



# Model order determination using the Hankel matrix of impulse responses<sup>☆</sup>

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## ABSTRACT

This letter studies identification problems of model orders using the Hankel matrix of impulse responses of a system and presents two order identification methods: one is based on the singularities or ratios of the Hankel matrix determinants and the other is based on the singular value decomposition of the Hankel matrix. A numerical example verifies the proposed methods.

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## 1. Introduction

Identification for linear systems contains the structure or order determination and parameter estimation. There exist a lot of parameter estimation methods for linear systems, e.g., the multi-innovation parameter identification methods [1–7], the iterative estimation methods [8,9], the data filtering based estimation methods [10], and the gradient based estimation methods [11–14], but most assume that the system orders or structure indices are known.

In the area of order identification, Duong and Landau presented an instrumental variable based criterion for model order selection [15]. Bauer studied the order estimation methods using the subspace technique [16]. Schoukens, Rolain and Pintelon discussed the modified AIC rule for model selection in combination with prior estimated noise models [17]. Ruan, Yang, Chen and Li considered the on-line order estimation and parameter identification problems for linear stochastic feedback control systems [18]. Lind and Ljung explored the regressor selection with the analysis of variance method [19]. Aladag, Egrioglu and Kadilar forecasted a nonlinear time series with a hybrid methodology [20]. Thavaneswaran, Appadoo and Ghahramani studied RCA models with GARCH innovations [21]. Gong and Thavaneswaran studied parameter estimation problems for continuous time stochastic volatility models [22]. Recently, Ding et al. presented a least squares parameter estimation algorithm with irregularly missing data [23] and a hierarchical estimation algorithm for non-uniformly sampled systems [24].

This letter studies the order estimation problems for linear systems using the Hankel matrix and the SVD decomposition.

## 2. The method of determining the Hankel Matrix's rank

Consider a linear discrete-time system described by the following state space model,

$$\begin{cases} \mathbf{x}(t+1) = \mathbf{A}\mathbf{x}(t) + \mathbf{b}u(t), \\ \mathbf{y}(t) = \mathbf{c}\mathbf{x}(t) + du(t), \end{cases} \quad (1)$$

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where  $\mathbf{x}(t) \in \mathbb{R}^n$  is the state vector,  $u(t) \in \mathbb{R}$  and  $y(t) \in \mathbb{R}$  are the input and output of the system,  $\mathbf{A} \in \mathbb{R}^{n \times n}$ ,  $\mathbf{b} \in \mathbb{R}^n$ ,  $\mathbf{c} \in \mathbb{R}^{1 \times n}$  and  $d \in \mathbb{R}$  are constant matrix, vectors or number.

Order identification is determining the minimum dimensions of the controllability matrix and the observability matrix using the input–output data  $\{u(t), y(t)\}$  of the system. Let  $z^{-1}$  be a unit backward shift operator [ $z^{-1}\mathbf{x}(t) = \mathbf{x}(t-1)$ ]. From (1), we have

$$\begin{cases} \mathbf{x}(t) = (z\mathbf{I} - \mathbf{A})^{-1}\mathbf{b}u(t), \\ y(t) = \mathbf{c}\mathbf{x}(t) + du(t) \\ \quad = [\mathbf{c}(z\mathbf{I} - \mathbf{A})^{-1}\mathbf{b} + d]u(t), \end{cases}$$

where  $\mathbf{I}$  is an identity matrix of appropriate size. Thus, the transfer relation from the system input to output is given by

$$G(z) := \frac{y(t)}{u(t)} = \mathbf{c}(z\mathbf{I} - \mathbf{A})^{-1}\mathbf{b} + d. \quad (2)$$

Using long division, we have

$$G(z) = z^{-1}\mathbf{c} \left( \mathbf{I} - \frac{\mathbf{A}}{z} \right)^{-1} \mathbf{b} + d = d + \mathbf{c}\mathbf{b}z^{-1} + \mathbf{c}\mathbf{A}\mathbf{b}z^{-2} + \mathbf{c}\mathbf{A}^2\mathbf{b}z^{-3} + \dots.$$

From the above equation, we can obtain the impulse responses:

$$g(t) = \begin{cases} d, & t = 0, \\ \mathbf{c}\mathbf{A}^{t-1}\mathbf{b}, & t > 0. \end{cases} \quad (3)$$

The controllability matrix  $\mathbf{Q}_c$  and the observability matrix  $\mathbf{Q}_o$  of system (1) are defined by

$$\mathbf{Q}_c := [\mathbf{b}, \mathbf{A}\mathbf{b}, \mathbf{A}^2\mathbf{b}, \dots, \mathbf{A}^{n_0-1}\mathbf{b}, \dots, \mathbf{A}^{l-1}\mathbf{b}], \quad (4)$$

$$\mathbf{Q}_o := \begin{bmatrix} \mathbf{c} \\ \mathbf{c}\mathbf{A} \\ \mathbf{c}\mathbf{A}^2 \\ \vdots \\ \mathbf{c}\mathbf{A}^{n_0-1} \\ \vdots \\ \mathbf{c}\mathbf{A}^{l-1} \end{bmatrix}, \quad (5)$$

where  $n_0$  represents the true order of the system and is unknown, the number  $l$  is large enough and satisfies  $l \geq n_0$ .

For control, the system order is generally regarded as the dimension  $n$  of the state vector  $\mathbf{x}(t)$ . For identification, the system order is defined as the dimension of both controllable and observable subsystem and thus the system order is the minimum rank of the controllability matrix  $\mathbf{Q}_c$  and the observability matrix  $\mathbf{Q}_o$ , i.e.,  $n_0 := \min\{\text{rank}[\mathbf{Q}_c], \text{rank}[\mathbf{Q}_o]\}$ .

Suppose that  $\mathbf{A}$  is full rank (otherwise, take  $t = 1$ ). Assume that the controllability matrix  $\mathbf{Q}_c$  has rank  $n_1$  ( $n_0 \leq n_1$ ). That is, the successive  $n_1$  columns of  $\mathbf{Q}_c$  are linearly independent. From (4), we form the matrix

$$\mathbf{J}_c(t) := [\mathbf{A}^{t-1}\mathbf{b}, \mathbf{A}^t\mathbf{b}, \mathbf{A}^{t+1}\mathbf{b}, \dots, \mathbf{A}^{t+l-2}\mathbf{b}] \in \mathbb{R}^{n \times l}, \quad l \geq n_0, \quad (6)$$

which has rank  $n_1$ . Thus, there exists a series of numbers  $\alpha_{ij}$  (not all zeros) such that the following relation holds,

$$\mathbf{A}^{t+n_1-1+i}\mathbf{b} = \alpha_{i1}\mathbf{A}^{t-1+i}\mathbf{b} + \alpha_{i2}\mathbf{A}^{t+i}\mathbf{b} + \dots + \alpha_{in_1}\mathbf{A}^{t+n_1-2+i}\mathbf{b}, \quad i = 0, 1, \dots, (l - n_1 - 1).$$

Pre-multiplying both sides of the above equation by  $\mathbf{c}$  gives

$$\mathbf{c}\mathbf{A}^{t+n_1-1+i}\mathbf{b} = \alpha_{i1}\mathbf{c}\mathbf{A}^{t-1+i}\mathbf{b} + \alpha_{i2}\mathbf{c}\mathbf{A}^{t+i}\mathbf{b} + \dots + \alpha_{in_1}\mathbf{c}\mathbf{A}^{t+n_1-2+i}\mathbf{b}, \quad i = 0, 1, \dots, (l - n_1 - 1).$$

Using (3) gives

$$g(t + n_1 + i) = \alpha_{i1}g(t + i) + \alpha_{i2}g(t + i + 1) + \dots + \alpha_{in_1}g(t + n_1 - 1 + i), \quad i = 0, 1, \dots, (l - n_1 - 1). \quad (7)$$

Pre-multiplying both sides of (6) by  $\mathbf{c}$ , we have

$$\begin{aligned} \mathbf{H}(t) &:= \mathbf{c}\mathbf{J}_c(t) = [\mathbf{c}\mathbf{A}^{t-1}\mathbf{b}, \mathbf{c}\mathbf{A}^t\mathbf{b}, \mathbf{c}\mathbf{A}^{t+1}\mathbf{b}, \dots, \mathbf{c}\mathbf{A}^{t+l-2}\mathbf{b}] \\ &= [g(t), g(t+1), g(t+2), \dots, g(t+l-1)] \in \mathbb{R}^{1 \times l}. \end{aligned}$$

Replacing  $t$  with  $t + i$  gives

$$\mathbf{H}(t + i) = [g(t + i), g(t + i + 1), g(t + i + 2), \dots, g(t + 2i)] \in \mathbb{R}^{1 \times l}.$$

Taking  $i = 0, 1, \dots, l - 1$ , we obtain  $l$  equations which can be rewritten as the form of a Hankel matrix

$$\mathbf{H}(l, t) := \begin{bmatrix} \mathbf{H}(t) \\ \mathbf{H}(t+1) \\ \vdots \\ \mathbf{H}(t+l-1) \end{bmatrix} = \begin{bmatrix} g(t) & g(t+1) & \cdots & g(t+l-1) \\ g(t+1) & g(t+2) & \cdots & g(t+l) \\ \vdots & \vdots & \ddots & \vdots \\ g(t+l-1) & g(t+l) & \cdots & g(t+2l-2) \end{bmatrix} \in \mathbb{R}^{l \times l}. \tag{8}$$

According to (7), we can see that the matrix  $\mathbf{H}(l, t)$  has rank  $n_1$ , so we can determine the dimension of the controllable subsystem from the rank of the Hankel matrix  $\mathbf{H}(l, t)$ . Since the impulse response  $g(t)$  of a stable system approaches zero as  $t$  increases,  $t$  in  $\mathbf{H}(l, t)$  should not be too large.

Similarly, assume that the observability matrix  $\mathbf{Q}_o$  has rank  $n_2$  ( $n_0 \leq n_2$ ). That is, the successive  $n_2$  rows are linearly independent. From (5), we form the matrix,

$$\mathbf{J}_0(t) := \begin{bmatrix} \mathbf{cA}^{t-1} \\ \mathbf{cA}^t \\ \mathbf{cA}^{t+1} \\ \vdots \\ \mathbf{cA}^{t+l-2} \end{bmatrix} \in \mathbb{R}^{l \times n}, \quad l \geq n_0, \tag{9}$$

which has rank  $n_2$ . Thus, there exists a series of numbers  $\beta_{ij}$  (not all zeros) such that the following relation holds,

$$\mathbf{cA}^{t+n_2-1+i} = \beta_{i1}\mathbf{cA}^{t-1+i} + \beta_{i2}\mathbf{cA}^{t+i} + \cdots + \beta_{in_2}\mathbf{cA}^{t+n_2-2+i}, \quad i = 0, 1, \dots, (l - n_2 - 1).$$

Post-multiplying both sides by  $\mathbf{b}$  gives

$$\mathbf{cA}^{t+n_2-1+i}\mathbf{b} = \beta_{i1}\mathbf{cA}^{t-1+i}\mathbf{b} + \beta_{i2}\mathbf{cA}^{t+i}\mathbf{b} + \cdots + \beta_{in_2}\mathbf{cA}^{t+n_2-2+i}\mathbf{b}, \quad i = 0, 1, \dots, (l - n_2 - 1).$$

Using (3), it follows that

$$g(t + n_2 + i) = \beta_{i1}g(t + i) + \beta_{i2}g(t + i + 1) + \cdots + \beta_{in_2}g(t + n_2 - 1 + i), \quad i = 0, 1, \dots, (l - n_2 - 1). \tag{10}$$

Post-multiplying both sides of (9) by  $\mathbf{b}$ , we can write the Hankel matrix of (8) as follows:

$$\mathbf{H}(l, t) = [\mathbf{J}_0(t)\mathbf{b}, \mathbf{J}_0(t+1)\mathbf{b}, \dots, \mathbf{J}_0(t+l-1)\mathbf{b}] \in \mathbb{R}^{l \times l}.$$

According to (10), it is clear that  $\mathbf{H}(l, t)$  has rank  $n_2$ . Therefore, we can determine the dimension of the observable subsystem based on the rank of the Hankel matrix  $\mathbf{H}(l, t)$ .

Since the system order is equal to the minimum rank of the controllability matrix and the observability matrix, i.e.,  $n_0 := \min\{\text{rank}[\mathbf{Q}_c], \text{rank}[\mathbf{Q}_o]\}$ , we have

$$\text{rank}[\mathbf{H}(l, t)] = n_0, \quad l \geq n_0, \quad t \geq 1. \tag{11}$$

This indicates that the rank of the Hankel matrix equals the true order of the system when  $l \geq n_0$ . In other words, when  $l \leq n_0$ , the Hankel matrix has full rank and its determinant is not equal to zero. Thus, we can determine the system order according to the singularity of the Hankel matrix.

In practice, the impulse response data contain measurement errors (noises) and the determinant of the Hankel matrix cannot equal zero for  $l > n_0$ . In this case, the system order can be determined by observing the changing rates of the determinants of the Hankel matrices, i.e., the number  $l$  is the system order when  $\det[\mathbf{H}(l, t)] / \det[\mathbf{H}(l + 1, t)]$  is maximum for every  $l = 1, 2, \dots$

### 3. The SVD method

The singular value decomposition (SVD) theorem: Suppose that  $\mathbf{R}$  is an  $m \times n$  matrix with rank  $r$ , there exist an  $m \times m$  orthogonal matrix  $\mathbf{U}$  and an  $n \times n$  orthogonal matrix  $\mathbf{V}$  such that the following equality holds,

$$\mathbf{R} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^T, \tag{12}$$

where

$$\mathbf{\Sigma} = \begin{bmatrix} \sigma_1 & & & & & \\ & \sigma_2 & & & & \\ & & \ddots & & & \\ & & & \sigma_r & & \\ & & & & & \mathbf{0} \end{bmatrix} \in \mathbb{R}^{m \times n}, \tag{13}$$

**Table 1**The impulse response sequence ( $k = 5$ ).

$t$	$g(t)$	$t$	$g(t)$	$t$	$g(t)$	$t$	$g(t)$	$t$	$g(t)$	$t$	$g(t)$
1	1.00000	6	0.05128	11	-0.46126	16	0.31241	21	0.01087	26	-0.04296
2	2.96000	7	-0.41179	12	-0.31950	17	0.09593	22	0.06709	27	0.02292
3	3.24240	8	-0.49809	13	-0.04230	18	-0.10418	23	0.03638	28	0.06940
4	2.32986	9	-0.47469	14	0.24749	19	-0.17075	24	-0.03369	29	0.06068
5	1.04294	10	-0.47802	15	0.38849	20	-0.10164	25	-0.07109	30	0.00889

**Table 2**The impulse response sequence ( $k = 3$ ).

$t$	$g(t)$	$t$	$g(t)$	$t$	$g(t)$	$t$	$g(t)$	$t$	$g(t)$	$t$	$g(t)$
1	1.000	6	0.051	11	-0.461	16	0.312	21	0.011	26	-0.043
2	2.960	7	-0.412	12	-0.320	17	0.096	22	0.067	27	0.023
3	3.242	8	-0.498	13	-0.042	18	-0.104	23	0.036	28	0.069
4	2.330	9	-0.475	14	0.247	19	-0.171	24	-0.034	29	0.061
5	1.043	10	-0.478	15	0.388	20	-0.102	25	-0.071	30	0.009

$\sigma_i = \sqrt{\lambda_i}$  ( $i = 1, 2, \dots, r$ ) are the singular values of  $\mathbf{R}$ , and  $\lambda_i$  ( $i = 1, 2, \dots, n$ ) are the eigenvalues of the  $n \times n$  symmetric square matrix  $\mathbf{R}^T \mathbf{R}$  with  $\lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_r > 0, \lambda_{r+1} = \dots = \lambda_n = 0$ . Because  $\mathbf{U}$  and  $\mathbf{V}$  are the orthogonal matrices, we have

$$\text{rank}[\mathbf{R}] \leq \min\{\text{rank}[\mathbf{U}], \text{rank}[\mathbf{\Sigma}], \text{rank}[\mathbf{V}]\} = \text{rank}[\mathbf{\Sigma}] = r. \quad (14)$$

For the Hankel matrix  $\mathbf{H}(l, t)$  in (8) (take  $t = 1$  for singular  $\mathbf{A}$ ) and given  $l$  value ( $l$  is large enough and should be greater than the true order  $n_0$ ) and smaller  $t$ , we make the SVD decomposition to  $\mathbf{H}(l, t)$  in (12) and obtain the matrix  $\mathbf{\Sigma}$  like (13). Observing the structure of  $\mathbf{\Sigma}$ , the number  $r$  of the nonzero singular values  $\sigma_1, \sigma_2, \dots, \sigma_r$  is the rank of the Hankel matrix  $\mathbf{H}(l, t)$  and is also the model order.

For the noise-free ideal case, the SVD method is very simple and can determine the model's order. But for the cases with the impulse responses having the measurement errors (noise), none singular values equal zero, we compute and compare  $\sigma_i/\sigma_{i+1}$  for  $i = 1, 2, \dots$ . If  $\sigma_i/\sigma_{i+1}$  is maximum for some  $i = l$ , then  $l$  is the system order.

In theory,  $t$  in  $\mathbf{H}(l, t)$  may take any number more than unity, but the impulse response  $g(t)$  approaches zero for a stable system as  $t$  increases. Thus,  $t$  in  $\mathbf{H}(l, t)$  generally takes a small integer, i.e.,  $t = 1, t = 2$  or  $t = 3$ .

#### 4. Numerical example

Consider the following simulation plant:

$$\begin{aligned} y(t) - 2.44y(t-1) + 2.96y(t-2) - 1.86y(t-3) + 0.55y(t-4) \\ = 1.00u(t-1) + 0.52u(t-2) - 1.02u(t-3) + 1.32u(t-4). \end{aligned}$$

In a simulation, the input  $\{u(t)\}$  is taken as a unit impulse sequence and the corresponding impulse responses  $g(t)$  (i.e., the system output  $y(t)$ ) are shown in Fig. 1 and Tables 1–2, for keeping  $k = 5$  decimal places and  $k = 3$  decimal places, respectively. That is, the impulse responses contain different measurement errors (noises). The corresponding step responses  $y(t)$  are shown in Fig. 2. We apply the ratios of the Hankel matrix method and the SVD method to determine the order of this system.

##### 1. The Hankel matrix determinant method.

Take  $t = 1$ . When  $l = 1, 2, \dots, 8$ , computing  $D_l = |\det[\mathbf{H}(l, t)]|$  gives

$$\begin{aligned} D_l &= [D_1, D_2, D_3, D_4, D_5, D_6, D_7, D_8] \\ &= [1.00000, 5.51920, 0.55062, 0.02874, 0.00001, 0.00000, 0.00000, 0.00000] \quad \text{for } k = 5, \end{aligned}$$

$$\begin{aligned} D_l &= [D_1, D_2, D_3, D_4, D_5, D_6, D_7, D_8] \\ &= [1.00000, 5.51960, 0.54224, 0.02922, 0.00005, 0.00000, 0.00000, 0.00000] \quad \text{for } k = 3. \end{aligned}$$

Since  $D_5$  is very close to zero, the system order is 4.

##### 2. The SVD method.

Take  $t = 1$  and  $l = 8$ , using (12) and making the SVD composition of  $\mathbf{H}(l, t)$  give the diagonal matrices:

$$\begin{aligned} \mathbf{\Sigma} &= \text{diag}[\sigma_1, \sigma_2, \sigma_3, \sigma_4, \sigma_5, \sigma_6, \sigma_7, \sigma_8] \\ &= \text{diag}[8.46052, 3.94895, 1.10769, 0.80510, 0.00002, 0.00001, 0.00001, 0.00000] \quad \text{for } k = 5, \end{aligned}$$

$$\begin{aligned} \mathbf{\Sigma} &= \text{diag}[\sigma_1, \sigma_2, \sigma_3, \sigma_4, \sigma_5, \sigma_6, \sigma_7, \sigma_8] \\ &= \text{diag}[8.46031, 3.94899, 1.10700, 0.80449, 0.00130, 0.00124, 0.00122, 0.00096] \quad \text{for } k = 3. \end{aligned}$$

Since  $\sigma_4/\sigma_5$  is maximum, we say that the system order is 4.

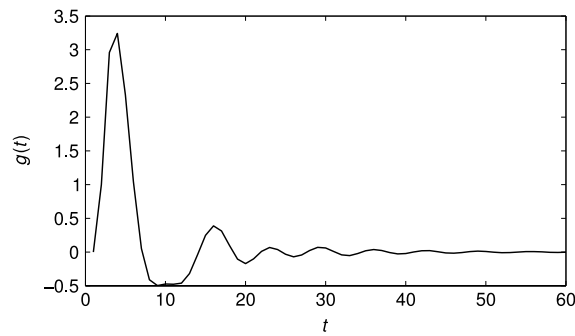


Fig. 1. The impulse response  $g(t)$ .

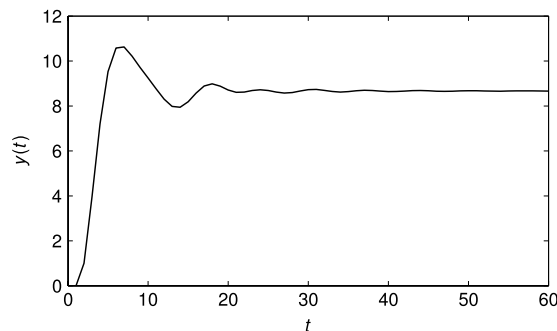


Fig. 2. The step response  $y(t)$ .

## 5. Conclusions

Two order identification methods are developed for linear systems using the impulse response sequences of the systems according to the singularities or ratios of the Hankel matrix determinants and the singular value decomposition of the Hankel matrix. The numerical results indicate that the proposed approaches are effective for determining the system orders.

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