Abstract

In this paper, an analytic model is present for evaluating the MAC layer Collision Probability and mean delay at wireless nodes using the Distributed Coordination Function of IEEE 802.11 MAC Model. This model is valid for finite loads and can account for arbitrary arrival patterns, packet size distributions and number of nodes. Each node is modeled as a discrete time M/G/1 queue and impact of packet collisions, the resulting back-offs as well as the packet size distribution. We analyze this Model for different number of nodes in any network. The impact of arrival pattern on collision probability and delay is also analyzed.

Keywords: Queuing Probability; DCF; M/G/1; Arrival rate; Back-off; Collision Probability; Mean Delay;

1. Introduction

The IEEE 802.11 MAC [5] has gained widespread popularity as a layer-2 protocol for wireless local area networks. While efforts have been made to support the transmission of real time traffic in such networks they primarily use centralized scheduling and polling techniques based on the point coordination function (PCF). For ad
hoc scenarios, a more reasonable model of operation is that of random access and the distributed coordination function (DCF).

In this paper we develop an analytic model for discrete time M/G/1 queue for modelling nodes in a random access network based on the 802.11 MAC. Network is assumed with N nodes using the DCF of IEEE 802.11 to schedule their transmissions. Arbitrary packet arrival process and packet length distribution are assumed. The use of RTS and CTS messages for channel reservation is assumed. The analysis can be easily extended for the cases where such messages are absent.

We propose and use a detailed model which accounts for the effect of finite load on the collision rates and the mean delay in network. This improved characterization allows a more accurate model for the service time distribution to be developed. This model provides closed form expressions for the queue length in the presence of arbitrary arrival patterns, packet size distributions and network load. The model accounts for the collision avoidance and exponential back off mechanism of 802.11, the delays in the channel access due to other nodes transmitting and the delays caused by collisions. The results obtained from this model are used to show the behavior of collision probability that how it varies with change of arrival rates. This model also analyzes the behavior of mean delay in network.

The rest of the paper is organized as follows. In Section 2 presents a brief overview of the IEEE 802.11 MAC protocol. Section 3 presents the detailed queuing model and back off mechanism. Section 4 presents the analysis of results. Finally, Section 5 presents a discussion of the results and concluding remarks.

### Nomenclature

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>CFP</td>
<td>Contention Free Period</td>
</tr>
<tr>
<td>CSMA/CA</td>
<td>Carrier Sense Multiple Access with Collision Avoidance</td>
</tr>
<tr>
<td>CSMA/CD</td>
<td>Carrier Sense Multiple Access with Collision Detection</td>
</tr>
<tr>
<td>RTS</td>
<td>Ready to send</td>
</tr>
<tr>
<td>CTS</td>
<td>Clear To Send</td>
</tr>
<tr>
<td>CW</td>
<td>Contention Window</td>
</tr>
<tr>
<td>DCF</td>
<td>Distributed Coordination Function</td>
</tr>
<tr>
<td>DIFS</td>
<td>Distributed Coordination Function IFS</td>
</tr>
</tbody>
</table>

### 2. Overview of IEEE802.11MAC

The IEEE 802.11 MAC layer is responsible for a structured channel access scheme and is implemented using a Distributed Coordination Function based on the Carrier Sense Medium Access with Collision Avoidance (CSMA/CA) protocol. The CSMA/CA based MAC protocol of IEEE 802.11 is designed to reduce the collisions due to multiple sources transmitting simultaneously on a shared channel.

In a network employing, the CSMA/CA MAC protocol, each node with a packet to transmit first senses the channel to ascertain whether it is in use. If the channel is sensed to be idle for an interval greater than the Distributed Inter-Frame Space (DIFS), the node proceeds with its transmission. If the channel is sensed as busy, the node defers transmission till the end of the ongoing transmission. The node then initializes its back off timer with a randomly selected back off interval and decrements this timer every time it senses the channel to be idle. The timer has the granularity of a back off slot and is stopped in case the channel becomes busy and the decrementing process is restarted when the channel becomes idle for a DIFS again. The node is allowed to transmit when the back off timer reaches zero. Since the back off interval is chosen randomly, the probability that two or more stations will choose the same back off value is very low.

Along with the Collision Avoidance, 802.11 use a positive acknowledgment (ACK) scheme. All the packets received by a node implementing 802.11 MAC must be acknowledged by the receiving MAC. After receiving a packet the receiver waits for a brief period, called the Short Inter-Frame Space (SIFS), before it transmits the ACK. The basic operation of the CSMA/CA based MAC protocol of IEEE 802.11 is shown in figure 1(a) and figure 1(b) it
shows the exchange of various packets involved in each successful transmission and the spacing between these packets.

Fig.1 (a) transmission mechanism;                                           (b) inter frame space relationship

Whenever a data frame is to be sent, the station senses the medium. If it is free for at least a DCF inter-frame space (DIFS) period of time, the frame is transmitted. Otherwise, if the medium is busy, a back-off time $B$ (measured in time slots which depends upon the characteristic of physical layer) is chosen randomly in the interval $[0, CW]$, where $CW$ is the contention window. After the medium has been detected idle for at least a DIFS, the back off timer is decremented by one for each time slot the medium remains idle. If the medium becomes busy during the back off process, the back off timer is paused, and is restarted when the medium has been sensed idle for a DIFS again. When the back off timer reaches zero, the frame is transmitted. Upon detection of a collision (which is detected by the absence of an acknowledgment frame to the data frame), the contention window is redefined according to given equation $CW[i] = 2^{(k+i)-1}$ where $i$ is the number of attempts (including the current one) to transmit the frame that has been done, and $k$ is a constant defining the minimum contention window, $CW_{min} = 2^k - 1$. A new back off time is then chosen and the back off procedure starts over. The back off mechanism is also used after a successful transmission before sending the next frame. After a successful transmission, the contention window is reset to $CW_{min}$.

3. Queuing model for IEEE802.11 MAC

In this section we introduce a discrete time M/G/1 queue for modelling nodes in a random access network based on the 802.11 MAC. We assume a network with $N$ nodes using the DCF of IEEE 802.11 to schedule their transmissions. Arbitrary packet arrival process and packet length distribution are assumed. We assume the use of RTS and CTS messages for channel reservation.

3.1 Modeling of back off mechanism

In order to model the MAC layer queuing delays and losses, we first analyse the mechanism associated with the exponential back-off of 802.11 MAC protocol’s Collision Avoidance algorithm [9]. In the following analysis, we denote the probability that an arbitrary packet transmission (i.e. an RTS transmission) results in a collision by $p$. The lower and upper bounds on the contention window associated with back-offs are denoted by $CW_{min}$ and $CW_{max}$ and we use the notation

$$m = \log_2 \left( \frac{CW_{max}}{CW_{min}} \right) \quad (1)$$

Once a node goes into collision avoidance or the exponential back-off phase, we denote the number of slots that it waits beyond a DIFS period before initiating transmission by $BC$. This back-off counter is calculated from [9]

$$BC = \text{int} \left( r \text{nd} \left( \right) \times CW \left( k \right) \right)$$

Where the function $r \text{nd} ()$ returns a pseudo-random number uniformly distributed in $[0; 1]$ and $CW \left( k \right)$ represents...
the contention window after k unsuccessful transmission attempts. Note that in case the \( \text{int}() \) operation is done using a \( \text{ceil}() \) function, the effective range for BC becomes \( 1 \cdot BC\cdot\text{CW}(k) \) since the probability of \( \text{rand}(k) = 0 \) is 0 assuming a continuous distribution. For the rest of this paper we assume that a \( \text{ceil}() \) function is used to do the \( \text{int}() \) operation. The first attempt at transmitting a given packet is performed assuming a CW value equal to the minimum possible value of \( \text{CW}_{\min} \) [5]. For each unsuccessful attempt, the value of CW is doubled until it reaches the upper limit of \( \text{CW}_{\max} \) specified by the protocol. Then, at the end of k unsuccessful attempts, CW (k) is given by

\[
\text{CW}(k) = \min(\text{CW}_{\max}, 2^{k-1}\text{CW}_{\min})
\]

(2)

In the saturated case where each packet is backlogged immediately, each packet starts out with a window of \( \text{CW}_{\min} \). With probability \( 1 - P \) the transmission is successful and the average back-off window of such a packet is \( \text{CW}_{\min}/2 \). With probability \( P(1 - P) \) the first transmission fails and the packet is successfully transmitted in the second attempt (using a back-off window of \( 2\text{CW}_{\min} \)) which adds \( \text{CW}_{\min} \) to the average back-off window seen by the packet. Continuing along these lines for cases with larger number of losses, the average back-off window in the saturated case is given by

\[
\bar{W} = (1 - P) \frac{\text{CW}_{\min}}{2} + P(1 - P) \frac{2\text{CW}_{\min}}{2} + \ldots + P^n(1 - P) \frac{2^n\text{CW}_{\min}}{2} + P^{n+1} \frac{2^n\text{CW}_{\min}}{2}
\]

\[
\Rightarrow \bar{W} = \frac{1 - P - P(2P)^n}{1 - 2P} \frac{\text{CW}_{\min}}{2}
\]

(3)

Now consider a network with N nodes operating in discrete time where the packet arrival rate at each node is \( \lambda \) packets per slot, the channel service rate is \( \mu \) packets per slot and the queue utilization at a node is denoted by \( \rho \). Consider a tagged node which transmits in a given slot. Now, a collision occurs if one or more of the remaining N-1 nodes also transmit in this slot. Then, letting \( P[\text{NT}] \) [9] denote the probability that a node does not transmit in a slot, we have

\[
P = 1 - P[\text{NT}]^N
\]

(4)

Now, using QE to denote queue empty and QNE to denote queue not empty for ease of notation, \( P[\text{NT}] \) is given by

\[
P[\text{NT}] = P[\text{NT}|\text{QE}]P[\text{QE}] + P[\text{NT}|\text{QNE}]P[\text{QNE}]
\]

\[
= 1 - (1 - \rho) + \rho P[\text{NT}|\text{QNE}]
\]

A queue is non-empty in a slot either if is backlogged or if a new arrival occurs in that slot while the queue was empty. Now, considering the fact that we are interested in stable queues and back-off slots are two orders of magnitude smaller than typical data packet lengths, the probability of the latter case is quite small. Also, a backlogged queue will not transmit in a slot with probability \((W - 1)/W\).

Then, \( P[\text{NT}|\text{QNE}] \) can be approximated by \((W - 1)/W\). So,

\[
P[\text{NT}] = (1 - \rho) + \rho \frac{W - 1}{W} = 1 - \frac{\rho}{W}
\]

(5)

Now combining Equations (3), (4) and (5) the collision probability \( P \) can be given by:

\[
P = 1 - (1 - \rho) \left( 1 - \frac{2P}{1 - P - P(2P)^n} \frac{2}{\text{CW}_{\min}} \right)^{N-1}
\]
Here $\rho = \lambda / \mu$ and by taking constant value for $\mu$, and for different values of $\lambda$ we can calculate the collision probability.

### 3.2 Queuing model

To obtain the delays experienced by the packets, each node is modelled as a discrete time $M/G/1$. The following general formula we can use to derive the delay of system [7]

$$P(z) = \frac{P_0 (z-1) A(z)}{z - A(z)}$$

........................ (1)

Here $P(z)$ denoted the PGF of the state probability distribution at the imbedded instants. Let $A(z)$ denote the PGF at regime of the number of arrivals during the service time of a request. The value of $P_0$ can be calculated with equation $P_0 = 1 - E[a]$ The mean number of arrivals during the service time of a request, $E[a]$ can be derived by considering the mean number of Poisson arrivals [7] in a given interval $T = t$, $E[a|T=t] = \lambda t$, and then by removing the conditioning with the probability destiny function $g(t)$ of $T$:

$$E[a] = \int_0^T E[a|T=t]g(t)dt = \lambda \int_0^T tg(t)dt = \lambda E[X]$$

........................ (2)

The PGF in equation (1) has a singularity for $z=1$ that causes some problems both for the normalization test and for the derivation of the moments of the distribution [7]. We can use the theorem to prove that $p(z=1)=1$ (normalization). The moments of the state probability distribution can be easily obtained by taking subsequent derivatives at both sides of the right most expression in equation (1).

First derivation step: -

$$P'(z)[z-A(z)] + P(z)[1-A'(z)] = P_0 A(z) + P_0 (z-1) A'(z)$$

........................ (3)

If we evaluate equation (3) for $z=1$ we obtain $P_0 = 1 - A'(1)$

Now we derive again equation (3) again with respect to $z$ and we get:-

$$P''(z)[z - A(z)] + 2P'(z)[1 - A'(z)] + P(z)[-A''(z)] = 2P_0 A'(z) + P_0 (z-1) A''(z)$$

........................ (4)

If we evaluate equation (4) for $z=1$ and we use equation (2) and we have:

$$2P'(1)[1 - A'(1)] - A''(z) = 2P_0 A'(1)$$

........................ (6)
The mean number of requests at the imbedding instants, \( N \) depends on the first two derivatives of \( A(z) \) computed for \( z=1 \). The stability condition is assured for \( 1-A'(1)>0 \) i.e. traffic intensity in Erlang [4] lower than 1. To solve this equation we need the value of \( A'' \) and \( A'(z) \), and value of these two we can find out like \( A(z) \) can be computed considering the PGF of the number of arrivals in a given interval \( T=t \),

\[
A(z|t) = e^{A(t)(z-1)}
\]

And then removing the conditioning by means of probability density function of the service time, \( g(t) \):

\[
A(z) = \int_0^\infty e^{A(x)(z-1)} g(x) dx = \Gamma(s = -A(z-1))
\]

Where \( \Gamma(s) \) denotes the Laplace transform of the probability density function \( g(t) \). On the basis of the expression of \( A(z) \) we can evaluate \( A'(1) \) and \( A''(1) \) as follows:

\[
\frac{dA(z)}{dz} \bigg|_{z=1} = -\lambda \Gamma'(-\lambda(z-1)) \bigg|_{z=1} = \lambda \Gamma'(0) = \lambda E[X]
\]

\[
\frac{d^2 A(z)}{dz^2} \bigg|_{z=1} = \frac{d}{dz} \left[-\lambda \Gamma'(-\lambda(z-1)) \right] \bigg|_{z=1} = \lambda^2 \Gamma''(-\lambda(z-1)) \bigg|_{z=1} = \lambda^2 E[X^2]
\]

Now using the values of equation (7) and equation (8) and substituting these values in equation (5) we get this result:

\[
N = \lambda E[X] + \frac{\lambda^2 E[X^2]}{2[1-\lambda E[X]]}
\]

We can derive mean delay \( T \), for a request to cross the M/G/1 queuing system [7], by applying Little theorem to equation (9)

\[
T = \frac{N}{\lambda} = E[X] + \frac{\lambda E[X^2]}{2[1-\lambda E[X]]}
\]
This is the final equation which we can use to calculate the mean delay of system.

4. Parameter settings & analytical results

Analytical Result for Mean Delay:-

QualNet is a comprehensive suite of tools for modeling large wired and wireless networks. Here we have used this simulator for simulation and emulation to predict the behavior and performance of networks to improve their design, operation and management. By using Qualnet simulator we can calculate the End to End delay for each node. We can get Mean Delay by using end to end delay.

Mean Delay = Total delay at each node/No. of nodes.

Parameter setting at physical and MAC layer:-

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simulation Time</td>
<td>1800 sec</td>
</tr>
<tr>
<td>Routing Protocol</td>
<td>Bellmen ford</td>
</tr>
<tr>
<td>Antenna height</td>
<td>1.5m</td>
</tr>
<tr>
<td>Temperature</td>
<td>290k</td>
</tr>
<tr>
<td>Bandwidth</td>
<td>2mbps</td>
</tr>
</tbody>
</table>

By simulation we got some values of end to end delay for different no of nodes and by using those values we can calculate mean delay which is shown in table 2

<table>
<thead>
<tr>
<th>Number of nodes</th>
<th>Mean Delay = Total of Delay at each node/ number of nodes</th>
</tr>
</thead>
<tbody>
<tr>
<td>N=10</td>
<td>(0.2+0.088+0.225+0.0241+0.173+0.355+0.405+0.317+0.475+0.315)/10 = 0.24</td>
</tr>
<tr>
<td>N=8</td>
<td>(0.038+0.17+0.0446+0.186+0.096+0.0134+0.115+0.147)/8 = 0.18</td>
</tr>
<tr>
<td>N=7</td>
<td>(0.049+0.0456+0.0656+0.975+0.057+0.19+0.0695)/7 = 0.17</td>
</tr>
<tr>
<td>N=6</td>
<td>(0.09+0.36+0.28+0.24+0.24+0.12)/6 = 0.16</td>
</tr>
<tr>
<td>N=5</td>
<td>(0.085+0.072+0.097+0.074+0.098)/5 = 0.08</td>
</tr>
<tr>
<td>N=4</td>
<td>(0.032+0.053+0.065+0.069)/4 = 0.05</td>
</tr>
<tr>
<td>N=3</td>
<td>(0.028+0.0184+0.0084)/3 = 0.018</td>
</tr>
</tbody>
</table>

We can represent the behavior of mean delay with respect to no of nodes with this graph. The following graph shows that when no of nodes increases in any network the mean delay increases.
Analytical Result for Collision Probability:-

Collision probability varies with arrival rate, when different values of arrival rate is given we can find out different values of collision probability. Collision probability can be calculated with help of given formula. It can be calculated by putting the value of $\rho$ in given equation which is equal to $\lambda / \mu$.

$$ P = 1 - (1 - \rho) \left( \frac{1 - 2P}{1 - P - P(2P)^m} \right)^{m-1} \left( \frac{2}{CW_{min}} \right) $$

So we take some constant value, table 3 shows parameter and respected values.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>CWmax</td>
<td>960</td>
</tr>
<tr>
<td>CWmin</td>
<td>15</td>
</tr>
<tr>
<td>Service Rate($\mu$)</td>
<td>0.2</td>
</tr>
<tr>
<td>Arrival Rate($\lambda$)</td>
<td>Vary from 0.05 to 10</td>
</tr>
<tr>
<td>Average Inter-arrival Rate</td>
<td>$(1/P)$</td>
</tr>
<tr>
<td>m = log2 (CWmax / CWmin)</td>
<td>6</td>
</tr>
<tr>
<td>Number of nodes N</td>
<td>3,4,5,6,7,8,10</td>
</tr>
</tbody>
</table>

For different values of $\lambda$ different different values of P can be calculated with the help of maple software. Like in the following figure we have taken number of nodes N=10 m=6, and $\mu=0.2$.so for $\lambda=10$ we get the value for $P=0.87$ and when $\lambda=5$ then $P=0.79$

For 10 no of nodes we have calculated these values of collision probability P by changing different values of arrival rate from 0.05 to 10. Following table shows some different values of collision probability calculated. We can also calculate the average inter-arrival rate by dividing collision probability with 1.

Change in collision probability with respect to arrival rate and average Inter-arrival rate can be analysed with help of the given graph. Here we have taken different values of arrival rate ranging from 0.05 to 10 from different no of nodes ranging 2 to 10. By putting different values of collision probability for different nodes we plot this graph. It is clearly shown that as we increase the arrival rate the collision probability increases

Similarly when we plot a graph for collision probability and average inter-arrival rate we analyses that collision probability decreases with increase of average inter-arrival rate.
Fig. 3: maple software is used for calculating collision probability

<table>
<thead>
<tr>
<th>Arrival Rate(λ)</th>
<th>Collision Probability(P)</th>
<th>Average Inter-arrival Rate(1/P)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.05</td>
<td>0.2</td>
<td>4.9</td>
</tr>
<tr>
<td>0.07</td>
<td>0.25</td>
<td>4.0</td>
</tr>
<tr>
<td>7</td>
<td>0.83</td>
<td>1.20</td>
</tr>
<tr>
<td>10</td>
<td>0.87</td>
<td>1.13</td>
</tr>
</tbody>
</table>

5. Conclusion

In this paper we present a queuing model for each node in the network accounts for the intricacies of the MAC protocol and its behavior as a function of the number of users in the network. Each node is modeled as a discrete time M/G/1 queue and we allow for arbitrary number of nodes, arrival patterns and packet size distributions. Our model help to analyze that with increase the value of arrival rate (λ) the collision probability (P) increases and if we increase the average Inter-arrival rate (1/P) the collision probability decreases. This paper also conclude that in case of arrival rate, collision probability increases with increasing the arrival rate with increasing number of nodes, and in case of average inter-arrival rate collision probability decreases with increase of average Inter-arrival rate with increase number of nodes. For Mean Delay, with increase of the arrival rate, the Mean Delay (T) increases and if we increase the average Inter-arrival Rate (1/P) the Mean Delay decreases. And for Qualnet simulation Results, with increasing the number of nodes in network the mean delay increases.
Acknowledgements

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References