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Development of a Hub and Spoke Model for Bus Transit Route Network Design

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Abstract

For a large city bus transit service, a point to point route network, attempting to connect each node to every other node results in large number of routes in the network which makes it practically very complex to understand and operate the services. To overcome these limitations a different transit network design approach is required. A combination of traditional destination oriented routes along with direction oriented routes, which is called as Hub and Spoke network could be better for operating bus transit in large network. Previous research works done using hub-and-spoke model mainly concentrates on airline network, where the influencing factors and variables for network design could be substantially different than transit network. This paper presents an approach to develop hub and spoke route network for bus transit services. Whole process consists of obtaining potential hubs, finding optimum location of hubs and allocation of non-hub nodes to hub nodes, generating inter hub and intra-hub routes and the frequencies for running buses. The model developed in this study is tested on the Mandl's Swiss network of fifteen nodes. The Mandl's network is most widely used by many researchers to analyze their models developed for point to point bus transit route network. The results suggest noticeable savings in operator cost along with slight increase in user cost.

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Keywords: Urban transportation; Bus transit; Routes; Hub and spoke network; Optimization; Genetic algorithm.

1. Introduction

Public transport system forms an indispensable part of urban transport because of its major share in the social and economic growth of the area. A great emphasis should be given to the public transport system, as it can support future travel demands in urbanized areas in an efficient way. There is a widespread reliance on buses for public

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transport as they play an essential role in reducing urban travel congestion. Designing the routes of a transport network in an optimal way calls for attention in view of achieving user satisfaction as well as catering to the interests of service providers. So transport route network design problem may be stated as the determination of a set of routes and associated frequencies, subject to a set of feasibility constraints, to achieve the desired objectives. Usually the objective will be to reduce the overall system costs: both user and operator costs. User costs are often measured by the total time incurred to the users, which consists of waiting time, in vehicle time, transfer time. Operator costs often depend on the fleet size, transport vehicle size, transport vehicle km and vehicle operation hours required for a route configuration. Feasibility constraints may include, but are not limited to, maximum route length, maximum and minimum load factor, the fleet size, maximum allowable headway and available resources including capital and operating costs.

2. Hub and Spoke Transportation network

A transport network may be represented as a set of links and a set of nodes. A link connects two nodes and a node connects two or more links. Links may be either directed, in which case they specify the direction of movement, or undirected. A point to point network is a route network where in network focuses mainly on its origin-destination traffic. Due to the heavy demand in a large city, the destination oriented design methodology results in large number of very long transit bus routes. These zigzag routes, connecting each node to every other node, overlap on certain corridors resulting in bunching on various sections of the network. This bunching leads to high concentration of buses, thereby causing irregular distribution of headways on the stops of the route. This type of transit network in a large city is very complex to understand, inefficient and also difficult to operate. To minimize some of the major limitations encountered in the destination oriented bus routes, the large networks in a city area need to be designed on a different approach. A combination of traditional destination oriented along with direction-oriented approach, called Hub and Spoke network, is best suited for such type of large networks. A hub and spoke network aggregates multiple origin flows at a single hub node where the high volume aggregated flows travels from one hub to another via a hub-hub link, all hub nodes are assumed to be inter connected. Arriving at the second hub node the flow then gets split up and sorted to flow to each of its respective destination nodes. The routing through hubs takes advantage of economies of scale on inter hub links. Although this may increase travel distance or time, it decreases travel cost. Economies of Scale, economies of Scope, network coverage, seamless connection for passenger travel, reduced fleet size are the advantages of hub and spoke network.

3. Previous studies on hub and spoke network

O'Kelly (1987) was the first to introduce hub and spoke mathematical model, which was named as the single allocation p-hub median problem. He introduced a quadratic programming model of hub location-allocation. This problem is sufficiently difficult to solve. An upper bound on the objective function can be found by complete enumeration of the locational configurations with the assumption that each node is allocated to its nearest hub. A few years later O'Kelly (1992) introduced the single allocation hub location problem, which is similar to the p-hub median problem, but differing by accounting for some given fixed costs of setting up the hubs at specific nodes. This additional cost was added to the objective function, resulting in a solution suggesting the number of hubs to set up instead of using the predetermined p number of hubs. Campbell (1992) was the first to introduce the use of multiple allocations to the p-hub median problem and a couple of years later Campbell (1994) proposed two additional sets of problems, namely, the p-hub center problem and the hub covering problem. All these problems are NP hard. Heuristic methods are developed by many researchers. Ernst and Krishnamoorthy (1996) have developed a Simulated Annealing (SA) algorithm for the p-hub median problem. They were able to solve problem of size $n=200$ and $p=15$ in significantly less computing time. Problems of practical size are, unfortunately, larger than this size. Abdinnour-Helm and Venkataramanan (1998) proposed the first Genetic

Algorithm (GA) for HLP. The genetic search has proven to be robust in solving small as well as large (200nodes) problems.

Most of these methods mainly concentrate on airline network where in non-hub nodes are allocated to hub node resulting in spokes. Each spoke contains only two nodes, a node and a hub, and no intermediate nodes. But in case of bus transit road network each spoke (path), most often, consists of many nodes. Therefore it is required to take care of this aspect in to account. As on date very limited work has been done on hub and spoke network for bus transit operation. Parti R., Marwah B.R., Kalra P.K (2002) developed hub and spoke network for bus transit operation by using heuristic method. In this study an optimization model is used to develop hub and spoke network for bus transit operation. And genetic algorithm is used to solve the problem.

4. Proposed Model

The objective of the proposed model is to find a set of optimal inter hub routes and feeder routes while minimizing user and operator cost subject to maximum load factor to take care of user interest, minimum load factor to take care of operator interest, and fleet size available. Developing hub and spoke model for bus transit route network involves identifying potential hubs, finding optimal hub location and allocation of non-hub nodes to hub node, generating optimal inter hub routes, generating optimal feeder routes for each of hub area.

4.1 Identifying potential hubs

In this study uniform data envelopment analysis (DEA) method (Charnes et al, 1978) is used to identify potential hubs. In this method efficiencies of systems, called decision making unit (DMU) which have inputs and outputs, are compared. In this study all the nodes in the network are treated as DMU. Each input and output is associated with a common weight. The ratio of the weighted sum of output to the weighted sum of input is called the efficiency of the DMU. The score of each DMU is obtained, with a set of common weights, by maximizing the efficiency of ideal DMU and subjecting the efficiencies of other DMUs less than or equal to 1. The mathematical model is as follows.

$$\text{Max } AO \div BI \quad (1)$$

Subject to

$$a_j O \div b_j I \leq 1 \quad \forall j = 1, \dots, n \text{ DMUs} \quad (2)$$

$$AO \div BI \leq 1 \quad (3)$$

$$O > 0, I > 0$$

Where, O and I are common weights for output and input, A and B are output and input values of the ideal DMU, a_j and b_j are output and input values of DMU j . Any number of inputs and outputs can be considered for a DEA model. In this study hub establishment cost is taken as input to be minimized and demand at node is taken as output to be maximized. Here the nonlinear objective function is converted to linear by subjecting its denominator to equal to 1. Solving this linear programming we will get a set of common weights O and I . By applying these common weights on the efficiency equations of each DMU, their efficiencies are calculated. Now arranging these scores in descending order we can select topmost DMUs i.e. nodes having high efficiency as potential hubs.

4.2 Optimum location of hubs and allocation of nodes

Among potential hubs selected by DEA model, p hubs are to be selected as hubs and non-hub nodes are allocated to these p hubs. Objective of optimum location allocation model is to minimize the total demand weighted distance. Mathematical model is as follows.

$$\text{Min } \sum_{i=1}^n \sum_{j=1}^P Q_i d_{ij} a_{ij} \tag{8}$$

Subject to

$$\sum_{j=1}^P a_{jj} = P \tag{5}$$

$$\sum_{j=1}^P a_{ij} = 1 \quad \forall i = 1, \dots, n \tag{6}$$

Where, n =total number of nodes, p =number of hubs to be established, Q_i =demand at node i , d_{ij} = distance from node i to hub j , a_{ij} =1 if node i is allocated to hub j , and 0 otherwise. Constraint (5) ensures that only p numbers of hub are selected, and constraint (6) ensures that each node is allocated to only one hub.

4.2.1 Solution

Select m number of nodes from DEA efficiency scores. Among m nodes, p nodes are to be selected as hubs. There are ${}^m C_p$ combinations of hub nodes. Out of these one optimal combination is to be selected. This search space is sufficiently large for a large network. Genetic algorithm (GA) which is most suitable for combinatorial optimization problem is used to find optimal combination of hub nodes and allocation of non-hub nodes to hub. For details of GA reader can refer to any standard book on GA.

4.2.2 GA Coding

Most important part of GA is coding of solution vector. Most commonly used method of representing the solution vector is binary system i.e. by using 0_s and 1_s . In this problem variables are positions of p hub nodes in the array m consisting of potential hubs. Therefore the length of the string depends on the size of the array m and the value of p , i.e. string length = bits required to represent the size of array $m \times p$. Upper bound for hub position in array m =size of array m , lower bound=1.

11th element of array m				5th element of array m					
1	0	1	1	0	1	0	1		
1				2			 p	

Fig1. GA coding for optimal location-allocation problem

As shown in Fig.1. 11th and 5th nodes in array m are selected as 1st and 2nd hub nodes, and so on till p hubs.

4.2.3 Fitness function

GA is suitable for maximization problem as they work on the principle of survival of fittest. Minimization problem can be converted to maximization problem. In this problem, fitness is taken as $1 / (1 + \text{objective function value of location-allocation problem})$. Codes for GA are developed in MATLAB-R2010a.

4.3 Generating Inter-Hub Routes

Every hub has to be connected to every other hub through inter hub routes. There will be a total of $p(p-1)$ number of inter hub routes, where p is the total number of hubs. In this study we generated one way routes, instead of two ways, to consider one way links in the network. Following steps are involved in developing inter hub routes.

Step1. Identifying inputs required for the model, such as number of nodes in the network and links connecting these nodes along with their travel time, demand matrix for the network, design parameters like bus seating capacity, maximum and minimum load factors, and fleet size.

Step2. Find the inter hub demand between all pair of hubs.

$$\text{Inter hub demand from } hub_x \text{ to } hub_y = \sum_i \sum_j q_{ij} \quad \forall x, y = 1, \dots, P, \quad i \in hub_x, j \in hub_y$$

Step3. K -shortest paths are generated between each pair of hubs

Step4. Load on each path is obtained using multipath traffic assignment.

Step5. Mathematical formulation of the problem

$$\text{Min } c_1 \sum_k \sum_i \sum_j q_{ij}^k t_{ij}^k + c_2 \sum_k \sum_i \sum_j q_{ij}^k t_{wtij}^k + c_3 \sum_k f_k T_k \quad (7)$$

Subject to

$$\{q_{\max}^k \div f_k CAP\} \leq LF_{\max} \quad \forall K \in SR \quad (8)$$

$$\{q_{\max}^k \div f_k CAP\} \geq LF_{\min} \quad \forall K \in SR \quad (9)$$

$$\sum_k f_k T_k = W \quad (10)$$

Where, q_{ij} =demand from node i to node j , t_{ij} =in vehicle travel time from node i to node j , t_{wtij} =waiting time for the demand from node i to node j , T_k =bus operation time for route k , CAP =capacity of bus operating on the route, SR =set of routes, C_1 =invehicle travel time cost per minute, C_2 =waiting time cost per minute, C_3 =bus operation cost per minute, f_k =frequency on route k , LF_k =load factor on route k , q_{\max}^k =maximum link flow on route k , w =fleet size. Objective of this model is to minimize both user and operator cost, subject to maximum (8) and minimum load factors (9), fleet size available (10).

Step6. Solve the problem to find optimal route combination along with their frequency. This is a complex, multi objective combinatorial optimization problem. Therefore genetic algorithm (GA), most suitable for combinatorial optimization problem, is used to solve this problem.

4.3.1 GA coding

In this problem the task is to find the best route combination along with frequency which gives optimal objective function value. Hence the variables are path number and frequency of a route.

Hub pair # 1							Hub pair # 2							Hub pair # 3						p(p-1)																					
Route		Frequency					Route		Frequency					Route		Frequency																										
1	0	0	1	0	0	1	1	0	1	1	1	0	0	0	1	1	1	0	1	0	1	0	1																			
Path = 2							Frequency = 19							Path = 1							Frequency = 49							Path = 3							Frequency = 21							

Fig2. GA coding for inter hub route problem

$$\text{Decimal value of path} = LB_p + \left\{ \text{decoded value} (UB_p - LB_p) \div (2^{L_p} - 1) \right\}$$

$$\text{Decimal value of frequency} = LB_f + \left\{ \text{decoded value} (UB_f - LB_f) \div (2^{L_f} - 1) \right\}$$

Where, LB_p, LB_f, UB_p, UB_f are lower bound and upper bound for path number and frequency respectively, L_p and L_f are string length for path and frequency respectively.

4.4 Generating Feeder Routes

Feeder routes are generated for each hub area. Generation of feeder route, considering one hub at a time, involves the following steps.

- Step1. Identify terminal nodes in the hub area based on the minimum length of feeder route.
- Step2. Find feeder demand from each terminal to hub and vice versa.

$$(Feeder\ demand)_{i,hub} = \sum_j q_{ij} \forall i \in \text{set of nodes allocated to hub}, j \notin \text{hub}$$

$$(Feeder\ demand)_{hub,i} = \sum_j q_{ji} \forall i \in \text{set of nodes allocated to hub}, j \notin \text{hub}$$

- Step3. Generate K -shortest paths between each terminal node and hub.
- Step4. Follow the steps involved in developing inter hub routes.
- Step5. Repeat above steps for all other hubs.

This completes the development of hub and spoke bus route network.

5. Application of the Proposed Model

Developed hub and spoke model is applied on Mandl’s network. This is a benchmark network (Mandl 1980) used by many researchers to test their bus route model, for destination oriented network, and compare with others result. This network consists of 15 nodes and 21 links as shown in Fig.3. having total demand of 15570 person trips per hour. The same network is used by Mandl (1980), Baaj et al (1990), Gundaliya (2000). The results of the proposed model are compared with the previous researchers.

Table 1. Mandl's Network demand matrix

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
0	0	400	200	60	80	150	75	75	30	160	30	25	35	0	0
1	400	0	50	120	20	180	90	90	15	130	20	10	10	5	0
2	200	50	0	40	60	180	90	90	15	45	20	10	10	5	0
3	60	120	40	0	50	100	50	50	15	240	40	25	10	5	0
4	80	20	60	50	0	50	25	25	10	120	20	15	5	0	0
5	150	180	180	100	50	0	100	100	30	880	60	15	15	10	0
6	75	90	90	50	25	100	0	50	15	440	35	10	10	5	0
7	75	90	90	50	25	100	50	0	15	440	35	10	10	5	0
8	30	15	15	15	10	30	15	15	0	140	20	5	0	0	0
9	160	130	45	240	120	880	440	440	140	0	600	250	500	200	0
10	30	20	20	40	20	60	35	35	20	600	0	75	95	15	0
11	25	10	10	25	15	15	10	10	5	250	75	0	70	0	0
12	35	10	10	10	5	15	10	10	0	500	95	70	0	45	0
13	0	5	5	5	0	10	5	5	0	200	15	0	45	0	0
14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

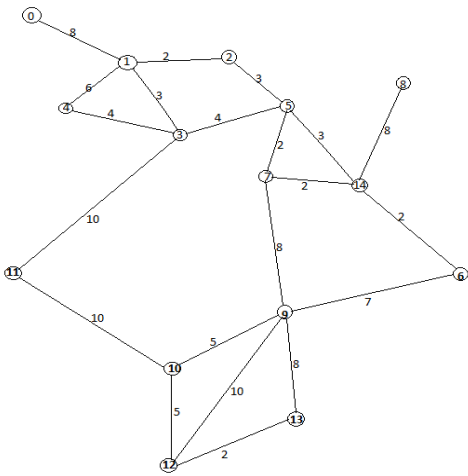


Fig 3. Mandl's Network

5.1 Potential hubs of Mandl network

In this study it is assumed that, for simplicity, hub establishment cost at each node as 10000units. Ideal DMU output $A=4154$ person trips, and ideal DMU input $B=10000$ units. Solving DEA model we obtained common weight for output $O=0.000241255$ and common weight for input $I=0.0001$. Using these common weights we selected nodes 9 5 0 and 1 as potential hubs which scored higher efficiency sorted in descending order.

5.2 Optimal Location-Allocation

From the four potential hubs obtained by DEA model, two hubs are to be selected and non-hub nodes are allocated to one of these two. GA converged within 12 generations with objective function value 65855, selecting nodes 9 and 5 as hubs.

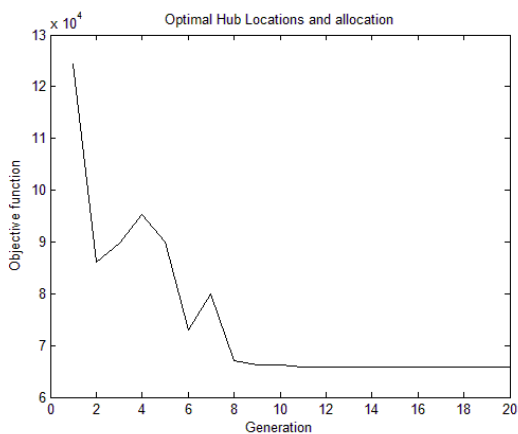


Fig4. Objective function v/s generations

Table 2. Allocation of nodes to hub

Nodes	Hubs	
	9	5
0	0	1
1	0	1
2	0	1
3	0	1
4	0	1
5	0	1
6	0	1
7	0	1
8	0	1
9	1	0
10	1	0
11	0	1
12	1	0
13	1	0
14	1	0

5.3 Inter-Hub Routes and Feeder Routes

In this study for violation of any constraint, square of the constraint multiplied by appropriate penalty parameter is added to the objective function. Cost of in-vehicle travel time is taken as Rs 0.22 per minute, cost of waiting time is taken as Rs 0.13 per minute, route operation cost of bus is taken as Rs 6.8 per minute, bus capacity as 50 (Gundaliya P.J 2000). Nodes which are 5 minutes away from hub are considered as terminals for feeder routes.

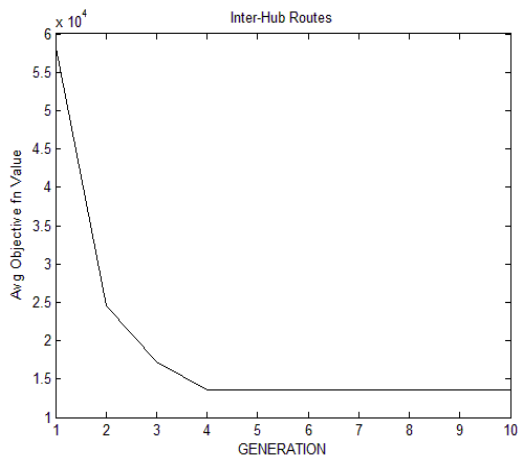


Fig5. Objective function v/s generation

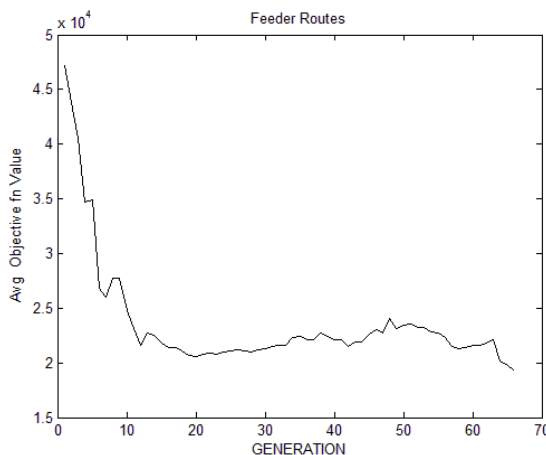


Fig6. Objective function v/s generation

GA for inter hub routes, having 2 pair of nodes, converged within 5 generation. And for feeder routes of hub node5, having 12 pair of nodes, converged within 70 generation. Following table3 gives the list of routes developed by hub and spoke network model.

Table.3. list of routes developed by hub and spoke model

Node pair	Route	Frequency	Buses
Pair 9-5	5-7-9	42	7
Pair 5-9	9-7-5	42	7
Pair 0-5	0-1-2-5	15	3
Pair 1-5	1-3-5	39	4
Pair 4-5	4-1-2-5	14	3
Pair 6-5	6-14-5	35	3
Pair 8-5	8-14-5	13	2
Pair 11-5	11-3-5	13	3
Pair 5-0	5-2-1-0	14	3
Pair 5-1	5-2-1	16	2
Pair 5-4	5-2-1-4	18	3

Node pair	Route	Frequency	Buses
Pair 5-6	5-14-6	28	2
Pair 5-8	5-7-14-8	28	5
Pair 5-11	5-3-11	12	4
Pair 10-9	10-9	19	2
Pair 12-9	12-13-9	21	4
Pair 13-9	13-9	16	2
Pair 14-9	14-6-9	13	2
Pair 9-10	9-10	55	5
Pair 9-12	9-12	14	2
Pair 9-13	9-13	13	2
Pair 9-14	9-6-14	49	7

6. Results and Discussion

Hub and spoke bus route network developed in this study was run by varying the various parameters like number of potential hubs, number of shortest paths, minimum length of feeder routes and observed the values of demand satisfied with different level of transfers, in vehicle travel time, transfer time, number of buses required. And the results obtained are compared with those of other researchers.

Table.4. Comparison of results

Set Of Routes	% Demand Satisfied through transfer				In Vehicle travel time	Transfer time	Waiting time	Total Travel time	Number of Buses
	Zero	One	Two	Three					
Mandl I	68.78	31.22	0	0	211210	24330	14672	250212	128
Mandl II	69.85	29.97	0.17	0	177752	23590	17598	218940	111
Baaj I	78.3	21.69	0	0	169138	16970	20007	206115	103
Baaj II	79.65	20.35	0	0	166461	15915	26675	209051	93
Baaj III	80.88	19.12	0	0	180620	14910	22156	217686	100
Mandl II	72.95	26.91	0.13	0	177006	21070	21070	216469	114
Baaj I	77.92	19.62	2.3	0	166167	18690	18690	205220	110
Baaj II	85.67	14.32	0	0	161540	11150	11940	199000	92
Baaj III	85.54	14.45	0	0	162680	11250	12430	199911	84
P J Gundaliya I	93.38	6.62	0	0	160521	5150	18154	183825	75
P J Gundaliya II	89.92	10.08	0	0	173394	7850	16446	197690	88
P J Gundaliya III	88.89	11.11	0	0	173616	8650	21049	203315	88
PROPOSED MODEL-I	52.41	36.13	9.22	2.25	196295	12703	18594	227592	70
PROPOSED MODEL-II	56.90	33.01	8.64	1.45	196020	9892	18907	224819	72
PROPOSED MODEL-III	53.18	36.54	8.61	1.67	191870	12240	18051	222161	73
PROPOSED MODEL-IV	58.57	36.22	5.20	0.00	179160	6744	17173	203077	94
PROPOSED MODEL-V	61.79	33.33	4.88	0.00	173985	8331	19638	201954	76

From the comparison it is observed that the hub and spoke bus route network developed in this study results in reduction of 12 vehicles i.e. 13.63% of buses required for destination oriented route network, with an increase in travel time by 2.13%. It shows that even with additional transfers, hub and spoke route network, travel time is not affected considerably. Required numbers of vehicles reduced mainly because of inter hub routes carrying dense flow with optimum load factor.

7. Summary and Conclusions

This paper deals with development of hub and spoke model for bus transit route network. Because of bunching of routes and difficulty in operation of destination oriented bus route network, hub and spoke network can be used for large city bus route operation taking the advantage of economy of scale for dense network. Review of past studies on hub and spoke network shows that it is focused only on airline and communication network, where the influencing factors and variables for network design could be substantially different than transit network. Therefore an attempt has been made, in this study, to develop hub and spoke model for bus route design.

Development of hub and spoke model involves identifying potential hubs, finding optimal location of hubs- allocation of nodes to hubs, generation of inter hub routes, generation of feeder routes. DEA model is used to

identify potential hubs. Location-allocation, inter hub route and feeder hub route problems are multi objective, combinatorial problems, hence Genetic algorithm is used to solve these three problems. Developed hub and spoke model is applied on Mandl's network and compared the results with destination oriented route network models developed by others.

The following conclusions can be made from this study.

- From the comparison of results it is found that the hub and spoke network model results in reduction of fleet by 13.64% satisfying same demand with not so much variation in total travel time.
- As we expected, because of transfer through hubs, demand satisfied through zero transfer is very less as compared to other models.
- Reduction in fleet size will reduce the fuel consumption and hence reduction in CO₂ emission which contributes a major part for global warming.
- Considering the reduced fleet size with very little increase in travel time together with potential reduction in CO₂, hub and spoke network, most suitable for large city bus route network operation, can be treated as a step towards sustainable bus transport system.

8. Future Scope of this Study

As a future extension of this work, the authors intend to apply the proposed model on the network of Bangalore city, where a large number of Traffic and Transit Management Centers (TTMCs) have been build and operated in recent past, which can serve as potential hubs.

Further study is also required on capacity of the hub, hub and spoke model for bus and metro rail integrated transportation system. Study can also be carried out to assess the impact of hub and spoke route network on transportation demand.

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