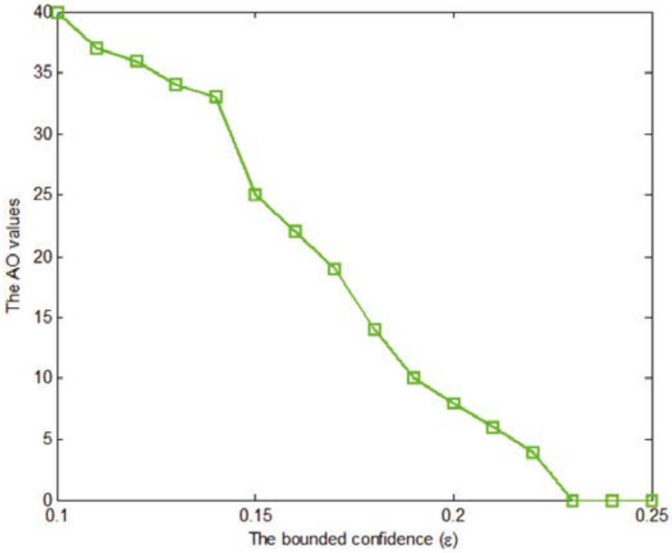

Fig. 4. The AO values under different ε values

From Figs. 3-4, we obtain the following observations: (i) The AOE values decrease as β increases. This implies that less amounts of adjusted opinions at each iteration will be yielded with the increase in the adjustment thresholds. (ii) The AO values decrease as ε increases. This implies that less amounts of adjusted opinions will be yielded with the increase in the bounded confidences.

5. Conclusion

In the present study, we propose the opinion control rule to support the consensus reaching. The primary contribution of this study are as follows:

(1) We propose the consensus model with the minimum adjustment. Then, based on the proposed consensus model, we propose the opinion control rule to support the consensus reaching.

(2) We provide a numerical example to illustrate the feasibility of the proposed opinion control rule. Through simulation experiments, the effects of adjustment thresholds and bounded confidences on the opinion control rule are investigated. The experimental results shows that: (i) less amounts of adjusted opinions at each iteration will be yielded with the increase in the adjustment thresholds, and (ii) less amounts of adjusted opinions will be yielded with the increase in the bounded confidences.

In practical opinion dynamics, due to the limitation of knowledge and experiences, the opinions of the agents often exhibit the uncertainty. However, in this paper, the proposed opinion control rule is based on the exact opinions. Therefore, it would be an interesting future topic to propose the opinion control rule based on the opinions with the uncertainty.

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