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Scalar–tensor cosmology with R^{-1} curvature correction by Noether symmetry

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ABSTRACT

We discuss scalar–tensor cosmology with an extra R^{-1} correction by the Noether symmetry approach. The existence of such a symmetry selects the forms of the coupling $\omega(\varphi)$, of the potential $V(\varphi)$ and allows to obtain physically interesting exact cosmological solutions.

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1. Introduction

Recent observational data indicate that $\simeq 70\%$ of the today cosmological energy density is dominated by some form of “dark energy” which can be described, in the simplest way, by the cosmological constant Λ [1–4]. Such an ingredient should explain the accelerated expansion of the observed universe, firstly deduced by luminosity distance measurements of SNeIa supernovae. However, even though the presence of a dark energy component is appealing in order to fit observational results with theoretical predictions, its fundamental nature still remains an open question.

Although several models describing the dark energy component have been proposed over the past few years, one of the first physical realizations of quintessence was a cosmological scalar field, which dynamically induces a repulsive gravitational force, causing an accelerated expansion of the universe.

The existence of such a large proportion of dark energy in the universe presents a large number of theoretical problems. Firstly, why do we observe the universe at exactly the time in its history when the vacuum energy dominates over matter (this is known as the *cosmic coincidence problem*). The second issue, which can be thought of as a *fine tuning problem*, arises from the fact that if the vacuum energy is constant, like in the pure cosmological constant scenario, then at the beginning of the radiation era the energy density of the scalar field should have been vanishingly small with respect to the radiation and matter components. This poses the problem that, in order to explain the inflationary behavior of the early universe and the late time dark energy dominated regime, the vacuum energy should evolve and should not simply be a *constant*.

Some recent works have shown that the fine-tuning problem could be alleviated by selecting a subclass of quintessence models, which admit a *tracking behavior* [5], and in fact, to a large extent, the study of scalar field quintessence cosmology is often limited to such a subset of solutions. In scalar field quintessence, the existence condition for a tracker solution provides a sort of selection rule for the potential $V(\phi)$ (see [6] for a critical treatment of this question), which should somehow arise from a high energy physics mechanism (the so-called *model building problem*). Also, adopting a phenomenological point of view, where the functional form of the potential $V(\phi)$

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can be determined from observational cosmological functions, for example the luminosity distance, we still cannot avoid a number of problems. For example, an attempt to reconstruct the potential from observational data (and also fitting the existing data with a linear equation of state) shows that a violation of the weak energy condition (WEC) is not completely excluded [7], and this would imply a *superquintessence regime*, during which $w_\phi < -1$ (*phantom regime*). However, it turns out that assuming a dark energy component with an arbitrary scalar field Lagrangian, the transition from regimes with $w_\phi \geq -1$ to those with $w_\phi < -1$ (i.e. crossing the so-called *phantom divide*) are probably physically impossible since they are either described by a discrete set of trajectories in the phase space or are unstable [8,9]. These shortcomings have been recently overcome by considering the *unified phantom cosmology* [10] which, by taking a generalized scalar field kinetic sector into account, allows one to achieve models with natural transitions between inflation, dark matter, and dark energy regimes. Moreover, in recent works, a dark energy component has been modelled also in the framework of scalar–tensor theories of gravity, also called extended quintessence (see for instance [11–20]).

It turns out that they are compatible with a *peculiar* equation of state $w \leq -1$, and provide a possible link to issues occurring in non-Newtonian gravity [12]. In such theoretical backgrounds, the accelerated expansion of the universe is due to the effect of the non-standard form of the gravitational action. In extended quintessence cosmologies (EQ) the scalar field is coupled to the Ricci scalar R in the Lagrangian density of the theory: the standard term $16\pi G_* R$ is replaced by $16\pi F(\phi)R$, where $F(\phi)$ is a function of the scalar field, and G_* is the *bare* gravitational constant, generally different from the Newtonian constant G_N measured in Cavendish-type experiments [11].

Of course, the coupling is not arbitrary, but it is subjected to several constraints, mainly arising from the time variation of the constants of nature [21]. In EQ models, a scalar field has indeed a double role: it determines at any time the effective gravitational constant and contributes to the dark energy density, allowing some different features from the minimally coupled case [21]. Actually, while in the framework of the minimally coupled theory we have to deal with a fully relativistic component, which becomes homogeneous on scales smaller than the horizon, so that standard quintessence cannot cluster on such scales, in the context of non-minimally coupled quintessence theories the situation is different, and the scalar field density perturbations behave like the perturbations of the dominant component at any time. This is referred to as *gravitational dragging* [13].

On the other hand, the cosmic speed up can be simply explained considering some sub-dominant terms of geometric origin like R^{-1} , where R is the Ricci curvature scalar, which becomes dominant toward small curvature regimes (see e.g. [18]). In fact, it is possible to show that, by adding these terms to the Hilbert–Einstein action and varying with respect to the metric, such modified field equations naturally produce the observed cosmological acceleration. The simplest action of these models is:

$$S = \frac{1}{8\pi G_N} \int \left(R - \frac{\mu_0^4}{R} \right) \sqrt{-g} d^4x \tag{1}$$

where G_N is the Newton's gravitational constant and μ_0 is a constant. The Palatini variation of this action is studied, for example, in Ref. [19]. However, we need some additional ingredient to fit the observed data and physical constraints at every redshift so a modified scalar–tensor theory could be a more suitable candidate to achieve the whole observed dynamics. In this perspective, the investigation of theories like

$$S = \int \left[\phi \left(R - \frac{\mu_0^4}{R} \right) + \frac{\omega(\phi)}{\phi} g^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi - V(\phi) \right] \sqrt{-g} d^4x \tag{2}$$

is in order. Here ϕ denotes a real scalar field, non-minimally coupled to gravity while $\omega(\phi)$ and $V(\phi)$ are the coupling function and a self-interacting potential, respectively.

A physical criterion to achieve general cosmological solutions could be by the *Noether symmetry approach* which revealed a useful tool to fix the forms of the coupling and the potential [22], and, very recently, also the form of $f(R)$ [23,24]. From a mathematical point of view, the method lies on the fact the presence of symmetries selects cyclic variables which allow to reduce the dynamics and then to integrate the equations of motion. From a physical point of view, any Noether symmetry is associated to some conserved quantity. This fact allows to select physically viable models (see for example [22]) and constitutes a criterion to select suitable effective Lagrangian (in particular, the forms of the coupling, of the self-interacting potential and the higher-order corrections).

Specifically, in this Letter, we work out the above action (2) searching for Noether symmetries in order to see if the coupling, the self-interacting potential and the R^{-1} can be related in physically viable models. Besides, as we will see, this procedure allows to exactly integrate the equations of motion.

The Letter is organized as follows. In Section 2, we search for Noether symmetries for the above action selecting the coupling and the potential. Section 3 is devoted to find out the cosmological solutions and the discussion of the various sub-cases. Concluding remarks and conclusions are reported in Section 4.

2. The Noether symmetry

In order to search for Noether symmetries, it is convenient to recast the action (2) by redefining $\phi = \varphi^2$ and $\mu_0^4 = -\mu$, that is

$$S = \int \left[\varphi^2 \left(R + \frac{\mu}{R} \right) + 4\omega(\varphi) g^{\mu\nu} \nabla_\mu \varphi \nabla_\nu \varphi - V(\varphi) \right] \sqrt{-g} d^4x. \tag{3}$$

Using the Friedmann–Robertson–Walker (FRW) metric, the scalar curvature takes the form $R = 6\left(\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{k}{a^2}\right)$, where $a(t)$ is the scale factor of the universe and the dot denotes the derivative with respect to time, with $k = \pm 1, 0$. Now, using the Lagrange multipliers method [26], one can rewrite the action (3) as follows

$$S = \int \left[\varphi^2 \left(R + \frac{\mu}{R} \right) + 4\varphi^2 \omega(\varphi) - V(\varphi) + \lambda_1 (R - 6) \left(\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{k}{a^2} \right) \right] \sqrt{-g} d^4x \tag{4}$$

where the scalar curvature R and scale factor a are considered as two independent variables, and λ_1 is a Lagrange multiplier. The parameter λ_1 can be determined by varying the action with respect to R , that is

$$\lambda_1 = \varphi^2 (\mu R^{-2} - 1). \quad (5)$$

Now, in order to apply the Noether symmetry approach, one can easily show that, in a FRW manifold, the Lagrangian related to the action (4) takes the point-like form [22]

$$\mathcal{L} = 2a^3 \varphi^2 \mu q + 6(\mu q^2 - 1)(2a^2 \varphi \dot{a} \dot{\varphi} + \varphi^2 \dot{a}^2 a) + 12\mu \varphi^2 a^2 q \dot{a} \dot{q} - 6\varphi^2 k a (\mu q^2 - 1) + a^3 (4\omega(\varphi) \dot{\varphi}^2 - V(\varphi)) \quad (6)$$

where $q = R^{-1}$ is a new variable. This means that we are considering an effective theory with two scalar fields. The corresponding Euler–Lagrange equations are given by

$$(\mu q^2 - 1) \left(2\varphi H^2 + \frac{\varphi k}{a^2} + \dot{H}\varphi \right) + \frac{1}{3} \frac{d\omega}{d\varphi} \dot{\varphi}^2 + \frac{2}{3} \omega(\varphi) \ddot{\varphi} - \frac{1}{3} \mu q \varphi + 2\omega(\varphi) \dot{\varphi} H + \frac{1}{12} \frac{dV}{d\varphi} = 0, \quad (7)$$

$$(\mu q^2 - 1) \left[2\varphi \dot{\varphi} H + \varphi^2 \left(\frac{3}{2} H^2 + \frac{k}{2a^2} + \dot{H} + \frac{d(\varphi \dot{\varphi})}{dt} \right) \right] + \frac{1}{4} V(\varphi) - \omega(\varphi) \dot{\varphi}^2 + \mu \varphi^2 \frac{d(q \dot{q})}{dt} + 2\mu \varphi q \dot{q} (2\dot{\varphi} + H\varphi) = 0 \quad (8)$$

with the constraint, derived from the definition of the scalar curvature R ,

$$6 \left(2H^2 + \dot{H} + \frac{k}{a^2} \right) = \frac{1}{q}. \quad (9)$$

Here $H = \frac{\dot{a}}{a}$ is the Hubble parameter. Eqs. (7) and (8) are equivalent to the second order Einstein equation and to the Klein–Gordon equation, respectively. Finally, one can choose the initial conditions of these field equations such that the energy function associated with the Lagrangian (6) vanishes, that is

$$E_{\mathcal{L}} = \dot{a} \frac{\partial \mathcal{L}}{\partial \dot{a}} + \dot{q} \frac{\partial \mathcal{L}}{\partial \dot{q}} + \dot{\varphi} \frac{\partial \mathcal{L}}{\partial \dot{\varphi}} - \mathcal{L} = 0, \quad (10)$$

or explicitly

$$(\mu q^2 - 1) \left(\varphi \dot{\varphi} H + \frac{1}{2} \varphi^2 H^2 + \frac{\varphi^2 k}{2a^2} \right) + \frac{1}{3} \omega(\varphi) \dot{\varphi}^2 + \frac{1}{12} V(\varphi) + \mu \varphi^2 H q \dot{q} - \frac{1}{6} \mu q \varphi^2 = 0 \quad (11)$$

which corresponds to the $\{0, 0\}$ Einstein equation. Now, let us introduce the *lift vector field* X [30] as an infinitesimal generator of the Noether symmetry in the tangent space $TQ \{a, \dot{a}, \varphi, \dot{\varphi}, q, \dot{q}\}$ related to the configuration space $Q = \{a, q, \varphi\}$ as follows

$$X = A \frac{\partial}{\partial a} + B \frac{\partial}{\partial \varphi} + C \frac{\partial}{\partial q} + \dot{A} \frac{\partial}{\partial \dot{a}} + \dot{B} \frac{\partial}{\partial \dot{\varphi}} + \dot{C} \frac{\partial}{\partial \dot{q}} \quad (12)$$

where A , B and C are unknown functions of the variables a , φ and q . The existence of Noether symmetry for the dynamics implies that the vector field X is non-trivial and the Lie derivative of the Lagrangian, with respect to this vector field, vanishes

$$L_X \mathcal{L} = 0.$$

Explicitly, this condition leads to the following differential equations

$$6a^2 \mu \varphi^2 q A - 3a^2 V(\varphi) A - Ba^3 \frac{dV}{d\varphi} + 4\mu a^3 \varphi q B + 2a^3 \varphi^2 \mu C + 6k\varphi^2 A (1 - \mu q^2) + 12ka\varphi B (1 - \mu q^2) - 12\mu ka\varphi^2 q C = 0, \quad (13)$$

$$3\omega(\varphi) A + Ba \frac{d\omega}{d\varphi} + 3(\mu q^2 - 1) \varphi \frac{\partial A}{\partial \varphi} + 2\omega(\varphi) a \frac{\partial B}{\partial \varphi} = 0, \quad (14)$$

$$(\mu q^2 - 1) \left(\varphi A + 2Ba + 2a\varphi \frac{\partial A}{\partial a} + 2a^2 \frac{\partial B}{\partial a} \right) + 2\mu q a \varphi \left(C + a \frac{\partial C}{\partial a} \right) = 0, \quad (15)$$

$$(\mu q^2 - 1) \left(2\varphi A + Ba + a\varphi \frac{\partial A}{\partial a} + \varphi^2 \frac{\partial A}{\partial \varphi} + \varphi a \frac{\partial B}{\partial \varphi} \right) + \frac{2}{3} \omega(\varphi) a^2 \frac{\partial B}{\partial a} + \mu q a \varphi \left(2C + \varphi \frac{\partial C}{\partial \varphi} \right) = 0, \quad (16)$$

$$(\mu q^2 - 1) \left(\varphi \frac{\partial A}{\partial q} + a \frac{\partial B}{\partial q} \right) + 2\mu q A \varphi + 2\mu q a B + \mu a \varphi \left(C + q \frac{\partial A}{\partial a} + q \frac{\partial C}{\partial q} \right) = 0, \quad (17)$$

$$\mu q \varphi^2 \frac{\partial A}{\partial \varphi} + \frac{2}{3} \omega(\varphi) a \frac{\partial B}{\partial q} + \varphi (\mu q^2 - 1) \frac{\partial A}{\partial q} = 0 \quad (18)$$

and

$$\frac{\partial A}{\partial q} = 0. \quad (19)$$

Putting (19) into (18) implies

$$3\mu q \varphi^2 \frac{\partial A}{\partial \varphi} + 2\omega(\varphi) a \frac{\partial B}{\partial q} = 0. \quad (20)$$

By choosing $A = A_0 a^n \varphi^m$, $B = B_0(q) a^l \varphi^s$ and substituting them into Eq. (20), we get

$$B_0(q) = -\frac{3}{4}\mu \frac{mA_0}{\omega_0}q^2 + k_1, \tag{21}$$

$$\omega(\varphi) = \omega_0\varphi^{m-s+1} \tag{22}$$

where A_0 , ω_0 and k_1 are constant. By substituting this results into (14) we get

$$\omega_0 = m = 1 \quad \text{and} \quad s = 2. \tag{23}$$

Taking into account Eqs. (21), (22) and (23), we get the solutions

$$\omega(\varphi) = 1, \tag{24}$$

$$A = A_0a^n\varphi, \tag{25}$$

$$B = \left(-\frac{3}{4}\mu A_0q^2 + k_1\right)a^{n-1}\varphi^2. \tag{26}$$

An important remark is in order at this point. In the case $\mu = 0$, such solutions are ruled out, if φ is massless, by gravity tests on Solar System. This is not true for $\mu \neq 0$. In this case, the previously mentioned tests strongly constrain the allowed masses for φ and therefore the parameters in the potential $V(\varphi)$. For a detailed discussion on the effective scalar field mass constrained by Solar System tests see [27–29].

In view of these solutions, Eqs. (15), (16) and (17) read

$$\left[\left(\frac{7}{2} + \frac{2k_1}{A_0} + n\right) - 3\mu q^2\right]a^n\varphi^2\mu A_0q + \mu a\varphi\left(C + q\frac{\partial C}{\partial q}\right) = 0,$$

$$(\mu q^2 - 1)A_0a^n\varphi^2\left[\left(3 + n + \frac{k_1}{A_0}\right) - \frac{9}{4}\mu q^2\right] + \frac{(1-n)}{2}\mu q^2 A_0a^n\varphi^2 + a\varphi\mu q\left(2C + \varphi\frac{\partial C}{\partial \varphi}\right) = 0$$

and

$$(\mu q^2 - 1)A_0a^n\varphi^2\left[\left(1 + 2n + \frac{2k_1}{A_0}\right) - \frac{3}{2}n\mu q^2\right] + 2\mu q\varphi\left(C + a\frac{\partial C}{\partial a}\right) = 0.$$

These equations are satisfied if

$$q^2 = q_0^2 = \frac{1}{\mu}, \tag{27}$$

$$C = f_0a^{n-1}\varphi, \tag{28}$$

$$f_0 = \frac{\beta_0}{q_0} + \frac{A_0q_0}{4}\left(3\mu q_0^2 - 7 - 2n - \frac{4k_1}{A_0}\right),$$

$$G = \frac{2n - \frac{3}{n} + \frac{2k_1}{A_0}\left(1 - \frac{3}{n}\right)}{n - \frac{3}{n} + 1 + \frac{2k_1}{A_0}\left(1 - \frac{3}{n}\right)},$$

$$\beta_0 = \frac{A_0}{2\mu} \frac{2n(n^2 + n - 1) - 5 + \frac{k_1}{A_0}(6n^2 - 4n - 16) + \frac{k_1^2}{A_0^2}(4n - 12)}{n^2 + n - 3 + \frac{2k_1}{A_0}(n - 2)}$$

where G , f_0 , q_0 and β_0 are constant. In conclusion, the Noether symmetry for the Lagrangian (6) exists and the vector field X is determined by (25), (26) and (27) and (28) while the functional form of $\omega(\varphi)$ is given by (24).

It is straightforward to obtain a general self-interaction potential from Eq. (13) as

$$V(\varphi) = \lambda\varphi^2 + k_2\varphi^{\frac{1}{\Lambda_2}} \tag{29}$$

where we have used the definitions

$$k_2 = \frac{1}{12}\left(9A_0\mu q_0^2 - 6A_0 + \frac{12\mu f_0}{1 - \mu q_0^2}\right), \quad \lambda = \frac{\Lambda_1}{1 - 2\Lambda_2},$$

$$\Lambda_1 = \left(2q_0 - 3q_0^3 + \frac{2f_0}{3A_0}\right)\mu, \quad \Lambda_2 = \frac{1}{4}\mu q_0^2 - \frac{k_1}{3A_0}. \tag{30}$$

The existence of the Noether symmetry means that there exists a constant of motion. In this case, the conserved quantity corresponding to the Noether symmetry can be obtained using the Cartan one-form associated with the Lagrangian (6), that is

$$\theta_{\mathcal{L}} = \frac{\partial \mathcal{L}}{\partial \dot{a}}da + \frac{\partial \mathcal{L}}{\partial \dot{\varphi}}d\varphi + \frac{\partial \mathcal{L}}{\partial \dot{q}}dq.$$

By contracting $\theta_{\mathcal{L}}$ with X one obtains the following required constant of motion

$$F_0 = i_X\theta_{\mathcal{L}} = A_0\varphi a^n((\mu q^2 - 1)(12a^2\varphi\dot{\varphi} + 12\varphi^2a\dot{a} + 72\mu a^2\varphi^2q\dot{q}))$$

$$+ 12f_0a^{n+1}\varphi^3\mu\dot{a}q + a^{n-1}\varphi^2\left(k_1 - \frac{3}{4}\mu A_0q_0^2\right)(12(\mu q^2 - 1)a^2\dot{a}\varphi + 8a^3\omega(\varphi)\dot{\varphi}). \tag{31}$$

3. The cosmological solution

Starting from (9) and (27), it is straightforward to get the following general solution for the scale factor

$$a(t) = \sqrt{6kq_0 + \alpha_1 \exp\left(\frac{t}{\sqrt{3q_0}}\right) + \alpha_2 \exp\left(\frac{-t}{\sqrt{3q_0}}\right)}$$

where α_1 and α_2 are arbitrary constants. In special case, by choosing $k = \alpha_2 = 0$ and $\alpha_1 = a_0^2$, this solution takes the standard de Sitter form

$$a(t) = a_0 \exp(\alpha t) \quad (32)$$

where $\alpha = \frac{1}{2\sqrt{3q_0}}$. Clearly this is a singularity free solution evolving as an hyperbolic cosine. It shows accelerated phases for $t \rightarrow \pm\infty$ so both inflationary and dark energy behaviors are easily achieved.

Some interesting sub-cases can be obtained considering the field potential (29). For $k_2 = 0$, it takes the form $V(\varphi) = \lambda\varphi^2$. In addition, one can use the constant of motion (31) and the scale factor (32) to find a solution for $\varphi(t)$. To this purpose, we rewrite (31) as

$$F_0 = a^{n+2} \left(A_0(\mu q_0^2 - 1)(12\varphi^2 \dot{\varphi} + 12\varphi^3 \alpha) + 12f_0\varphi^3 \mu \alpha q_0 + \varphi^2 \left(k_1 - \frac{3}{4} \mu A_0 q_0^2 \right) (12(\mu q_0^2 - 1)\alpha\varphi + 8\dot{\varphi}) \right) \quad (33)$$

and then

$$\varphi(t) = \varphi_0 \exp(-\vartheta_0 t) \quad (34)$$

with

$$\vartheta_0 = \frac{(n+2)\alpha}{3}, \quad \varphi_0 = \left(\frac{F_0}{u_0} \right)^{\frac{1}{3}}$$

and

$$u_0 = a_0^{n+2} \left(12A_0(\mu q_0^2 - 1)(12\vartheta_0 + 12\alpha) + 12f_0\mu\alpha q_0 + \left(k_1 - \frac{3}{4} \mu q_0^2 A_0 \right) (12\alpha(\mu q_0^2 - 1) + 8\vartheta_0) \right).$$

It must be stressed that these results have been obtained due to the existence of the Noether symmetry, and one can easily check that these solutions are consistent with the corresponding field equations. In this case ($k_2 = 0$), solutions (32) and (34) satisfy Eqs. (7) and (8) which now assume the forms

$$2\varphi H^2(\mu q_0^2 - 1) + \frac{2}{3}\ddot{\varphi} - \frac{1}{3}\mu q_0 \varphi + 2\dot{\varphi}H + \frac{\lambda}{6}\varphi = 0,$$

$$(\mu q_0^2 - 1) \left(2\varphi \dot{\varphi}H + \frac{3}{2}H^2\varphi^2 + \frac{d(\varphi\dot{\varphi})}{dt} \right) + \frac{1}{4}\lambda\varphi^2 - \dot{\varphi}^2 = 0,$$

respectively, with the following definitions of the constants

$$q_0^2 = \frac{1}{\mu} \frac{348n + 108n^2 + 16n^3 + 473}{(4n + 4n^2 + 73)(13 + 2n)}, \quad \lambda = \frac{583n + 327n^2 + 16n^3 - 4n^4 + 1508}{27q_0(4n + 4n^2 + 73)}. \quad (35)$$

Clearly n is a free parameter depending on the constant of motion.

Another interesting case is for $\lambda = 0$ and $A_2 = \frac{1}{4}$. The self-interaction potential takes the form $V(\varphi) = k_2\varphi^4$. As it is well known, this potential is widely used in the discussion of vibrations of polyatomic molecules [25] and it is widely used in chaotic inflationary models also if, strictly speaking, the $V(\varphi) \sim \varphi^2$ potential is the prototype of chaotic inflationary potentials [22,31,32]. However, we have to say that quartic φ^4 potentials are almost ruled out by current observations (see, for instance, [33]).

In this case, the general solution of the equations of motion is achieved numerically while particular solutions have a power-law form as discussed in [34] for non-minimally coupled theories without the R^{-1} correction.

4. Concluding remarks

We have explored the conditions for the existence of a Noether symmetry in a scalar–tensor theory of gravity, with an extra R^{-1} term, in which the coupling function and the generic potential are unknown. The motivation for this study is that we want to construct cosmological models capable of achieving inflationary and dark energy phases. To this goal, we need two fields leading the two eras which, in our case, are φ and $q = R^{-1}$.

We have shown that the existence of the symmetry fixes the coupling and the self-interacting potential which have physically interesting forms. Furthermore, it allows to achieve exact cosmological solutions which are singularity free and suitable to mimic inflationary and dark energy behaviors.

A more physically appealing model should consider the role of standard perfect fluid matter and should fit also the dust dominated phase [23], but in this cases the Noether symmetry cannot always be achieved.

However, also if we have considered a phenomenological model, the important lesson of this research is that, as shown also in other contexts [22,24], the Noether symmetry is a powerful approach to select physically motivated solutions.

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