Evaluation of the mathematical and economic basis for conversion processes in the LEAP energy-economy model

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An evaluation was made of the mathematical and economic basis for conversion processes in the LEAP energy-economy model. Conversion processes are the main modelling subunit in LEAP used to represent energy conversion industries and are supposedly based on the classical economic theory of the firm. The study arose out of questions about the uniqueness and existence of LEAP solutions and their relation to classical equilibrium economic theory. An analysis of classical theory and LEAP model equations was made to determine their exact relationship. The conclusions drawn from this analysis were that LEAP theory is not consistent with the classical theory of the firm. Specifically, the capacity for factor formalism used by LEAP does not support a classical interpretation in terms of a technological production function for energy conversion processes. The economic implications of this inconsistency are suboptimal process operation and short term negative profits in years where plant operation should be terminated. A new capacity factor formalism, which retains the behavioural features of the original model, is proposed to resolve these discrepancies.

Key words: mathematical model, conversion processes, energy economy

Introduction

The present study grew out of an investigation of the uniqueness and existence of solutions to the underlying equations of the LEAP energy modelling system.[1–3] In the previous work, the investigation of uniqueness was based on the behaviour of the supply curves arising in the generic LEAP processes for conversion and allocation of energy products. The particular proof of uniqueness for solutions to the conversion process equations required certain conditions to be met related to profitability in process plant operation. Although these conditions are usually met in the classical economic theory of the firm, the behaviour of certain solutions to large LEAP modelling problems[4,5] sheds some doubt on the mathematical and economic basis of the LEAP conversion process equations. This study was therefore undertaken to improved understanding of the connection between LEAP conversion process modelling assumptions and the classical economic theory of the firm.

Although other theoretical discussions of the economic basis for the equations in LEAP have already been published[6,7] it is not at all clear that the actual modelling assumptions implemented in the GEMS[1] or LEAP-EMS[2] codes are consistent with this theory. This confusion makes it difficult to understand the results of some major LEAP results[4,5] and affects the proofs of existence and uniqueness of LEAP model solutions.[3] Since the conversion process is central to all large scale models, this problem clearly needs resolution.

To illustrate the theoretical problems encountered in the LEAP conversion process, the basis for this process module will first be developed along classical lines. This entails reviewing the classical economic theory of the firm in the context of generalized equilibrium modelling. The central conclusions concerning plant profitability and operation under classical assumptions will be analysed in some detail. This theoretical foundation will be developed. The particular manner in which this theory is applied in the LEAP code

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will then be discussed. The possible weaknesses of this latter approach, compared with classical theory, will then be examined and the economic consequences of these deficiencies in solving the LEAP model equations will be explored. A modified theory retaining both the essence of the practical LEAP approach and the conditions needed to meet classical constraints is offered. Conclusions will be drawn about the usefulness of the current version of LEAP and the potential for its improvement with implementation of the proposed changes.

Economic basis of conversion process equations

Classical theory of the firm

The basis for the LEAP conversion process equations has been discussed at great length.[1,2,6–8] The model used is intended to represent an optimized aggregate production unit consistent with the classical theory of the firm.[9] Since details of this theory can be found elsewhere,[6,7,9] it is only appropriate here to review this theory briefly to establish notation and understand the optimal conditions which must exist in the equilibrium market.

The LEAP code system consists of a series of generic processes linked together to form a network of interacting economic activities. The LEAP conversion process, as it might be imbedded in such a network, appears in *Figure 1*. This module takes a vector of input quantities $\boldsymbol{Q}_I$, at prices $\boldsymbol{w}$, per unit of input and builds plant capacity N_w, sufficient to convert the inputs into a vector quantity of final products Q, at prices $\boldsymbol{p}$, per unit of output in order to meet a specified demand.

In the LEAP model the problem solved by the basic conversion process can be stated in terms of the following optimization problem for the behaviour of the process firm. Given the unit input prices of the factors of production $\boldsymbol{w}$, the final product prices $\boldsymbol{p}$, and the unit cost of capital N_c, the conversion process in LEAP attempts to model an optimally operated firm which seeks to find the quantities of input factors $\boldsymbol{Q}_I$, and the plant capacity per unit final product N_w, which maximizes long-term profit π, over the life of the plant.

For the sake of notational simplicity, the discussion will deal with the specific case of an energy conversion process plant with a two-year lifetime. The plant will be considered to have been built in year 1 and operated in years 1 and 2. The profitability of the plant can be written for this case as follows:

$$\begin{aligned}\pi = {} & p_1 Q_1 - w_1 Q_{I,1} - \sum_j w_{1,j} Q_{I,1,j} - N_c N_w \\ & + p_2 Q_2 - w_2 Q_{I,2} - \sum_j w_{2,j} Q_{I,2,j}\end{aligned} \quad (1)$$

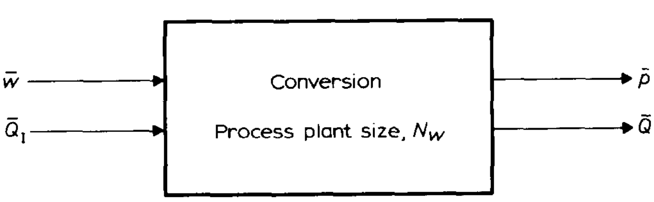

Figure 1 LEAP conversion process module

In this notation the subscripts 1 and 2 refer to the year of plant operation and j denotes factors of production other than the primary energy source. The first four terms on the right-hand side of equation (1), represent the plant profit in the first year of operation and the next three terms represent the profit for the second year. The form of the equation is such that all capital investments are made in the initial year of operation and only the prices and quantities of energy input and output linked to other LEAP process modules are considred in the optimization process. Thus, the two summation terms over j represent the costs of factors of production other than energy and are specified exogeneously to the process module. Also, π is considered to be the net present value of profit in LEAP with a discount factor for capital being specified exogeneously. For simplicity in this case, the discount factor was assumed to be unity (i.e. a zero discount rate) and therefore does not appear explicitly in the equation.

In order to formulate a classical profit optimization problem from the profit definition given in equation (1), the conversion process must obey a technology or production constraint. This constraint determines the maximum output which the conversion technology can produce from a given quantity of the factors of production fed into the process. It therefore represents a relationship between input and output for the particular technology of production. For all LEAP conversion processes, this technology production constraint is assumed to be one with constant returns-to-scale.[6] The definition of a constant returns-to-scale production function in the ith year is, for this case, is given by the following:

$$f_i(Q_i, Q_{I,i}, N_w) = f_i\left(\frac{Q_i}{N_w}, \frac{Q_{I,i}}{N_w}, 1\right) = 0 \quad (2)$$

This is, the relationship between input and output in any given year is independent of plant size N_w; it depends solely on the technological constraints relating unit input to unit output.

Now define the following parameters and variables:

$$\chi_i = Q_i / N_w \quad (3a)$$

$$y_i = Q_{I,i}/N_w \equiv g_i(\chi_i) \quad (3b)$$

$$\phi_i = \sum_j w_{i,j} Q_{I,i,j} / Q_i \quad (3c)$$

where χ_i is the output per unit plant size, y_i is the input per unit plant size, $g_i(\chi_i)$ is the functional form relating input to maximum output (i.e. in this form it is the inverse production function) and ϕ_i is the constant operating cost per unit output for the input factors of production other than energy, all in the ith year. In terms of these new variables, the classical profit maximization problem for a firm with a constant returns-to-scale production function can be written as follows:

$$\underset{Q,\, Q_I,\, N_w}{\text{Maximize}} \quad \pi = N_w[(p_1 - \phi_1)\chi_1 - w_1 y_1 - N_c + (p_2 - \phi_2)\chi_2 - w_2 y_2] \quad (4)$$

subject to the constraints:

$$f_1(\chi_1, y_1) = 0 = y_1 - g_1(\chi_1) \quad (5a)$$

$$f_2(\chi_2, y_2) = 0 = y_2 - g_2(\chi_2) \quad (5b)$$

Eliminating $\boldsymbol{Q}_I$ from consideration by using the constraint relationships (5a) and (5b) directly, equation (4) can be written in terms of χ and N_w in its simplest form as:

$$\underset{\chi, N_w}{\text{Maximize}} \quad \pi = N_w[(p_1-\phi_1)\chi_1 - w_1 g_1(\chi_1) - N_c + (p_2-\phi_2)\chi_2 - w_2 g_2(\chi_2)] \quad (6)$$

The conditions under which a local extrenum exists for equation (6) are the following:

$$\frac{\partial\pi}{\partial\chi_1}=0 \quad \frac{\partial\pi}{\partial\chi_2}=0 \quad \frac{\partial\pi}{\partial N_w}=0 \quad (7)$$

The equations for extremum solutions can therefore be derived by differentiating equation (6) with respect to the independent variables χ_1, χ_2 and N_w. This results in the following equations for the extremum solutions χ_1^*, χ_2^* and N_w^*:

$$\frac{\partial\pi}{\partial\chi_1} = 0 = (p_1-\phi_1) - w_1 g_1'(\chi_1^*) \quad (8)$$

$$\frac{\partial\pi}{\partial\chi_2} = 0 = (p_2-\phi_2) - w_2 g_2'(\chi_2^*) \quad (9)$$

$$\frac{\partial\pi}{\partial N_w} = 0 = (p_1-\phi_1)\chi_1^* - w_1 g_1(\chi_1^*) - N_c + (p_2-\phi_2)\chi_2^* - w_2 g_2(\chi_2^*) \quad (10)$$

where $g_i'(\chi_i) \equiv dg_i/d\chi_i$.

Rewriting these equations substituting the relationships given in equations (8) and (9) in (10), the final conditions for optimal production can be written as follows:

$$g_1'(\chi_1^*) = \frac{p_1-\phi_1}{w_1} \quad (11)$$

$$g'(\chi^*) = \frac{p_2-\phi_2}{w_2} \quad (12)$$

$$w_1[g_1'(\chi_1^*)\chi_1^* - g_1(\chi_1^*)] + w_2[g_2'(\chi_2^*)\chi_2^* - g_2(\chi_2^*)] = N_c \quad (13)$$

The three equations here are similar to the classic Kuhn-Tucker conditions[10] for operation of the firm at maximal profit. The first two conditions (i.e. equations (11) and (12)) are the short run optimal profit conditions of the prices being equal to the marginal cost of production. The third condition (i.e. equation (13)) represents the long-range profit optimum of zero profit in firms with production functions displaying constant returns-to-scale.

It is important to note here that the present value of profit (i.e. π in equation (6)) is a linear function of plant size N_w as a result of the production function having constant returns-to-scale. In this case, therefore, no real extremum condition represented by the partial derivative $\partial\pi/\partial N_w$ exists. Equation (13) simply represents a relationship which must exist between the prices $\boldsymbol{p}$ and $\boldsymbol{w}$, so that any plant size yields an optimum. Prices are therefore constant with respect to plant size and N_w can be eliminated when considering optimal plant operation.

If all the prices are given (i.e. $\boldsymbol{p}$ and $\boldsymbol{w}$ are known), then equations (11)-(13) represent three equations in the two unknowns χ_1^* and χ_2^*. All the prices $\boldsymbol{p}$ and $\boldsymbol{w}$ cannot therefore be independent. The optimality conditions allow only three of the four prices p_1, p_2, w_1 or w_2 to be specified independently in order to solve for the fourth price and χ_1^* and χ_2^*. Note also that the unknowns y_1^* and y_2^* can be obtained directly from χ_1^* and χ_2^* once the functional form of the inverse production functions $g_1(\chi)$ and $g_2(\chi)$ are given.

A closer look at equations (11)-(13) reveals that these are very general conditions for an extremum in the profit function. In order for a profit maximum (as opposed to a minimum) to exist and be unique and for profits to be positive to offset the plant capitalization N_c, certain additional constraints must be placed on the behaviour of the inverse production function $g(\chi)$. It is clear from equation (13) that the profits in each individual year are governed by the terms $g'(\chi^*)\chi^* - g(\chi^*)$ and for yearly positive profits to exist under all circumstances it is clear that:

$$g'(\chi^*)\chi^* > g(\chi^*) \quad \text{for all } \chi^* > 0 \quad (14)$$

In addition, equations (11) and (12) require $g'(\chi^*) > 0$ for yearly positive profits to exist. For a profit maximum to occur the second derivatives of the production function must be such that:

$$g_1'(\chi_1^*) > 0 \quad \text{and} \quad g_2'(\chi_2^*) > 0 \quad \text{for all } \chi^* > 0 \quad (15)$$

The inverse production function must therefore be a monotically increasing function of χ and $g'(\chi)$ must take on all values from 0 to ∞ in the region of optimum output χ^* in order for a positive profit maximum to occur. If these conditions are not satisfied, the possibilities exist for multiple extrema, negative profits, and profit minimization.

When all three extremal conditions are satisfied (i.e. equations (11)-(13)), together with the yearly positive profit maximization conditions given by equations (14) and (15), the resulting maximization problem can be displayed graphically as shown in *Figure 2*. The solution to this set of equations is unique and requires only that the input prices w_1 and w_2 together with one additional variable (either p_1 or p_2 or p_1/p_2) be specified to solve for the remaining price and χ_1^*, χ_2^*, y_1^* and y_2^*. The case displayed also exhibits the desirable property of decreasing returns-to-scale for output as a function of input. That is, it takes increasingly more units of input to produce a unit of output as the quantity of output increases.

LEAP form of classical theory of firm

To put the optimality conditions of the classical theory of the firm into the framework of the LEAP code, equations

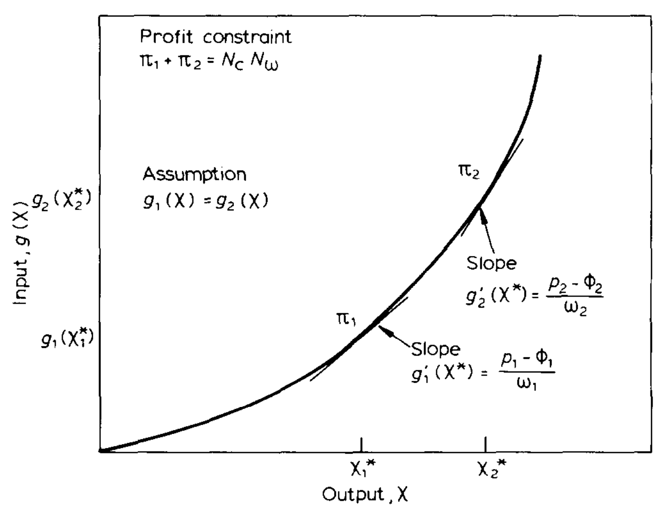

Figure 2 Generic behaviour of equilibrium solutions to conversion process equations for classical inverse production $g(x)$

(11)-(15) must first be expressed as a single first year equation for plant operation. Since all future years are assumed to satisfy a positive profit constraint and a short run optimization condition of price equals marginal cost, the first year equations equivalent to equations (11)-(13) are the following (dropping first year subscripts),

$$(p-\phi)\,\chi^* - wg(\chi^*) = k \tag{16}$$

$$g'(\chi^*) = \frac{p-\phi}{w} \tag{17}$$

where:

$$k \equiv N_c - [(p_2-\phi_2)\,\chi_2^* - w_2 g_2(\chi_2^*)] \tag{18}$$

In LEAP the above equations are simplified even further by defining the following variables:

$$\hat{\phi} \equiv \phi + \frac{wg(\chi^*)}{\chi^*} \qquad \text{average operating cost} \tag{19}$$

$$C_f \equiv \frac{Q}{N_w} = \chi^* \qquad \text{average capacity factor} \tag{20}$$

where $\hat{\phi}$ is the average constant operating cost for non-energy factors of production (i.e. labour, materials, etc.) and C_f is the plant capacity factor. The final form of the LEAP equations for the first year of operation are, therefore:

$$(p-\hat{\phi})\,C_f = k \tag{21}$$

$$g'(\chi^*) = \frac{p-\phi}{w} \tag{22}$$

where:

$$\chi^* = C_f \qquad y^* = g(\chi^*) \tag{23}$$

These equations are now in the form in which the authors of LEAP interpreted the classical theory of the firm for use in conversion process modelling.[1,11]

For the sake of further discussions, it should be noted that equations (21) and (22) constitute two equations in two unknowns (i.e. χ^* and p) which can be solved once w, k and ϕ are known. Given such a solution, equation (23) can then be used to calculate y^*, while N_w can be obtained from the condition of constant price with respect to plant size in meeting the final demand Q. The procedure for determining k given w_2 is the only complication in this approach. This, however, is handled by a separate model for future year prices beyond the range of years with which the LEAP model deals directly, called the terminal value model.[1,11]

Although considerable documentation on the relationship between the classical theory of the firm and LEAP conversion process equations existed before the current study began, the theoretical foundation of the actual code equations was never made explicit. The major reason for the difficulty in making this connection lies in the manner in which LEAP defines and uses the capacity factor C_f. In most classical theory applications, the basis for any particular model has traditionally been the choice of the functional form for the conversion process production function. Several familiar classical forms used in modelling production are, for instance, the Cobb-Douglas, CES, or Leontief models.[9] Given such functional forms, the defining equations for either marginal or average capacity factors can easily be derived. This approach, however, was not taken in the case of LEAP. Instead a functional form for the capacity factor itself was defined, thereby implying an underlying functional form for the basic production function. This choice has profound affects on the results produced by this model. The rationale for this choice together with its economic and mathematical implications for LEAP are discussed more fully in the next section.

Implementation of classical theory of the firm in LEAP

The major divergence of LEAP modelling from other classical approaches is the choice of a functional form for the capacity factor C_f. In LEAP, the authors attempted to build a model which had a certain behavioural relationship not present in other models. In particular, a capacity factor was sought which was bounded by some maximum operation level and one which varied as a function of prices and operating costs so as to increase capacity when profitability was high and decrease capacity when profitability was low. To this end a behavioural relationship for the capacity factor of the following form was chosen:[11]

$$C_f \equiv \frac{\beta}{1+\left(\dfrac{\alpha\hat{\phi}}{p}\right)^{\delta}} \tag{24}$$

where β is the maximum fraction of plant capacity which can be used under optimum conditions (usually $0 \leqslant \beta \leqslant 1$), p and $\hat{\phi}$ are the price of final product and average constant operating cost for the plant, and α and δ are behavioural parameters which determine how sensitive the capacity factor is to the ratio $\hat{\phi}/p$.

As can be seen from equation (24), the capacity factor is a fractional quantity (a fraction of the plant total capacity) which is strictly positive and has an upper bound of β (i.e. $0 \leqslant C_f \leqslant \beta$). When prices are much higher than average operating costs and plant profitability is high (i.e. $p \gg \hat{\phi}$), then $C_f \to \beta$ and the plant operates at maximum capacity. Likewise when $p \ll \hat{\phi}$, the plant shuts down as $C_f \to 0$. This model also has the flexibility, through the δ parameter, to approach a pure Leontief input-output model[9] in the limit as $\delta \to \infty$. This feature is an important one for comparing LEAP results to those derived from the more traditional approach of choosing a production function to model the conversion process.

Looking more closely at the form of C_f given in equation (24), it becomes clear that a conventional analysis of the LEAP conversion process is possible if the capacity factor form chosen implies a conventional underlying production function (i.e. if the inverse production function $g(\chi)$ can be derived from equation (24)). The procedure for uncovering the inverse production function is to couple equation (24) to the short run optimality condition for $g'(\chi^*)$ (i.e. equation (22)) to form a differential equation for $g(\chi)$. It should be noted, however, that in order for $g(\chi)$ to be a classical inverse production function it should only involve a relationship between output, input and any parameters which describe the technology of the conversion process itself (i.e. no costs or prices should be involved).

In order to uncover the inverse production function implied by equation (24), this equation must first be rewritten in terms of χ^* and $g(\chi^*)$ for use in setting up a differential equation for $g(\chi)$. Noting the definitions of $\hat{\phi}$ from equation (19) and C_f from equation (20), the capacity factor equation can be rewritten as follows:

$$\chi^* = \frac{\beta}{1 + \left\{\dfrac{\alpha\left[\phi + \dfrac{wg(\chi^*)}{\chi^*}\right]}{p}\right\}^{\delta}} \tag{25}$$

This equation is trancendental in χ^* but can easily be solved for $g(\chi^*)$ to give:

$$g(\chi^*) = \chi^*\left\{\frac{p}{w}\frac{1}{\alpha}\left(\frac{\beta - \chi^*}{\chi^*}\right)^{1/\delta} - \frac{\phi}{w}\right\} \tag{26}$$

If several new variables are now defined as follows:

$$\eta \equiv \frac{1}{\delta} \qquad r \equiv \frac{p}{w} \qquad s \equiv \frac{\phi}{w} \tag{27}$$

Equations (26) and (22) can be finally written as:

$$g(\chi^*) = \chi^*\left[\frac{r}{\alpha}\left(\frac{\beta - \chi^*}{\chi^*}\right)^{\eta} - s\right] \tag{28}$$

$$g'(\chi^*) = r - s \tag{29}$$

From these two equations it is clear that the single equation defining the capacity factor (i.e. equation (20)) is not sufficient to uniquely define a classical production function which is not a function of prices or operating costs. If $g(\chi^*)$ were a function of $r - s$ (i.e. $g(\chi^*) = g(r - s, \chi^*, \text{parameters})$), then equations (28) and (29) would be sufficient, since $g'(\chi^*)$ in equation (29) could be substituted into (28) generating a differential equation in $g(\chi^*)$, χ^*, and the parameters α and η which could be solved for $g(\chi)$. The functional form of equation (28), however, is not sufficient for this procedure and another independent equation in r and s is needed to uniquely define a production function. This additional equation together with equation (29) could then be used to solve for r and s in terms of $g'(\chi^*)$, $g(\chi^*)$ and χ^*, which could then be substituted into equation (28) to obtain the necessary differential equation to solve for $g(\chi)$.

The LEAP conversion process does have a third equation associated with it to complete the set of equations needed to solve for the three unknowns p, χ^* and y^* given w. The complete set of equations are as follows:

$$(p - \hat{\phi})\, C_f = k \tag{30}$$

$$\chi^* \equiv C_f = \frac{\beta}{1 + \left(\dfrac{\alpha\hat{\phi}}{p}\right)^{\delta}} \tag{31}$$

$$y^* = g(\chi^*) = C\chi^* \tag{32}$$

where equation (32) is the new defining equation for output y^* in terms of a fixed input-output coefficient C. Thus, LEAP uses a long term zero profit condition, equation (30), a capacity factor definition, equation (31), and a fixed input–output relationship, (32), to form a complete set of equations to solve for prices and quantities in conversion processes.

The two key questions which immediately arise are: what is the relationship between the fixed input/output equation and the equation defining the capacity factor, and are both of these consistent with the short run optimality condition given in equation (29)? To see these relationships more clearly (31), (29) and (32) are rewritten in terms of r and s as follows:

$$g(\chi^*) = \chi^*\left[\frac{r}{\alpha}\left(\frac{\beta - \chi^*}{\chi^*}\right)^{\eta} - s\right] \tag{33}$$

$$g'(\chi^*) = r - s \tag{34}$$

$$g(\chi^*) = C\chi^* \tag{35}$$

It should be obvious that equation (35) is not a function of r and s and therefore does not satisfy the criteria needed for a third independent equation in r and s to complete the set of equations defining a differential equation for the production function $g(\chi)$. Since the parameter C is an exogeneous constant, $g(\chi)$ can be inferred directly from equation (35) to be a linear inverse production function without the need for any differential equation. This, however, is totally inconsistent with any short run optimality condition given by equation (34).

The inescapable conclusion to be drawn from this analysis is that since LEAP uses equations (30)–(32), no short run optimization condition is in use in the LEAP conversion process. Furthermore, since short run optimization is unavailable, no production function within the framework of classic economic theory is consistent with the defining equation for the capacity factor. Neither observation has ever been clearly explained in the documentation of the LEAP code. On the contrary, all material published to date strongly implies that the equations in the conversion process are consistent with classical economical theory. The implications of these inconsistencies are of great practical importance and are explained more fully in the next section.

Economic implications of use of LEAP equations

The most immediate consequence of the choice of equations (30)–(32) for use in LEAP is the fact that the short run optimality condition (i.e. equation (34)) is not consistent with the three conversion process equations. This clearly implies that LEAP can operate plants at sub-optimal production levels in the short run. Also, strict positive profit constraints no longer apply to the short run behaviour of LEAP solutions. Since short run optimality conditions lie at the heart of many theoretical proofs of existence and uniqueness in equilibrium economics, the failure of LEAP to meet these conditions can give rise to economic behaviour which is not usually found in solutions to equilibrium modelling problems.

The implications of the use of the LEAP conversion process equations (31) and (32) are best explored graphically. *Figure 3* shows the generic behaviour of these two relationships for an arbitrary set of exogeneous parameters. Note that while the $g_2(\chi^*)$ curve (representing the capacity factor equation) changes in magnitude as a function of prices p and w (i.e. r and s), the general shape of the curve remains the same. Since $g_2(\chi^*)$ represents the locus of all solutions to the conversion process equations as $g_1(\chi^*)$ traces out all values of the input–output coefficients C, its shape gives it the property of increasing returns-to-scale up to the peak of the curve and the somewhat unsatisfactory behaviour of decreasing to negative returns-to-scale past the peak. Note also the range of profits on the $g_2(\chi^*)$ curve vary from maximum losses at $\chi^* = 0$ to maximum profits at $\chi^* = \beta$ with zero profit somewhere in between near the peak of the curve.

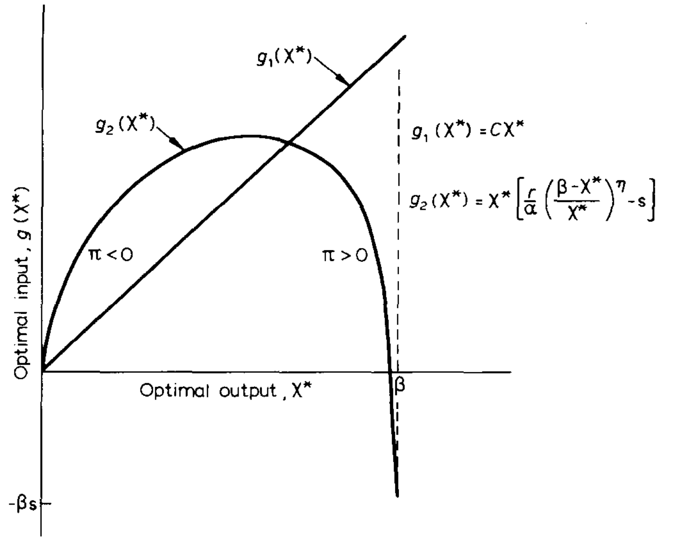

Figure 3 Generic behaviour of equilibrium solution of conversion process equations for LEAP production function $g_2(\chi)$ and input-output function $g_1(\chi)$

From this figure it is clear that for some values of the parameter C, LEAP admits losses in the operation of a plant in its startup year. This in general will be the case when the parameter C is large and the slope of $g_1(\chi^*)$ is steep. Since equation (30) is also used by LEAP, long run profit will still be zero because short run losses will be made up by profits in future years. This behaviour is not typically that predicted by simple classical theories, although it is more realistic in some instances. In the case of LEAP, however, this situation arises from poorly conceived economics rather than more realistic marketplace behaviour. Plants are operated at a short term loss in LEAP because of sub-optimal behaviour rather than the optimal balancing of future profits and short term losses.

While the modelling of sub-optimal short run losses early in plant life is in some sense a tolerable practical feature of the LEAP equations, the reverse situation is equally possible and this latter situation is most unsatisfactory as far as the economic theory of the firm is concerned. In the most widely used models constructed by the LEAP system,[4] this latter effect is clearly seen. High positive profits early in plant life are generated, eventually to be offset by losses in all future years of plant operation.[4,5] Such behaviour is clearly economically unsound, since the plant should be shut down immediately upon incurring the first future year loss so as to avoid further losses. It is difficult to imagine an economic theory which would justify running plants at losses for the remainder of their useful lifetime without the possibility of further profitability. The distortion caused in equilibrium price solutions as a result of such plant operation clearly needs further investigation if LEAP results are to be useful in energy-economy modelling.

Economic consistency within LEAP framework

Behaviour of exogenous parameters

The major conclusions of the last section are that, (1), the LEAP conversion process equations yield sub-optimal solutions in which short term losses are possible, and (2), no classical production function defining the technology of plant operation is implied by the form of the LEAP equations. Looking at equations (30)-(32), however, it is clear that under certain circumstances the LEAP model can be made consistent with classical theory. In particular, the limit as the parameter $\delta \to \infty$ (or $\eta \to 0$) is one in which the capacity factor approaches the discontinuous behaviour characteristic of a Leontief or input-output production function of classical economics.[9] In this limiting case, when $p < \hat{\phi}$ the capacity factor is zero, and when $p > \hat{\phi}$ it is equal to its maximum, β. If, in addition, the parameter $\alpha = 1$, then both equations (33) and (34) become consistent since $g(\chi^*)$ can be made a function of $r - s$. This fact allows the short run optimality condition (i.e. equation (34)) to again become an operative constraint in the LEAP formalism (i.e. equations (30)-(32)), restoring classical optimal behaviour to LEAP solutions. This latter fact prevents short term losses from occurring and maintains both short and long run optimality and short term positive profits. In many important economic sectors of large scale modelling problems like Model 22C[4] these conditions are closely approximated and the solutions behave much like those arising from a classical Leontief model of the firm.

Alternative formulation of capacity factor

To avoid the difficulties present in the current formulation of the LEAP conversion process equations, an alternate approach can be taken. This alternative preserves the positive behavioural features of the original LEAP capacity factor formulation while at the same time unifying the model with classical economic theory. In this light, an appropriate behavioural relationship defining the capacity factor will be chosen which is consistent with the short run optimality condition of classical equilibrium economics. Once this is accomplished a classical production function can be derived for this particular form of the capacity factor and a completely consistent classical set of equations for optimal short run production can be obtained.

The following functional form, closely akin to the original LEAP formalism, is suggested for the behaviour of the capacity factor:

$$C_f \equiv \frac{\beta}{1 + \dfrac{wg(\chi^*)/\chi^*}{\delta\left[p - \hat{\phi} - \dfrac{\alpha w g(\chi^*)}{\chi^*}\right]}} \tag{36}$$

where β, δ and α have the same definitions as parameters that they had in the original capacity factor equation, except that α is assumed to be a very small positive number (i.e. $\alpha \cong 0$) for reasons explained later. It should be noted that this capacity factor definition has the same behavioural characteristics as the original one. Namely, for $p \gg \hat{\phi}$ the capacity factor approaches its maximum (i.e. $C_f \to \beta$) and when $p \to \hat{\phi}$ (with $\alpha \cong 0$) the plant shuts down (i.e. $C_f \to 0$). Also, when $\delta \to \infty$ the capacity factor approximates a Leontief model as was observed earlier.

Rewriting equation (36) in terms of the variables r and s the differences between the new capacity factor and the old one became more obvious:

$$C_f \equiv \frac{\beta}{1 + \dfrac{1}{\delta\left[\dfrac{\chi^*(r - s)}{g(\chi^*)} - (1 + \alpha)\right]}} \tag{37}$$

From equation (37) it is clear that the major difference between the new and old form is the functional dependence on r and s. Equations (37) is a simple function of $r - s$. This simple behaviour makes it possible to couple equation (37) with the short run optimality condition given in equation (34) to derive a unique production function for LEAP which is consistent with classical economic theory.

The two equations of interest here are:

$$g'(\chi^*) = r - s \tag{38}$$

$$C_f = \chi^* \equiv \frac{\beta}{1 + \dfrac{1}{\delta\left[\dfrac{\chi^*(r-s)}{g(\chi^*)} - (1+\alpha)\right]}} \tag{39}$$

which when coupled together yield the following differential equation for the underlying inverse production function $g(\chi)$:

$$g'(\chi) = \frac{g(\chi)}{\chi}\left[\eta\frac{\chi}{\beta - \chi} + (1+\alpha)\right] \tag{40}$$

Note here that, as before $\eta \equiv 1/\delta$.

Solving this equation for $g(\chi)$ with the initial condition $g(0) = 0$ gives the following inverse production function:

$$g(\chi) = \frac{\chi^{1+\alpha}}{(\beta - \chi)^\eta} \tag{41}$$

The behaviour of this function is such that it takes on all values from $0 \leqslant g(\chi) < \infty$ when χ varies between 0 and β. It is also easily shown that the slope of this function varies between $0 \leqslant g'(\chi) < \infty$ when χ varies between 0 and β for $\alpha > 0$. The production function, defining output χ^* as a function of input, y^* therefore has decreasing returns-to-scale over the whole range of equilibrium output values $0 < \chi^* < \beta$. This latter property allows both short- and long-term profit optimization conditions to be satisfied at all operating price levels, thereby assuring the consistency of the new model with positive optimal profit conditions and classical economic theory.

Using the results given in equation (41), the final alternative LEAP conversion process equations are the following:

$$(p - \hat{\phi})\, C_f = k \qquad \text{long-run zero profit} \tag{42}$$

$$C_f = \frac{\beta}{1 + \dfrac{1}{\delta\left[\dfrac{\chi^*(r-s)}{g^*(\chi)} - (1+\alpha)\right]}} \qquad \text{capacity factor} \tag{43}$$

$$g(\chi^*) = \frac{(\chi^*)^{1+\alpha}}{(\beta - \chi^*)^\eta} \qquad \text{inverse production function at optimum} \tag{44}$$

$$g'(\chi^*) = r - s \qquad \text{short run profit optimum} \tag{45}$$

The major differences between the equations here and those currently used in LEAP are not behavioural in nature but the fact that equations (44) and (45) are consistent with classical economic theory.

Conclusions

The major finding of this study is that the particular functional form chosen for the capacity factor in LEAP is not consistent with classical economic theory. This fact leads to sub-optimal production levels and the possibility of plant operation at negative profits. The key element missing in the LEAP conversion process is an underlying classical production function for the conversion technology. A valid production function would allow the LEAP equations to satisfy both short and long run optimal profit conditions which would preclude negative profits.

Under certain conditions related to the limiting values of several parameters, however, the LEAP conversion process can be made to approximate a Leontief model of the firm. In this limit all optimality conditions are restored and losses do not occur. Unfortunately the Leontief model is restrictive and reduces the capabilities of LEAP to model more complex behavioural processes.

The easiest way to remedy the problems in the code is to slightly modify the behavioural relationship for the capacity factor. An alternative was presented in this paper by choosing a similar but functionally different form for the capacity factor and then deriving the underlying production function consistent with classical economic theory. The form suggested is close in behavioural characteristics to the original LEAP capacity factor definition, but satisfies all the optimality conditions in both the short and long run. Its implementation in LEAP, although time consuming, should not be a major chore.

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