A novel two-stage extended Kalman filter algorithm for reaction flywheels fault estimation

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Abstract This paper investigates the problem of two-stage extended Kalman filter (TSEKF)-based fault estimation for reaction flywheels in satellite attitude control systems (ACSs). Firstly, based on the separate-bias principle, a satellite ACSs with actuator fault is transformed into an augmented nonlinear discrete stochastic model; then, a novel TSEKF is suggested such that it can simultaneously estimate satellite attitude information and actuator faults no matter they are additive or multiplicative; finally, the proposed approach is respectively applied to estimating bias faults and loss of effectiveness for reaction flywheels in satellite ACSs, and simulation results demonstrate the effectiveness of the proposed fault estimation approach.

1. Introduction

The separate-bias estimation algorithm is used to estimate the state and constant bias of linear systems. The basic principle of this algorithm, which is also called two-stage Kalman filter (TSKF), is to estimate system states and constant biases separately, then obtain the optimal estimate using the coupling relationship between them. In 1969, Friedland firstly proposed the separate-bias estimation algorithm\cite{1} and made a further investigation\cite{2}. During the past four decades, many research achievements have been reported on this algorithm. At present, the main research on this topic is concerned with estimation of constant/time-varying bias and all kinds of engineering applications.

Recently, many scholars have paid considerable attention to separate-bias estimation algorithm for linear systems\cite{4,5}. Keller and Darouach\cite{4} suggested an optimal solution of the TSKF, which can be used to estimate optimal state and optimal random bias, and further a two-stage optimal strategy was developed for discrete-time stochastic linear systems subject to intermittent unknown inputs.\cite{5} Khabbazii and Esfanjani\cite{5} proposed a constrained TSKF for tracking control problem in an uncertain linear system and its main advantage is the improvement of estimation accuracy.

For fault/bias estimation of nonlinear systems, much progress has been made on EKF-based approaches. EKF-based sub-optimal algorithm was proposed in Ref.\cite{6}. Zhou et al. investigated a pseudo separate-bias estimation algorithm for nonlinear time-varying stochastic systems.\cite{7,8} In Ref.\cite{9}, fuzzy

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Kalman filter-based approach was proposed for nonlinear stochastic discrete time systems.

Based on the general TSKF, Hsieh\textsuperscript{11} proposed a general two-stage extended Kalman filter (GTSEKF)-based constant parameter estimation approach. In Ref.\textsuperscript{12}, an adaptive TSEKF like Ref.\textsuperscript{11} was proposed to estimate the closed-loop position and speed of sensorless control for permanent magnet synchronous motor. Kim\textsuperscript{13} proposed an adaptive TSEKF for INS-GPS loosely coupled systems and its main advantage was it cost less computation time due to the introduction of the forgetting factor. In Ref.\textsuperscript{14}, an adaptive TSEKF algorithm based on Ref.\textsuperscript{13} was introduced to the application of geomagnetic aided inertial navigation filtering. By introducing strong tracking multiple fading factors and embedding EKF into an optimal TSKF, a novel robust filter-based bearings-only maneuvering target tracking problem was investigated, which can provide an optimal estimation of the target state and the unknown statistical parameters of virtual noises.\textsuperscript{15}

The faults of actuators and sensors in control systems can be represented as biases via separate-bias estimation algorithm. Fault estimation for dynamic systems has attracted considerable attention during the past two decades. When estimating the additive actuator/sensor faults, the algorithm can be implemented easily since the biases representing faults in these models have specific physical meanings and the principles are clear.\textsuperscript{16} When estimating the multiplicative faults in actuators, it is necessary to use other parameters to represent the fault models, such as control effectiveness factors, which can be used to indicate the fault degree of control systems. By introducing forgetting factors into the optimal TSKF in Ref.\textsuperscript{5}, an adaptive TSKF was exploited to estimate the abrupt reduction of control effectiveness in dynamic systems by Wu et al.\textsuperscript{17} This algorithm is applied for the identification of impairment in its control surfaces in an aircraft model. In Refs.\textsuperscript{18–22}, a further investigation on this algorithm was made and widely applied it on fault diagnosis and fault-tolerant control. In Ref.\textsuperscript{23}, control effectiveness factor estimation was extended to the estimation of control distribution matrix elements, and the TSKF was applied to actuator/surface fault diagnosis and fault-tolerant control of F-16. In recent years, this method has been applied to the fault diagnosis and fault-tolerant control of sensors and actuators in satellite attitude control system.\textsuperscript{16,24–26} The value of the effectiveness factors can be derived via this method and system fault degree can be analyzed to obtain biases for the fault-tolerant control purpose. In addition, to the best of our knowledge, separate-bias principle never considers additive faults and multiplicative faults simultaneously.

In view of this, this paper investigates TSEKF-based fault estimation for reaction flywheels in satellites. A novel TSEKF is suggested such that it can simultaneously estimate satellite attitude information and reaction flywheel faults no matter they are additive or multiplicative. It is respectively applied to estimating bias faults and loss of effectiveness for reaction flywheels in a satellite ACSs, and the simulation results demonstrate the effectiveness of the proposed fault estimation approach.

2. System fault model

Consider the following nonlinear discrete-time stochastic system with bias vector of unknown magnitude $b_k \in \mathbb{R}^p$:

$$
\begin{align*}
    x_{k+1} &= f_k(x_k, b_k) + w^z_k, \\
    y_k &= h_k(x_k, b_k) + v_k
\end{align*}
$$

where $x_k \in \mathbb{R}^n$ is the system state; $y_k \in \mathbb{R}^m$ is the measurement vector; the noise sequence $w^z_k, w^x_k$ and $v_k$ are zero-mean uncorrelated Gaussian random sequences with $E\left[ w^z_k \right] = 0$ and $E\left[ w^x_k \right] = 0$.

The states of system Eq. (1) appears in the following form:

$$
\begin{align*}
    x_{k+1} &= A_k x_k + F^b_k b_k + w^z_k + M_k, \\
    y_k &= C_k x_k + F^h_k b_k + v_k + N_k
\end{align*}
$$

where $A_k \in \mathbb{R}^{nxn}$ and $C_k \in \mathbb{R}^{mxn}$ are state transition matrix and observation matrix, respectively, and we have

$$
\begin{align*}
    A_k &= \left. \frac{\partial f}{\partial x} \right|_{x=x_k} F^b_k = \left. \frac{\partial f}{\partial b} \right|_{x=x_k} \\
    b_k &= \left. \frac{\partial h}{\partial x} \right|_{x=x_k} F^h_k = \left. \frac{\partial h}{\partial b} \right|_{x=x_k}
\end{align*}
$$

The vector function $w = [\omega_x, \omega_y, \omega_z]^T$ represents the inertially referenced satellite angular rate vector of the satellite body relative to the inertial coordinate system and the corresponding Euler angles are $\phi, \theta$ and $\psi$. Define $\Phi(x) = \begin{bmatrix} \cos \theta & \cos \phi \sin \theta & \sin \phi \sin \theta & -\sin \phi \cos \theta & -\sin \phi \cos \theta \\ \sin \phi \sin \theta & \cos \phi & \sin \phi \cos \theta \\ -\sin \theta & 0 & \cos \theta \end{bmatrix}$, the state equation of satellite attitude control system based on the satellite attitude dynamics equation can be given as

$$
\dot{x} = g(x) + Bu(x)
$$

where

$$
\begin{align*}
    g(x) &= \left[ I^{-1}(-\omega \times I, \omega) \right] \\
    \Phi(x) &= \frac{1}{\cos \phi} \begin{bmatrix} \cos \phi & \cos \theta & \sin \phi \sin \theta & -\sin \phi \cos \theta & -\sin \phi \cos \theta \\ \sin \phi \sin \theta & \cos \phi & \sin \phi \cos \theta \\ -\sin \theta & 0 & \cos \theta \end{bmatrix} \\
    B &= \left[ I^{-1} \right] \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}
\end{align*}
$$

$u(x) \in \mathbb{R}^l$ is the known control input vector; $B \in \mathbb{R}^{m \times l}$ is the control input matrix; $I_k$ is the moment of inertial matrix of the satellite.
Observation equation is

\[ y = Cx \]  

(8)

where \( C = I_n \) is the output matrix, and \( I_n \) is an \( n \times n \) identity matrix.

Discretize the combination of Eqs. (6) and (8), then the system model without fault is established like Eq. (1).

(1) Additive fault model

The bias \( b_k \) represents additive faults of actuators, and the system model with additive faults of actuators can be described as

\[
\begin{align*}
\dot{x}_{k+1} &= g_k + B_k (u_k + b_k) + w_k, \\
y_k &= C_k x_k + v_k
\end{align*}
\]

(9)

where \( g_k, B_k, u_k \) and \( C_k \) are the corresponding discrete system matrices of Eqs. (6) and (8), the corresponding matrix of Eq. (1) is

\[
A_k = \left[ \frac{\partial g_k}{\partial x} \right]_{x=x_k} + \left[ \frac{\partial B_k u_k}{\partial x} \right]_{x=x_k}, \quad F_k = B_k F_k = 0
\]

(10)

(2) Multiplicative fault model

Here, \( b_k \) represents the degree of the multiplicative faults of actuators, and the bias \( A_k b_k \) represents the multiplicative faults of actuators, in which diagonal matrix \( A_k = \text{diag}(b_k) \). The system model with multiplicative faults of actuators can be described as

\[
\begin{align*}
\dot{x}_{k+1} &= g_k + B_k (I - A_k) u_k + w_k, \\
y_k &= C_k x_k + v_k
\end{align*}
\]

(11)

the corresponding matrix of Eq. (1) is

\[
\begin{align*}
A_k &= \left[ \frac{\partial g_k}{\partial x} \right]_{x=x_k} + \left[ \frac{\partial B_k (I - A_k) u_k}{\partial x} \right]_{x=x_k} \\
F_k &= - B_k \text{diag}(u_k), \quad F_k = 0
\end{align*}
\]

(12)

In Eq. (11), the vector function \( b_k \) is also called the control effectiveness factors, representing the possible loss of control effectiveness in the model,\(^{21-23}\) whose value is 0 in the absence of multiplicative faults.

The objective of this paper is to design a TSEKF such that it can estimate actuator faults no matter they are additive or multiplicative. That is, it can give a solution for system Eq. (1) with the unknown bias and system states, a solution for additive fault system Eq. (9) with additive faults and system states, and also a solution for multiplicative fault system Eq. (11) with multiplicative faults and system states.

3. TSEKF-based actuator fault estimation approach

In this section, a novel TSEKF is designed to simultaneously estimate satellite attitude information and actuator faults.

**Theorem 1.** A discrete-time TSEKF is given by the following coupled difference equations when the nonlinear discrete-time stochastic system with bias vector of unknown magnitude is given by Eq. (1).

\[
\dot{x}_{k+1} = \hat{x}_{k+1 | k} + \beta_{k+1 | k} \hat{b}_{k+1 | k} \]

(13)

\[
P_{k+1 | k} = \hat{P}_{k+1 | k} + \beta_{k+1 | k} \hat{b}_{k+1 | k} \hat{F}_{k+1 | k}
\]

(14)

where \( \hat{x}_{k+1 | k}, \hat{x}_{k+1 | k} \) and \( \hat{b}_{k+1 | k} \) are the state vectors of the TSEKF, the unknown bias free filter and the unknown bias filter, respectively; \( \beta_{k+1 | k} \) is the result of coupling Eq. (27); \( \hat{P}_{k+1 | k} \) and \( \hat{P}_{k+1 | k} \) are the estimation error covariance matrices of \( \hat{x}_{k+1 | k} \) and \( \hat{b}_{k+1 | k} \) respectively. \( \hat{P}_{k+1 | k} \) is the estimation error covariance matrix of \( \hat{x}_{k+1 | k} \).

The unknown bias-free filter is

\[
\dot{\hat{x}}_{k+1 | k} = \hat{x}_{k+1 | k} + \hat{K}_{k+1 | k} (y_{k+1} - C_k \hat{x}_{k+1 | k} - N_{k+1})
\]

(15)

\[
\hat{P}_{k+1 | k} = (I - \hat{K}_{k+1 | k} C_k) \hat{P}_{k+1 | k}
\]

(16)

\[
\dot{\hat{K}}_{k+1 | k} = \hat{P}_{k+1 | k} C_k^T (C_k \hat{P}_{k+1 | k} C_k^T + V)^{-1}
\]

(17)

\[
\dot{\hat{x}}_{k+1 | k} = f_k (x_{k | k}, \hat{b}_{k | k}) - \beta_{k+1 | k} \hat{b}_{k+1 | k} \]

(18)

\[
\dot{\hat{P}}_{k+1 | k} = A_k \hat{P}_{k | k} A_k^T + W^a + \hat{r}_k \hat{P}_{k+1 | k} \hat{r}_k^T - \beta_{k+1 | k} \hat{b}_{k+1 | k} \hat{P}_{k+1 | k} \hat{b}_{k+1 | k}
\]

(19)

The unknown bias filter is

\[
\dot{\hat{b}}_{k+1 | k} = \hat{b}_{k+1 | k}
\]

(20)

\[
\hat{P}_{k+1 | k} = \hat{P}_{k+1 | k} + W^b
\]

(21)

\[
\dot{\beta}_{k+1 | k} = \hat{b}_{k+1 | k} + \hat{K}_{k+1 | k} (y_{k+1} - \beta_{k+1 | k} (\hat{x}_{k+1 | k}, \hat{b}_{k+1 | k}))
\]

(22)

\[
\dot{\hat{x}}_{k+1 | k} = f_k (x_{k | k}, \hat{b}_{k | k})
\]

(23)

\[
\dot{\hat{K}}_{k+1 | k} = \hat{P}_{k+1 | k} H_k^T (H_k \hat{P}_{k+1 | k} H_k^T C_k + C_k \hat{P}_{k+1 | k} H_k^T + V)^{-1}
\]

(24)

\[
\dot{\hat{P}}_{k+1 | k} = (I - \hat{K}_{k+1 | k} H_k) \hat{P}_{k+1 | k} (I - \hat{K}_{k+1 | k} H_k)
\]

(25)

with the coupling equations

\[
H_{k+1 | k} = F_k + C_k \beta_{k+1 | k}
\]

(26)

\[
\dot{\beta}_{k+1 | k} = \beta_{k+1 | k} - \hat{K}_{k+1 | k} H_k \beta_{k+1 | k}
\]

(27)

\[
r_k = A_k \beta_{k | k} + F_k
\]

(28)

\[
\beta_{k+1 | k} = r_k \hat{P}_{k+1 | k} (\hat{P}_{k+1 | k})^{-1}
\]

(29)

To facilitate the derivations, the following notations are used:

\[
\begin{align*}
X_k &= [\hat{x}_k \hat{b}_k], \quad A_k = \begin{bmatrix} A_k & F_k \\ 0_{N \times N} & I_n \end{bmatrix} \\
C_k &= [C_k F_k], \quad \Gamma = \begin{bmatrix} I_n \\ 0_{N \times N} \end{bmatrix} \\
\hat{w}_k &= [w_k^T \hat{w}_k^T], \quad \hat{W} = \begin{bmatrix} W^a & 0 \\ 0 & W^b \end{bmatrix}
\end{align*}
\]

(30)

then the model Eq. (1) could be written as

\[
\begin{align*}
X_{k+1} &= A_k X_k + \hat{w}_k + \Gamma M_k \\
y_k &= C_k X_k + \hat{r}_k + N_k
\end{align*}
\]

(31)
A common approach to estimate the system state of system Eq. (31) is using the following well-known KF:

\[ X_{k+1|k} = X_{k+1|k} + K_{k+1}(Y_{k+1} - C_{k+1}X_{k+1|k} - \bar{N}_{k+1}) \]  

\[ X_{k+1|k} = A_kX_{k+1|k} + \Gamma M_k \]  

\[ P_{k+1|k} = A_kP_{k+1|k}A_k^T + W \] 

\[ K_{k+1} = P_{k+1|k}C_{k+1}^T(C_{k+1}P_{k+1|k}C_{k+1} + V)^{-1} \] 

\[ P_{k+1|k+1} = (I_{k+N} - K_{k+1}C_{k+1})P_{k+1|k} \]

where

\[ P_{k+1,k|k} = \begin{bmatrix} P_{k+1,k|k}^b & P_{k+1,k|k}^b \n P_{k+1,k|k}^b & P_{k+1,k|k}^b \end{bmatrix} \]  

(37)

where \( P_{k+1,k|k}^b \) and \( P_{k+1,k|k}^b \) are nondiagonal matrices.

The one-step prediction value \( X_{k+1|k} \) is then transformed to promote the one-step prediction variance into a diagonal matrix. The orthogonal transformation matrix is chosen as

\[ T = \begin{bmatrix} I_N & -\beta_{k+1|k} \\ 0 & I_N \end{bmatrix}, \quad \beta_{k+1|k} = P_{k+1,k|k}^b (P_{k+1,k|k}^b)^{-1} \]  

(38)

thus

\[ TX_{k+1|k} = \begin{bmatrix} \hat{x}_{k+1|k} - \beta_{k+1|k}\hat{h}_{k+1|k} \\ \hat{h}_{k+1|k} \end{bmatrix} \] 

\[ TP_{k+1|k}T^T = \begin{bmatrix} P_{k+1,k|k}^b - \beta_{k+1|k}P_{k+1,k|k}^b \beta_{k+1|k}^T & 0 \\ 0 & P_{k+1,k|k}^b \end{bmatrix} \]  

(39)

Define new variables

\[ \hat{x}_{k+1|1} = \hat{x}_{k+1|k} - \beta_{k+1|k}\hat{h}_{k+1|k} \]  

\[ \hat{P}_{k+1|1} = P_{k+1,k|k}^b - \beta_{k+1|k}P_{k+1,k|k}^b \beta_{k+1|k}^T \]  

(40)

Then, from Eqs. (33) and (34), the corresponding covariance matrix \( \hat{P}_{k+1|1} \) of Eq. (19) can be obtained and Eq. (40) can be changed to

\[ \hat{X}_{k+1} = A_k\hat{x}_{k+1} + M_k + r_1\hat{h}_{k+1} - \beta_{k+1}\hat{h}_{k+1} \]  

(42)

Then from Eqs. (38) and (41), we can get Eq. (29). Eq. (21) can be derived from Eq. (34) directly. From Eqs. (34)–(36) and Eq. (4), we have

\[ P_{k+1|k+1} = (I_{k+N} - K_{k+1}C_{k+1})P_{k+1|k} = \begin{bmatrix} x_1 & x_2 \\ x_3 & x_4 \end{bmatrix}^{-1} \]  

(43)

in which

\[ x_1 = (P_{k+1,k|k}^b)^{-1} + C_{k+1}^T V^{-1} C_{k+1} \] 

\[ x_2 = (z_1)^T = - (P_{k+1,k|k}^b)^{-1} \beta_{k+1} + C_{k+1}^T V^{-1} F_{k+1} \] 

\[ x_4 = (P_{k+1,k|k}^b)^{-1} + \beta_{k+1} (P_{k+1,k|k}^b)^{-1} \beta_{k+1} + (F_{k+1})^T V^{-1} F_{k+1} \]  

(44)

According to the rule of finding the inverse of partitioned matrix and Eq. (37), we can get

\[ P_{k+1|k+1} = \begin{bmatrix} P_{k+1,k+1}^i + \beta_{k+1} P_{k+1|k+1}^i \beta_{k+1} & P_{k+1,k+1}^p \\ P_{k+1,k+1}^p & P_{k+1,k+1}^p \end{bmatrix} \]  

(45)

Then it can be changed to Eq. (14), and in Eq. (45), we have

\[ \beta_{k+1|k+1} = x_1^{-1} \]  

(46)

\[ P_{k+1|k+1}^p = (x_3 - x_1 x_4^{-1} x_2)^{-1} \]  

(48)

From Eqs. (44) and (46), we can get Eq. (17). Eq. (27) can be derived from Eq. (47) directly.

From Eqs. (43) and (45), Eq. (35) can be changed to

\[ K_{k+1} = \begin{bmatrix} P_{k+1,k+1}^i + \beta_{k+1} P_{k+1|k+1}^i \beta_{k+1} & P_{k+1,k+1}^p \\ P_{k+1,k+1}^p & P_{k+1,k+1}^p \end{bmatrix} \] 

(49)

then we can calculate \( K_{k+1}^c \) and \( K_{k+1}^b \):

\[ K_{k+1}^c = \begin{bmatrix} \bar{P}_{k+1,k+1}^i + \beta_{k+1} \bar{P}_{k+1|k+1}^i \beta_{k+1} \bar{P}_{k+1,k+1}^i C_{k+1}^T V^{-1} + \beta_{k+1} \bar{P}_{k+1|k+1}^i \bar{P}_{k+1|k+1}^i C_{k+1}^T V^{-1} \end{bmatrix} \]  

\[ K_{k+1}^b = \bar{P}_{k+1,k+1}^i (P_{k+1,k+1}^i)^T V^{-1} \]  

(50)

\[ H_{k+1,k+1} = C_{k+1} \beta_{k+1|k+1} + F_{k+1} \]  

(51)

Then Eq. (26) can be obtained from Eq. (51).

Defining

\[ \tilde{K}_{k+1}^c = \bar{P}_{k+1,k+1}^i C_{k+1}^T V^{-1} \]  

(52)

Substituting Eq. (51) into Eq. (50), we have

\[ K_{k+1}^c = \tilde{K}_{k+1}^c + \beta_{k+1|k+1} K_{k+1}^c \]  

(53)

Eq. (17) can be obtained from Eqs. (16) and (52).

Based on Eq. (32), \( \hat{b}_{k+1|k+1} \) and \( \tilde{\hat{b}}_{k+1|k+1} \) can be calculated as

\[ \tilde{\hat{b}}_{k+1|k+1} = \bar{b}_{k+1|k} + K_{k+1}^c (y_{k+1|k} - N_{k+1} - C_{k+1} \tilde{\hat{X}}_{k+1|k} - H_{k+1} b_{k+1|k}) \]  

(54)

\[ \hat{b}_{k+1|k+1} = \tilde{\hat{b}}_{k+1|k+1} + \beta_{k+1} \bar{b}_{k+1|k} + (\bar{b}_{k+1|k} - K_{k+1}^c H_{k+1|k} \bar{b}_{k+1|k}) \]  

(55)

Then Eq. (22) is obtained from Eqs. (54) and (23). Eq. (15) can be derived from Eq. (32) directly. From Eqs. (15) and (47), Eq. (55) can be simplified into Eq. (13).

From Eq. (48), we have

\[ \begin{bmatrix} \bar{P}_{k+1, k+1|k}^i \end{bmatrix}^{-1} = \begin{bmatrix} \bar{P}_{k+1, k+1|k}^i \end{bmatrix}^{-1} + H_{k+1}^T C_{k+1}^T H_{k+1|k} \]  

(56)

Eq. (25) can be derived from Eq. (29) directly. Eq. (24) is obtained by substituting Eq. (56) into Eq. (51). Eq. (42) is obtained by substituting Eq. (4) into Eq. (18).
(1) The TSEKF, which is given by Eqs. (13)–(29), is equivalent to the TSKF when \( M_k = 0 \) and \( N_l = 0 \) in Eq. (5).

(2) System states and bias of Eq. (1) can be estimated based on Theorem 1.

(3) System states and additive faults of Eq. (9) can be estimated based on Theorem 1.

(4) System states and multiplicative faults of Eq. (11) can be estimated based on Theorem 1.

(5) The basic idea of this theorem is TSKF, so the TSEKF has the same advantages of low computational cost and high estimation precision as TSKF.

(6) The TSEKF processes the nonlinear terms in system model Eq. (1) with the same way of EKF. This way can motivate the generalization of the linear TSKF to nonlinear systems.

### 4. Fault simulation on closed-loop satellite attitude control systems

Actuators in closed-loop ACSs are three reaction flywheels. Sensors in closed-loop ACSs are three gyros and two star sensors. Without loss of generality, we assume that the fault happens in the reaction flywheel along \( x \) axis.

Here, we consider two simulation backgrounds: attitude stabilization control and attitude tracking control. The former is considered for additive faults estimation. The latter is mainly considered for multiplicative fault estimation. To estimate multiplicative fault for reaction flywheels, satellite ACSs must satisfy the persistent excitation condition. In other words, reaction flywheels should be activated to ensure satellite attitude maneuver or tracking.

#### 4.1. Faults conditions and simulation parameters

The conditions of the additive faults and the multiplicative faults are given as follows.

**Condition 1.** The first fault \( b_1 = [0.005, 0, 0]^T \) N · m exists during the time interval \( 200 \leq t_1 < 400 \) s, and the second fault \( b_2 = [0.010, 0, 0]^T \) N · m exists during the time interval \( 400 \leq t_2 < 600 \) s.

**Condition 2.** The control effectiveness factors of the first fault \( b_1 = [0.3, 0, 0]^T \) exist during the time interval \( 200 \leq t_1 < 400 \) s, and the control effectiveness factors of the second fault \( b_2 = [0.5, 0, 0]^T \) exist during the time interval \( 400 \leq t_2 < 600 \) s.

The conditions of the simulation and initial values are chosen as follows.

(1) The maximum output torque of the flywheels is 0.15 N · m and the maximum angular momentum is 15 N · m · s.

(2) The moment of inertia of the satellite is \( I_s = \text{diag}(200, 100, 300) \) kg · m².

(3) The PD controller is chosen with its gains: \( u = -[K_p, K_d]_[(x - x_d)] \), with \( K_D = [36, 18, 54]^T \) and \( K_p = [2, 1, 3]^T \).

(4) For attitude stabilization control, \( x_d = 0_{6 \times 1} \).

(5) For attitude tracking control,

\[
x_d = [0_{1,3} \text{ rad/s}, \quad \frac{1}{6} \sin (\frac{4}{3}t) \text{ rad}, \quad \frac{1}{6} \sin (\frac{4}{3}t) \text{ rad}, \quad 0 \text{ rad}]^T
\]

(6) The initial state is \( x_0 = 0_{6 \times 1} \).

(7) The initial transfer matrix is \( P_0 = I_6 \).

(8) The process noise covariance matrix is \( W = \text{diag}(\sigma_n, \sigma_e) \), where \( \sigma_n = 2 \times 10^{-4} \text{ rad/s} \), \( \sigma_e = 5 \times 10^{-3} \text{ rad} \).

(9) The measurement noise covariance matrix is \( V = 0.001^2 I_6 \).

(10) The sampling period \( k \) is 0.01 s.

#### 4.2. Simulation results

(1) Fault estimation under attitude stabilization control.

The TSEKF algorithm proposed in this paper is applied for fault estimation of attitude stabilization under Condition 1. The simulation results are shown in Fig. 1.

For attitude stabilization control, the fault estimation results using TSEKF algorithm and TSKF algorithm are the same: 0.005 N · m and 0.010 N · m.

The estimation results of multiplicative faults cannot be obtained under the same attitude stabilization control background under Condition 2. That is, the precondition of estimating the multiplicative faults of actuators is that the control torque output of actuators must not be approximately equal to zeroes. So, we have to estimate the multiplicative faults of actuators under attitude tracking control with a persistent excitation.

(2) Fault estimation under attitude tracking control.

The system model of attitude tracking control is nonlinear, which makes the normal TSKF useless. The TSEKF algorithm proposed in this paper is applied to fault estimation of attitude maneuver under Condition 1 and Condition 2. The simulation results are shown in Figs. 2 and 3.

For attitude tracking control, the fault estimation results using TSEKF algorithm are 0.005 N · m and 0.010 N · m. When the fault occurs, the trough and peak of the attitude angle curve are \(-58.942^\circ\) and \(59.372^\circ\) respectively, while the trough and peak of the attitude angle curve are \(-59.085^\circ\) and \(59.085^\circ\) respectively without faults. That is, after the first fault occurs, a bias of \(-0.143^\circ\) is added to the attitude angle, and after the second fault occurs, a bias of \(-0.287^\circ\) is added to the attitude angle.

For attitude tracking control, the estimation results of control effectiveness factors using TSEKF are 0.3 and 0.5 respectively, and the MSE is \(7.8 \times 10^{-4}\).

According to the simulation results, there is no obvious difference between the TSEKF and the TSKF in fault estimation of linear system (such as attitude stabilization control). While in fault estimation of nonlinear system (such as attitude tracking control), in which the TSKF cannot be applied, we can get good performances on the basis of TSEKF.
Fig. 1  Simulation results of attitude stabilization under Condition 1.

Fig. 2  Simulation results of attitude tracking control under Condition 1.
5. Conclusions

This paper has investigated the problem of two-stage extended Kalman filter-based reaction flywheel fault estimation.

(1) Based on the separate-bias principle, motivated by the optimal TSKF and EKF, employed EKF to process nonlinear terms in nonlinear system model, a novel TSEKF algorithm is designed such that it cannot only estimate satellite attitude angular rates and Euler angles, but also estimate reaction flywheel faults no matter they are bias ones or loss of effectiveness.

(2) It is respectively applied to estimating bias faults and loss of effectiveness for reaction flywheels in a satellite ACSs. Simulation results validate the feasibility and effectiveness in the cases of both attitude stabilization and attitude tracking control.

References


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