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String breaking and monopoles: a case study in the 3D Abelian Higgs model

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Abstract

We study the breaking of the string spanned between test charges in the three-dimensional Abelian Higgs model with compact gauge field and fundamentally charged Higgs field at zero temperature. In agreement with current expectations we demonstrate that string breaking is associated with pairing of monopoles. However, the string breaking is not accompanied by an ordinary phase transition.

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1. Introduction

The lattice Abelian Higgs model with compact gauge field (cAHM) in three dimensions is of a broad interest both for high energy physics [1,2] and condensed matter physics [3–5]—where it was suggested to describe high- T_c superconductors and strongly correlated electron systems. Nowadays, it has even entered the physics of cognitive networks [6].

Due to compactness of the gauge field the model possesses Abelian monopoles which are instanton-like excitations in three space-time dimensions. The Abelian monopoles are able—if they are in the plasma state—to accomplish confinement of electrically

charged particles. This is well known from cQED₃ where opposite charged particles are bound by a linear potential [7]. The confinement is arranged by monopoles forming an opposite charged double sheet along the surface spanned by the trajectories of the external test charges. This surface is usually considered as the world surface of a string. Due to screening, the free energy increases only proportional to the area of the surface such that an area law for the Wilson loop emerges.

However, if dynamical matter fields in the same representation as the external test charges are added to the confining theory, linear confinement may be lost. This should be so, irrespective whether the dynamical matter field is fermionic (the quarks in QCD) or bosonic (the Higgs particle in our case). The string breaking phenomenon has been extensively studied in non-Abelian gauge theories with matter fields [8]

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or with test charges in the adjoint representation [9]. Here we want to investigate string breaking in cAHM₃ with a $q = 1$ charged Higgs field, a model whose permanently confining counterpart, cQED₃, is well understood. The general, intuitive picture says in the present case that the string breaks *because* of Higgs particle pairs popping up out of the vacuum at a definite inter-particle separation between the external, infinitely heavy test charges. Thus, the physical state corresponding to a *broken string* would consist of two heavy–light mesonic states plus some number of light–light Higgs pairs.

In order to destroy the linearly rising potential in cAHM₃, the coupling between the Higgs field and the gauge field must be sufficiently strong. One might be tempted to associate the string breaking with a phase transition between confinement and Higgs phases. Indeed, Ref. [4] proposes to associate the string breaking with a Berezinsky–Kosterlitz–Thouless type transition. In this Letter we demonstrate that and how the expected string breaking happens in a part of the phase diagram where a first or second order phase transition can definitely be excluded.

Abelian monopoles play the crucial role in the dual superconductivity scenario [10] of confinement in QCD. There, the monopole degrees of freedom need to be defined with the help of Abelian projections [11] (see, e.g., reviews [12]). The condensed magnetic currents were shown to make a dominant contribution to the string tension between quarks, in pure $SU(2)$ gauge theory [13] as well as in $SU(3)$ gluodynamics and also in full finite-temperature QCD with $N_f = 2$ flavors of dynamical quarks [14]. Moreover, in full QCD with dynamical quarks the contribution of Abelian monopoles to the heavy-quark potential QCD shows the property of string breaking [15]. The breaking of the adjoint string in pure gluodynamics as well as the breaking of the fundamental string in full QCD can both be described within the Abelian projection formalism [16]. The back-reaction of the dynamical fermions on the gauge field should modify the dynamics of monopoles in such a way that this dynamics incorporates the above qualitative picture [15].

Therefore, guided by the analogy to QCD, we focus our interest in the present Letter on the monopole degrees of freedom in compact AHM in three dimensions under the influence of a scalar matter field. We would like to elucidate the changing role of

monopoles under the particular aspect of string breaking. As in QCD, the string tension in this model is exclusively due to monopoles. Therefore one can expect that monopoles also encode the back-reaction of the matter field causing the string breaking phenomenon. Here we want to demonstrate that (i) the monopole part of the potential indeed incorporates the effect of string breaking and (ii) that it is monopole pairing which is the reason for the breakdown of the monopole confinement mechanism. We are aware of the incompleteness of the analogy to QCD and the relative simplicity of monopole dynamics in 3 instead of 4 dimensions.

It seems that there is only one possibility to explain string breaking in three space–time dimensions. We assume that, in the presence of matter fields, monopoles are increasingly bound into neutral pairs (magnetic dipoles). The size of a typical pair should be of the order of the string breaking distance R_{br} . Indeed, if the distance R between the test charges is much larger than R_{br} then the test charges do not recognize individual monopoles inside the dipoles (in other words, the fields of the monopoles from the same magnetic dipole effectively screen each other) and the vacuum is basically composed of neutral particles. Therefore, at large inter-particle separations there should be no string tension. However, if $R \ll R_{br}$ then the test charges do recognize individual monopoles even if they are bound in dipoles, and the monopole fields may induce a piecewise linearly rising potential. These simple considerations can be made more rigorous by analytical calculations [17] for a gas of infinitely small-sized dipoles.

Recently, it was found that the matter fields in the Abelian Higgs model lead to a logarithmic attraction between monopoles and antimonopoles [4] which results in the formation of monopole–antimonopole bound states and string breaking. The formation of dipoles can also be explained as due to the existence of Abrikosov–Nielsen–Olesen vortices [18], the string tension of which gets increased as we move in the parameter space deeper into the Higgs region.¹ Massless quarks also force the Abelian monopoles

¹ Note that the division of the parameter space of the model into Higgs and confinement regions is only loose since these regions—as we discuss below—are analytically connected.

to form bound states [19]. Note that the origin of monopole binding in the zero temperature case of cAHM₃ is physically different from the monopole binding observed at the finite temperature phase transition in compact QED [20,21]. It is different as well from the Z₂ vortex mechanism in the Georgi–Glashow model [22].

In this Letter we numerically establish a relation between string breaking on one hand and the occurrence of monopole–antimonopole bound states on the other by studying some properties of the monopole ensembles provided by the compact Abelian Higgs model. In Section 2 we recall the definition of the model and discuss its missing phase transition. In Section 3 flattening of the potential is described. Here we also introduce the η angle as a parameter which defines the “effectiveness” of string breaking. Section 4 is devoted to an investigation of the cluster structure of the monopole ensembles. Our conclusions are presented in the last section.

2. The model and its crossover

We consider the 3D Abelian gauge model with a compact gauge field $\theta_{x,\mu}$ and a Higgs field Φ_x with unit electric charge. The coupling between the gauge and the Higgs fields is $S_{x,\mu} \propto \Re(\Phi_x^\dagger e^{i\theta_{x,\mu}} \Phi_{x+\hat{\mu}})$. To simplify calculations we consider the London limit of the model, which corresponds to an infinitely deep potential on the Higgs field. In this limit the radial part of the Higgs field, $|\Phi_x|$, is frozen and the only dynamical variable is the phase φ_x of this field, $\Phi_x = |\Phi_x| e^{i\varphi_x}$. Thus the Higgs-gauge coupling reduces to the simple interaction $S_{x,\mu} \propto \cos(\varphi_{x+\hat{\mu}} - \varphi_x + \theta_{x,\mu})$. However, the model can be simplified even further by fixing the unitary gauge, $\varphi_x = 0$ leading to $S_{x,\mu} \propto \cos\theta_{x,\mu}$. Thus we consider the model with the action

$$S[\theta] = -\beta \sum_P \cos\theta_P - \kappa \sum_l \cos\theta_l, \quad (1)$$

where β is the gauge (Wilson) coupling, κ is the hopping parameter and θ_P is the plaquette angle. We study the model at zero temperatures on lattices of size L^3 , with $L = 12, 16, 24, 32$.

The phase structure of the model on the boundaries of the phase diagram in the β – κ plane can be established using the following simple arguments. At zero

value of the hopping parameter κ the model (1) reduces to the pure compact Abelian gauge theory which is known to be confining at any coupling β due to the presence of the monopole plasma [7]. This argument extends to the low- κ region of the phase diagram. Therefore we call this the “confinement region”. At large values of κ (also called the “Higgs region”) the monopoles should disappear because the gauge field in this limit is increasingly restricted to the trivial vacuum state: $\theta_{x,\mu} = 0$.

At large β the model reduces to the three-dimensional XY model which is known to have a second order phase transition at $\kappa_c^{XY} \approx 0.453$ [23]. Indeed, in this limit we get the condition $d\theta_l \equiv \theta_P = 0$ which forces the gauge field to be a gauge transformation of the vacuum, $\theta_{x,\mu} = -\phi_{x+\hat{\mu}} + \phi_x + 2\pi l_{x,\mu} \in (-\pi, \pi]$, $l_{x,\mu} \in \mathbb{Z}$, $\phi_x \in (-\pi, \pi]$. The scalar fields ϕ are the spin fields in that model.

Despite the phase structure on the boundary of the coupling plane is well established, the structure of its interior is still under debate. Indeed, in Ref. [3] arguments were given that the interior is trivial (i.e., there is no phase transition for finite values of β and κ) while the XY-phase transition takes place in an isolated point at $\beta = \infty$. In Ref. [24] it has been suggested that the phase diagram of cAHM₃ resembles the vapor-liquid diagram with a critical end-point. Finally, in Ref. [2] it was argued that the phase diagram contains a “pocket” in which a Coulomb phase could be realized. Arguments given in Ref. [1] do not allow to distinguish between these three possibilities.

In a numerical study on rather small lattices [25] no hint for an ordinary phase transition at finite coupling constant β has been found. However, for simulations allowing fluctuating Higgs lengths, sufficiently away from the London limit, the phase diagram has been seen to become nontrivial [26]. Recently, the phase structure of the cAHM₃ has been studied by the authors of Ref. [27] in connection with the nature of the transition in the type-I and the type-II region. The alleged second order transition in the type-II region away from the London limit still remained inconclusive.

Here we are not going to study the whole phase diagram of cAHM₃ although this question would be still interesting. As we describe below, we observed that at moderately small β the Higgs and confinement

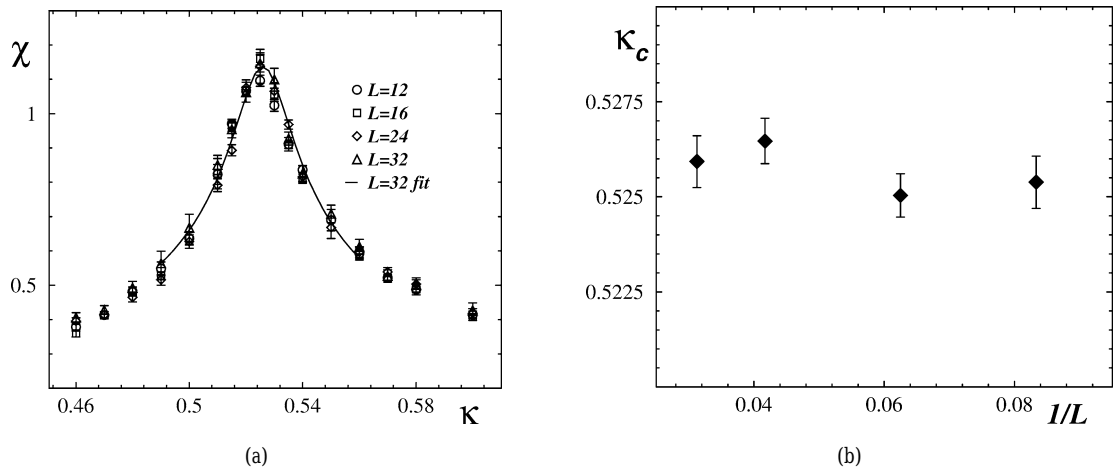


Fig. 1. (a) The susceptibility of the hopping term (2) as a function of κ ; (b) the crossover point κ_c as a function of the inverse lattice size, L^{-1} .

regions are connected analytically by a crossover as predicted in Ref. [1]. We concentrate on the changing role of monopoles under the aspect of the string breaking phenomenon accompanying the crossover at relatively small β with increasing hopping parameter κ . For the simulations we use a Monte Carlo algorithm similar to the one described in Ref. [21] and have considered 5×10^3 to 5×10^4 independent configurations per data point, depending on the lattice size and the set of coupling constants. We vary the value of the hopping parameter κ at a fixed value of gauge coupling constant $\beta = 2.0$. To locate a (pseudo)-critical point we use the susceptibility of the hopping term

$$\chi = \langle S_H^2[\theta] \rangle - \langle S_H[\theta] \rangle^2, \quad (2)$$

$$S_H[\theta] = - \sum_l \cos \theta_l,$$

which is shown² in Fig. 1(a) for $L = 12, 16, 24, 32$. The height of the peak is practically independent on the lattice size. We have observed a very similar volume independence also of the susceptibility of the gauge term, $S_G[\theta] = - \sum_P \cos \theta_P$. Thus, in agreement with Ref. [25] we conclude that there is *no ordinary phase transition* between the Higgs and confinement regions of the parameter space of the model.

The crossover point $\kappa_c(L)$ is located fitting the susceptibility (2) in the vicinity of the peak by the following function:

$$\chi^{\text{fit}}(\beta) = \frac{C_1}{[C_2^2 + (\kappa - \kappa_c)^2]^\alpha}, \quad (3)$$

where $C_{1,2}$, κ_c and the power α are fitting parameters. In Fig. 1(a) we show the fit of the susceptibility data for the 32^2 lattice. The fit parameters practically do not depend on the lattice size. We depict the critical value of the hopping parameter κ_c vs. the inverse lattice size L^{-1} in Fig. 1(b). The value of the power α is very close to $1/4$. In the next sections we will work with the lattice 32^2 which passes the crossover at $\kappa_c = 0.526(1)$ along a line with fixed $\beta = 2.0$.

3. The flattening of the potential

String breaking manifests itself in the flattening of the potential between test particles with (opposite) electric charges $q = \pm 1$. In principle, we can separate the contributions to the potential from monopoles and from the rest (“photon contribution”). Monopoles are responsible for the string tension. Therefore one can expect that the monopole contribution alone will signal the onset of string breaking when the monopole dynamics starts changing. It would be much more demanding to extract the string part from the full potential and to study its change over the parameter

² Note that all figures in this Letter are shown for $\beta = 2.0$.

space of the model. The full potential contains also the perturbative photon contribution which—being logarithmically large at small distances—shadows the eventually linearly rising part. Any statement about the string part would require a careful fit of full potential. On the more technical side, the monopole contribution alone, calculated separately according to the configurations generated in the simulation of the AHM, has a much better signal/noise ratio compared to the full potential. All this justifies to proceed directly to the evaluation of the monopole contributions to the external-charge potential.

To this end we have divided the gauge field θ_l into a regular (photon) part and a singular (monopole) part [28]:

$$\theta = \theta^{\text{phot}} + \theta^{\text{mon}}, \quad \theta^{\text{mon}} = 2\pi \Delta_3^{-1} \delta p[j]. \quad (4)$$

The 0-form $*j \in \mathbb{Z}$ is nonvanishing on the sites dual to the lattice cubes c which are occupied by monopoles [29]:

$$j_c = \frac{1}{2\pi} \sum_{P \in \partial c} (-1)^P [\theta_P]_{\text{mod } 2\pi}, \quad (5)$$

where the factor $(-1)^P$ takes the plaquette orientations relative to the boundary of the cube into account. In Eq. (4) the 2-form $p_P[j] = [\theta_P]$ (the notation $[\dots]$ means taking the integer part) corresponds to the Dirac strings living on the links of the dual lattice, which are either closed or connecting monopoles with anti-monopoles, $\delta * p[j] = *j$. While $*j$ is gauge invariant, the 2-form $p_P[j]$ is not. For the Monte Carlo configurations provided by the simulation of (1) we have located the Dirac strings, $p[j] \neq 0$, and constructed the monopole part θ^{mon} of the gauge field according to the last equation in (4). The operator Δ_3^{-1} in Eq. (4) is the inverse lattice Laplacian defined for a three-dimensional lattice L^3 :

$$\Delta_d^{-1}(\vec{x}; L) = \frac{1}{2L^d} \sum_{\vec{p}^2 \neq 0} \frac{e^{i(\vec{p}, \vec{x})}}{d - \sum_{i=1}^d \cos p_i}, \quad (6)$$

where $p_i = 2\pi k_i/L_i$ for $k_i = 0, \dots, L_i - 1$, with $i = 1, \dots, d$ and $L_i = L$.

We define the potential between test particles with the help of the following correlator of two Polyakov loops:

$$\langle P(\vec{0}) P^\dagger(\vec{R}) \rangle = e^{-LV(R)}, \quad (7)$$

located at two-dimensional points $\vec{0}$ and \vec{R} . The potential V depends on $R = |\vec{R}|$. The use of the Polyakov loop has clear advantages compared to the Wilson loops. The construction of the Polyakov loops is not only possible for finite-temperature but also for finite-volume cases. $L = L_i$ is the common length of the zero-temperature box in all three directions. Due to the absence of space-like links joining the Polyakov loops the correlator (7) defines the static component of the potential. Note that the monopole contribution to the Polyakov loop correlator (7) does not depend on the precise form of the Dirac string $*p[j]$. Therefore this contribution is gauge-invariant.

We discuss the results for the potential using the following fitting function:

$$\begin{aligned} & e^{-LV^{\text{fit}}(R)} \\ &= C_0 \left[\sin^2 \eta + \cos^2 \eta \frac{\cosh(\sigma L(L/2 - R))}{\cosh(\sigma L^2/2)} \right] \\ & \times \exp\{\gamma L[\Delta_2^{-1}(R) - \Delta_2^{-1}(0)]\}, \end{aligned} \quad (8)$$

where C_0 , η , σ and γ are fitting parameters and Δ_2^{-1} is the inverse lattice Laplacian in two dimensions.

The meaning of the expression (8) is quite simple. In the absence of string breaking and in an infinite two-dimensional volume the leading contribution to the function in the right-hand side of Eq. (8) should be just $\text{const} \times e^{-\sigma LR}$ where σ is the effective string tension. Here “effective” means that this term gives rise to a linear part in the potential at short distances.

The string breaking manifests itself in the appearance of an additional constant term, $\text{const}_1 + \text{const}_2 \times e^{-\sigma LR}$. Next, the finiteness of the two-dimensional volume reduces the exponential to the cosh-function which takes care of the symmetry $R \rightarrow L - R$. Finally, we introduced a Coulomb term in order to take into account sub-leading corrections.

The dimensionless parameter $\eta \in [0, \pi/2]$ —which we call a “breaking angle”—has a sense only as long as $\sigma \neq 0$. It can be considered as a kind of “order parameter” for string breaking: if $\eta = 0$, no string breaking occurs, and if $\eta = \pi/2$, the potential does not contain a linear piece at all. An intermediate value of the breaking angle implies the existence of the finite distance R_{sb} at which the string between the test particles breaks. Note that we have introduced a normalizing cosh-factor in the second term in the brackets in order to keep the $V^{\text{fit}}(R = 0)$ value

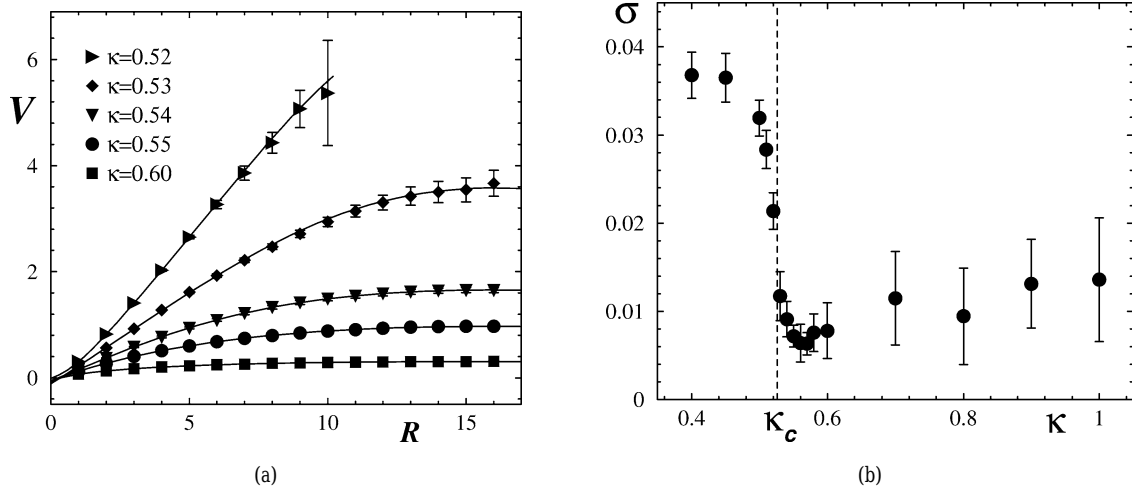


Fig. 2. (a) The potential for $\kappa = 0.52$, $\kappa = 0.53$, $\kappa = 0.54$, $\kappa = 0.55$ and $\kappa = 0.60$ extracted from the monopole contribution to the Polyakov loop correlator by Eq. (7). The fits by the function (8) are shown by solid lines. (b) The string tension vs. κ . In this and all subsequent figures the string breaking transition at κ_c for $\beta = 2.0$ is marked by a vertical line.

independent on η . This definition is a matter of conventions.

To justify the presence of the Coulomb-like term in the fitting function (8) let us consider three-dimensional compact QED. It is well known that in the Villain representation the Polyakov loop correlator factorizes into the photon and monopole contribution. The monopole contribution can be evaluated exactly and it contains a massless pole, $\Delta_2^{-1}(R)$, corresponding to the Coulomb potential between test particles. The total correlator should not contain the massless pole due to the massiveness of the photon. Therefore the monopole contribution to the correlator must contain—in addition to the linear term—the difference between the Yukawa and Coulomb potentials, $\Delta_2^{-1}(R; m) - \Delta_2^{-1}(R; m = 0)$ corresponding to the exchange by “real” (massive) and “bare” (massless) photons. Here $\Delta_2^{-1}(R; m)$ is the propagator of a particle with the mass m . The mentioned above sub-leading term is small at distances smaller than the inverse photon mass. However, this term gives a significant (logarithmically growing) contribution at larger separations between test particles. Thus the largest deviation from the linear behaviour of the monopole contribution to the potential is expected to come from large distances due to exchange of a massless (bare) photon.

Similar arguments should apply to the case of the compact AHM. The bare photon here, however, is not

massless due to the spontaneous breaking of the $U(1)$ symmetry. Therefore the fitting function (8) should be modified: the Coulomb potential should be replaced by the Yukawa one. We have found that such fits do not work well because the corresponding Yukawa mass turns out to be consistent with zero within huge error bars. On the other hand, the mass of the bare photon should be small at the QED side of the crossover where the form of the fit (8) is obviously justified. We have found numerically that this fitting function works well also at the Higgs side of the crossover. Therefore in Eq. (8) we restrict ourselves to the Coulomb term only.

The fits of the numerical data for the potential $V(R)$ due to monopoles by the expression (8) are shown in Fig. 2(a) for five values of the hopping parameter from $\kappa = 0.52$ (below string breaking) to $\kappa = 0.60$ (far from the transition on the Higgs side) including $\kappa = 0.53 \approx \kappa_c$ (in the vicinity of the transition). In the fits of the potential the point $R = 0$ was excluded. One can clearly recognize a linear part in the potential near the transition point. As κ increases (this corresponds to moving deeper into the Higgs region) the linear part gradually disappears. This can also be seen from the properties of the string tension σ shown in Fig. 2(b). The string tension itself, which on the confinement side amounts roughly to 50% of the QED₃ string tension (corresponding

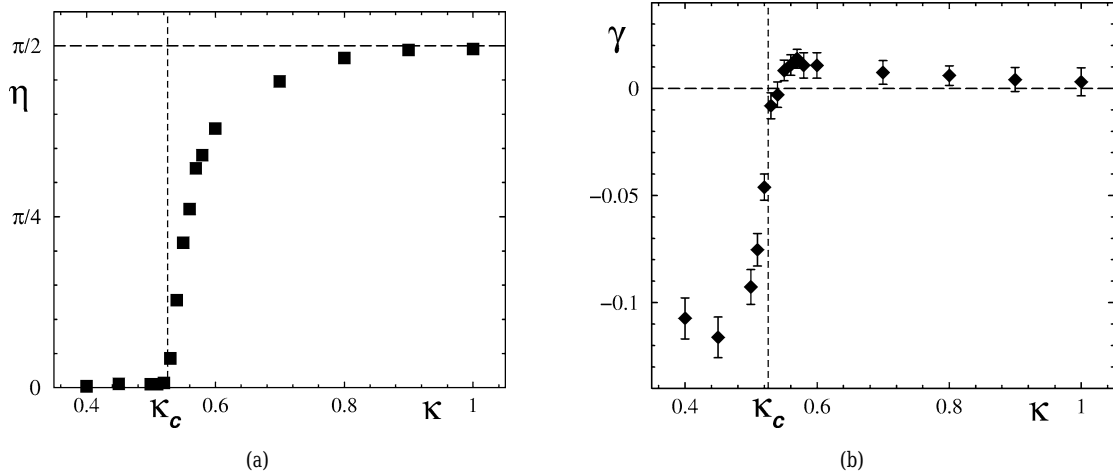


Fig. 3. (a) The breaking angle η and (b) the parameter γ appearing in the fitting function (8) vs. the hopping parameter κ .

to $\kappa = 0$), drops to a smaller value over a very narrow κ region. The described behaviour of the potential is consistent with the expected disappearance of isolated monopoles on the Higgs side of the string breaking transition. The residual string tension, which is accompanied by a short string breaking length R_{sb} , can be accounted for by the monopole–antimonopole dipoles of finite size. With $\eta \rightarrow \pi/2$ the fit error of σ increases.

The breaking angle η is shown in Fig. 3(a) as a function of κ . It clearly shows an “order-parameter-like” behaviour: it is close to zero for $\kappa < \kappa_c$ and it is finite at $\kappa > \kappa_c$. Small values of η imply that the string breaking distance is still large. At $\kappa \sim 1$ the value of $\eta \sim \pi/2$ indicates that the area-law term in the Polyakov loop correlator (8) has become irrelevant.

The parameter γ , shown in Fig. 3(b), seems to vanish on the Higgs side of the string breaking transition. This may indicate that in the Higgs region the “bare” photon mass becomes significant and that the corrections to the linear potential gets concentrated at small distances.³ Thus, long distance corrections should be zero, i.e., $\gamma \sim 0$.

4. The cluster structure of the monopole ensemble

In this section we turn to the monopole clustering aspect of the Monte Carlo configurations which have been used in the last section to work out the monopole part of the external-charge potential. We closely follow Ref. [21] where the cluster analysis of the monopole configurations in the case of compact QED₃ at nonzero temperature was performed.

The simplest quantity describing the behaviour of the monopoles is the monopole density, $\rho = \sum_c |j_c|/L^3$, where j_c is the integer valued monopole charge inside the cube c defined in Eq. (5). The density of the total number of monopoles is a decreasing function of the hopping parameter κ as it is shown in Fig. 4(a) by diamonds. The density sharply drops down at κ_c , which has been recognized as the string breaking transition point, but the density does not vanish on the Higgs side of the crossover. The binding of monopoles into dipoles should show up as an increase of the number of monopoles enclosed in neutral clusters. We call a monopole cluster neutral if the charges of the corresponding constituent monopoles sum up to zero. Clusters are connected groups of monopoles and antimonopoles where each object is separated from at least one neighbor belonging to the same cluster by a distance less or equal than some R_{max} . The smallest clusters are isolated (anti)monopoles. In our analysis we have used $R_{max}^2 = 3a^2$ which means that

³ We remind the reader that the smallest distance, $R = 0$, is excluded from the fit.

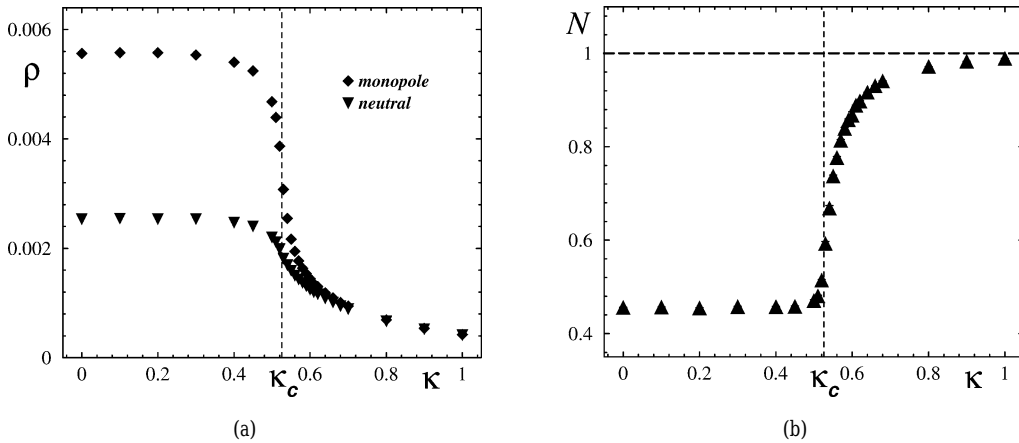


Fig. 4. (a) Total density of monopoles (diamonds) and density of the monopoles in neutral clusters (triangles), and (b) the fraction of neutral clusters among all clusters, both as functions of κ .

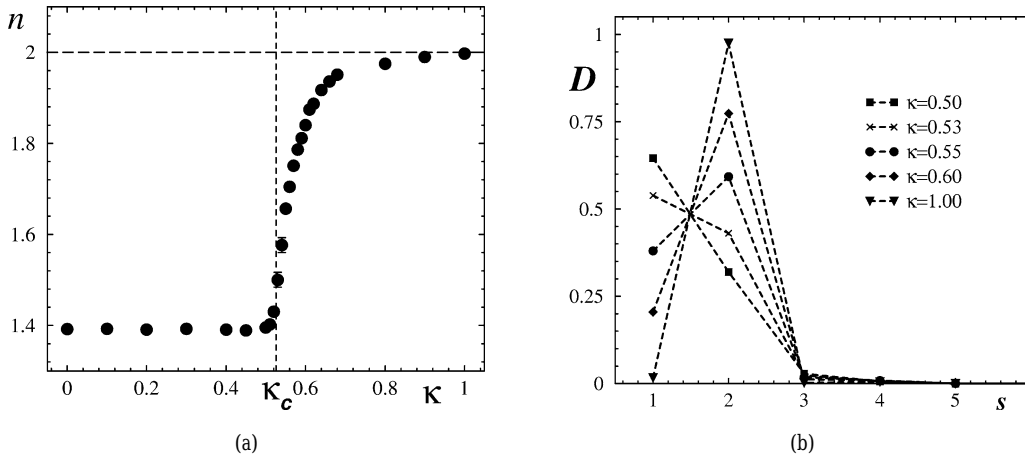


Fig. 5. (a) The average number of monopoles and antimonopoles per cluster as function of κ and (b) the (normalized) cluster size distribution $D(s)$ for a few values of κ .

monopoles are considered as neighbors if their cubes share at least one single corner.

We show also in Fig. 4(a), symbolized by triangles, the density of monopoles in neutral clusters which almost covers the total density on the Higgs side of the string breaking transition. If we take into account that also bigger dipoles—which cannot be identified by our procedure—may be formed, this clearly signals the binding transition.

In an alternative, perhaps more clear way this is illustrated by the fraction of monopoles belonging to neutral clusters, $N = \rho_{\text{neutral}}/\rho_{\text{total}}$, which is shown in Fig. 4(b). Being constant on the confinement side

of the string breaking transition, this quantity starts suddenly to rise at the transition. This indicates that at the transition point (crossover) the binding process rapidly takes place. At large κ the fraction is very close to unity. Then all monopoles are bound.

Finally, in Fig. 5(a) we present the average number of (anti)monopoles per cluster and in Fig. 5(b) the (normalized) cluster size distribution $D(s)$ where s is the number of (anti)monopoles in the cluster, for a few values of the hopping parameter κ . On the confinement side of the string breaking transition ($\kappa \approx 0.5$) the vacuum consists to $\approx 70\%$ of *isolated monopoles*. At the crossover (the string breaking transition) at

$\kappa \approx 0.53$ the number of isolated monopoles decreases, and on the Higgs side ($\kappa > 0.53$) the vacuum is dominated by the dipole gas.

5. Conclusions

We have numerically observed that in the London limit of the three-dimensional Abelian Higgs model string breaking occurs and is accompanied by monopole recombination into dipoles, in agreement with arguments given in Ref. [4].

Our study shows that the monopole binding is not necessarily accompanied by an ordinary phase transition of first or second order. There is a proposition [4], however, that the string breaking may be associated with a Berezinsky–Kosterlitz–Thouless type transition due to the appearance of an anomalous dimension of the gauge field induced by the fluctuations of the matter fields. This possibility is not ruled out by our results. In the London limit (studied in this Letter) the fluctuations of the radial components of the matter field are suppressed, while far away from the London limit the fluctuations become significant such that an ordinary phase transition may exist [26,27].

Acknowledgements

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