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Aspects of electric and magnetic variables in SU(2) Yang–Mills theory

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Abstract

We introduce a novel decomposition of the four-dimensional SU(2) gauge field. This decomposition realizes explicitly a symmetry between electric and magnetic variables, suggesting a duality picture between the corresponding phases. It also indicates that at large distances the Yang–Mills theory involves a three component unit vector field, a massive Lorentz vector field, and a neutral scalar field that condenses which yields the mass scale. Our results are consistent with the proposal that the physical spectrum of the theory contains confining strings which are tied into stable knotted solitons.

In the infrared limit the phase structure of a four-dimensional Yang–Mills theory is expected to be nontrivial. In particular, there should be a mass gap and color should be confined [1]. Presumably this also bestows new collective variables, more appropriate for describing the long distance theory than the gauge field A_μ^a which relates to the short distance spectrum. In a series of articles [2,3] we have proposed that the infrared limit of SU(2) Yang–Mills theory involves a variant of the $(3+1)$-dimensional nonlinear σ-model with a dynamical field $\vec{\mathbf{r}}(x)$ being a unit three vector. The ensuing Lagrangian should contain the following three terms,

$$L = (\partial_\mu \vec{\mathbf{r}})^2 - \Lambda(\vec{\mathbf{r}} \cdot \partial_\mu \vec{\mathbf{r}} \times \partial_\nu \vec{\mathbf{r}})^2 - V(\vec{\mathbf{r}}). \tag{1}$$

Here the first term from the left is the standard nonlinear σ-model contribution. The second term was introduced in [4]. Together with the σ-model term it yields an energy functional that supports knotted solitons [5,6]. The third term is a potential term. With it, the Lagrangian has the functional form of a $(2+1)$-dimensional baby Skyrme model [7]. But in three spatial dimensions the potential term has no effect on the stability of the knotted solitons which are described by the first two terms in (1). The potential term does influence the shape of the energy density and the mutual interactions of these solitons. But more importantly, it breaks the global SO(3) symmetry of the two first terms which removes the massless Goldstone bosons that would be present otherwise.

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This is necessary since there are no massless particles in the infrared spectrum of the Yang–Mills theory.

In the present Letter we shall argue that the Lagrangian (1), in combination with a massive Lorentz vector and a neutral scalar, may indeed be relevant in describing the infrared limit of $(3+1)$-dimensional SU(2) Yang–Mills theory. Our starting point is a *novel* decomposition of the gauge field A^a_μ, a modification of our previous decomposition [2] that extended the earlier one by Cho [8]. The decomposition in [2,8] entails a three component unit vector $\vec{\mathbf{r}}$ with a natural magnetic interpretation. But the string-like excitations of a gauge theory should relate to variables with a natural electric interpretation [1]. For that reason we now introduce a *different* decomposition of the gauge field. In fact, the decomposition that we present here involves two sets of variables which can be viewed as electric and magnetic, respectively. These variables enter in a very symmetric manner, which leads to a duality picture between them. In the first part of the Letter we describe our new decomposition and show how it leads to structures akin those present in (1). We then continue with a somewhat more speculative discussion how one could actually relate the infrared SU(2) Yang–Mills theory to the Lagrangian (1).

We shall consider a four-dimensional (Euclidean space for the moment) SU(2) Yang–Mills theory. In the so-called maximal abelian gauge which is very popular in lattice studies, one treats the Cartan A^3_μ as a U(1) gauge field while $A^+_\mu = A^1_\mu + iA^2_\mu$ together with its complex conjugate are charged vector fields. The two vector fields A^1_μ and A^2_μ lie in a plane of a four-dimensional space, and this plane can be parametrized by a *twobein* $e^a{}_\mu$ $(a = 1, 2)$ with

$$e^a{}_\mu e^b{}_\mu = \delta^{ab}.$$

We can then write the $a = 1, 2$ components as

$$A^a_\mu = M^a{}_b e^b{}_\mu. \tag{2}$$

But the two off-diagonal components of A^a_μ describe eight field degrees of freedom while on the r.h.s. of (2) we have nine since the matrix $M^a{}_b$ has four independent elements and the two normalized vectors $e^a{}_\mu$ have five independent components. However, there is also an internal SO(2) $\sim$ U(1) rotation invariance between $M^a{}_b$ and $e^a{}_\mu$: if for a fixed μ we rotate $e^a{}_\mu$ according to

$$e^a{}_\mu \to \mathcal{O}^a{}_b e^b{}_\mu,$$

the decomposition (2) remains intact provided we also transpose $M^a{}_b$ from the right with the same $\mathcal{O}^a{}_b$. When we account for this gauge invariance, each side in (2) indeed involves eight independent field degrees of freedom.

We introduce the combination

$$\mathbf{e}_\mu = \frac{1}{\sqrt{2}}\left(e^1{}_\mu + ie^2{}_\mu\right), \tag{3}$$

so that we have

$$\mathbf{e}^2 = 0, \qquad \mathbf{e}\cdot\mathbf{e}^* = 1. \tag{4}$$

We rewrite the decomposition (2) as

$$A^1_\mu + iA^2_\mu = i\psi_1\mathbf{e}_\mu + i\psi_2\mathbf{e}^\star_\mu, \tag{5}$$

where we have arranged the four matrix elements of $M^a{}_b$ into two complex scalar fields ψ_1 and ψ_2. A diagonal SU(2) gauge transformation sends

$$A^3_\mu \equiv A_\mu \to A_\mu - \partial_\mu\xi \tag{6}$$

and multiplies both ψ_1 and ψ_2 by a common phase,

$$\psi_{1,2} \to e^{i\xi}\psi_{1,2}, \tag{7}$$

but leaves $e^a{}_\mu$ intact. This is the natural action of a vector-like, or electric U(1) gauge transformation with $\psi_{1,2}$ the electrically charged fields. On the other hand, under the internal U(1) rotation we have

$$\mathbf{e}_\mu \to e^{-i\zeta}\mathbf{e}_\mu$$

and

$$\psi_1 \to e^{i\zeta}\psi_1, \qquad \psi_2 \to e^{-i\zeta}\psi_2.$$

Now the decomposition (5) remains intact, while the composite vector field

$$C_\mu = i\mathbf{e}\cdot\partial_\mu\mathbf{e}^\star, \tag{8}$$

transforms according to

$$C_\mu \to C_\mu - \partial_\mu\zeta. \tag{9}$$

Hence C_μ can be viewed as a gauge field for the internal rotation. In particular, (9) admits a natural interpretation as an axial-like or magnetic U(1) gauge transformation.

We employ the complex vector (3) to define a real antisymmetric tensor

$$G_{\mu\nu} = i(\mathbf{e}_\mu \mathbf{e}_\nu^\star - \mathbf{e}_\nu \mathbf{e}_\mu^\star), \tag{10}$$

which is invariant under the electric and magnetic gauge transformations. We introduce the corresponding Maxwellian electric and magnetic combinations

$$\mathcal{E}_k = G_{k0}, \qquad \mathcal{B}_k = \frac{1}{2}\epsilon_{klm} G_{lm}. \tag{11}$$

Then

$$\vec{\mathbf{u}} = \vec{\mathcal{E}} + \vec{\mathcal{B}}, \qquad \vec{\mathbf{v}} = \vec{\mathcal{E}} - \vec{\mathcal{B}}, \tag{12}$$

are two independent three-component unit vectors. When we invert (11) to give $\mathbf{e}_\mu$ in terms of the vectors $\vec{\mathbf{u}}$ and $\vec{\mathbf{v}}$ we get

$$\mathbf{e}_\mu = \frac{e^{i\phi}}{\sqrt{2}}\left(e_0, \frac{1}{2e_0}\left[\vec{\mathbf{u}} \times \vec{\mathbf{v}} + i(\vec{\mathbf{u}} + \vec{\mathbf{v}})\right]\right). \tag{13}$$

Here ϕ is the phase of the $\mu = 0$ component of $\mathbf{e}_\mu$ while the normalization condition (4) yields for the modulus

$$e_0 = \sqrt{1 + \vec{\mathbf{u}} \cdot \vec{\mathbf{v}}}.$$

For the magnetic gauge field (9) this gives

$$C_\mu = \frac{1}{1 + \vec{\mathbf{u}} \cdot \vec{\mathbf{v}}}(\partial_\mu \vec{\mathbf{u}} + \partial_\mu \vec{\mathbf{v}}) \cdot \vec{\mathbf{u}} \times \vec{\mathbf{v}} + 2\partial_\mu \phi, \tag{14}$$

which identifies ϕ as the magnetic phase. We also introduce a pair of complex vectors

$$U_\mu = e^{i\phi} \frac{\partial_\mu \vec{\mathbf{u}} \cdot (\vec{\mathbf{v}} + i\vec{\mathbf{u}} \times \vec{\mathbf{v}})}{\sqrt{1 - (\vec{\mathbf{u}} \cdot \vec{\mathbf{v}})^2}}, \tag{15}$$

$$V_\mu = e^{i\phi} \frac{\partial_\mu \vec{\mathbf{v}} \cdot (\vec{\mathbf{u}} + i\vec{\mathbf{u}} \times \vec{\mathbf{v}})}{\sqrt{1 - (\vec{\mathbf{u}} \cdot \vec{\mathbf{v}})^2}}. \tag{16}$$

Then

$$\partial_\mu \mathbf{e} \cdot \partial_\mu \mathbf{e} = U_\mu V_\mu. \tag{17}$$

Finally, we set $\rho^2 = |\psi|^2$ and define the three-component unit vector

$$\vec{\mathbf{t}} = \frac{1}{\rho^2}(\psi_1^\star \psi_2^\star)\vec{\sigma}\begin{pmatrix}\psi_1 \\ \psi_2\end{pmatrix},$$

where $\vec{\sigma}$ are the standard Pauli matrices. This vector is invariant under the electric gauge transformation. The component t_3 is also invariant under the magnetic gauge transformation, but for the other two components we have

$$t_\pm = \frac{1}{2}(t_1 \pm i t_2) \to e^{\mp 2i\zeta} t_\pm. \tag{18}$$

With these definitions we now proceed to the Yang–Mills action where we impose a partial gauge fixing, only for the off-diagonal components. For this we consider the following gauge fixed Lagrangian [1]

$$L_{\mathrm{YM}} = \frac{1}{4}\left(F_{\mu\nu}^a\right)^2 + \frac{1}{2}\left[\left(\partial_\mu \delta^{ab} - \epsilon^{ab} A_\mu\right) A_\mu^b\right]^2. \tag{19}$$

Here we have a renormalizable background gauge condition for the off-diagonal components $A_\mu^\pm$, with respect to the diagonal Cartan component $A_\mu \equiv A_\mu^3$ of the gauge field. The ensuing Lagrangian for the ghosts is constructed in an entirely standard fashion. But since it only becomes relevant in computing radiative corrections, we do not write the ghost contribution explicitly.

We substitute the decomposed gauge field in (19) and find for the gauge fixed Lagrangian

$$L_{\mathrm{YM}} = \frac{1}{4}F_{\mu\nu}^2 + \left|D_\mu^{ab}\psi_b\right|^2 + \frac{1}{8}\left(|\psi_1|^2 - |\psi_2|^2\right)^2 \tag{20}$$

$$+ \frac{1}{2}\rho^2\left(|\partial_\mu \vec{\mathbf{u}}|^2 + |\partial_\mu \vec{\mathbf{v}}|^2\right)$$

$$+ \frac{1}{2}\rho^2 t_- U_\mu V_\mu + \frac{1}{2}\rho^2 t_+ U_\mu^* V_\mu^* \tag{21}$$

$$+ \frac{1}{2}\rho^2 t_3 F_{\mu\nu} G_{\mu\nu} + \text{ghosts}. \tag{22}$$

Here

$$D_\mu^{ab} = \delta^{ab}(\partial_\mu + iA_\mu) - i\sigma_3^{ab} C_\mu \tag{23}$$

is the U(1) × U(1) covariant derivative. Indeed, we note that (20)–(22) is invariant both under the (electric) U(1) of the SU(2) gauge group, and under the internal (magnetic) U(1).

We define the vector field

$$B_\mu = A_\mu + \frac{i}{2\rho^2}\left[\psi_a \widehat{D}_\mu^{ab}\bar{\psi}_b - \bar{\psi}_a \widehat{D}_\mu^{ab}\psi_b\right]$$

$$\equiv A_\mu + \frac{i}{2\rho^2} J_\mu, \tag{24}$$

where

$$\widehat{D}^{ab} = \delta^{ab}\partial_\mu - i\sigma_3^{ab} C_\mu$$

is the magnetic covariant derivative, i.e., (23) with A_μ removed; notice that B_μ is invariant under the electric

U(1) gauge transformation. With this we then get for the Lagrangian (20)–(22)

$$
\begin{aligned}
L_{\mathrm{YM}} = {} & \frac{1}{4}(H_{\mu\nu} + M_{\mu\nu} + K_{\mu\nu}t_3)^2 + \frac{1}{2}(\partial_\mu \rho)^2 \\
& + \rho^2 (\nabla_\mu^{ij} t_j)^2 + \rho^2 B_\mu^2 + \frac{1}{8}\rho^4 t_3^2 \qquad (25) \\
& + \frac{1}{2}\rho^2 (|\partial_\mu \vec{\mathbf{u}}|^2 + |\partial_\mu \vec{\mathbf{v}}|^2) \\
& + \frac{1}{2}\rho^2 \big(t_- U_\mu V_\mu + t_+ U_\mu^* V_\mu^* \\
& \qquad + t_3 [H_{\mu\nu} + M_{\mu\nu} + K_{\mu\nu} t_3] G_{\mu\nu}\big) \\
& + \text{ghosts}, \qquad (26)
\end{aligned}
$$

where

$$
\nabla_\mu^{ij} = \delta^{ij}\partial_\mu + 2\epsilon^{ij3} C_\mu, \tag{27}
$$

describes the action of the magnetic covariant derivative on a vector, and

$$
\begin{aligned}
H_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu, \\
M_{\mu\nu} &= \epsilon^{ijk} t_i \nabla_\mu^{jl} t_l \nabla_\nu^{km} t_m, \\
K_{\mu\nu} &= \partial_\mu C_\nu - \partial_\nu C_\mu.
\end{aligned} \tag{28}
$$

The Lagrangian (25)–(26) is our main result. Most notably, we have removed the electric U(1) gauge structure by writing the Lagrangian in terms of the manifestly invariant quantities B_μ and $\vec{\mathbf{t}}$. We have also exposed a manifest duality between the electric variable $\vec{\mathbf{t}}$ and the magnetic variables $\vec{\mathbf{u}}$ and $\vec{\mathbf{v}}$, which becomes particularly transparent when we specialize to static ground state configurations described by the ensuing Hamiltonian

$$
\begin{aligned}
H_{\mathrm{YM}} = {} & \frac{1}{4}(H_{ij} + M_{ij} + K_{ij}t_3)^2 + \frac{1}{2}(\partial_i \rho)^2 \\
& + \rho^2 (\nabla_i \cdot \vec{\mathbf{t}})^2 + \rho^2 B_i^2 + \frac{1}{8}\rho^4 t_3^2 \qquad (29) \\
& + \frac{1}{2}\rho^2 |\partial_i \vec{\mathbf{m}}|^2 \\
& + \frac{1}{2}\rho^2 \big(t_+ Q_i^2 + t_- \overline{Q}_i^2 \\
& \qquad + t_3 \epsilon_{ijk} m_i [H_{jk} + M_{jk} + K_{jk} t_3]\big) \\
& + \text{ghosts}. \qquad (30)
\end{aligned}
$$

Here $\vec{\mathbf{m}}$ is a three component unit vector that emerges in the static limit where $\vec{\mathbf{m}} = \vec{\mathbf{u}} = \vec{\mathbf{v}}$, and $Q_i = U_i = V_i$. This follows when we contract the full Euclidean rotation group $SO(4) = SU(2) \times SU(2)$ to the spatial rotation group SO(3). Notice in particular that in the static limit

$$
K_{ij} = \vec{\mathbf{m}} \cdot \partial_i \vec{\mathbf{m}} \times \partial_j \vec{\mathbf{m}}.
$$

The result (25)–(26), or its Hamiltonian form (29)–(30) is remarkably similar to (1), *including* the potential term. Indeed, for the vector field $\vec{\mathbf{t}}$ we find the following potential from (25)–(26)

$$
\begin{aligned}
V(\vec{\mathbf{t}}) = {} & \frac{1}{8}\rho^4 t_3^2 + \frac{1}{2}\rho^2 \big(t_- U_\mu V_\mu + t_+ U_\mu^* V_\mu^* \\
& \qquad + t_3 [H_{\mu\nu} + M_{\mu\nu} + K_{\mu\nu} t_3] G_{\mu\nu}\big).
\end{aligned} \tag{31}
$$

We note that this is an example of the general class of potentials that have been considered in the context of the baby Skyrme model [7]. The same applies to the potential term for the vector $\vec{\mathbf{m}}$, for which we get from (29)–(30)

$$
V(\vec{\mathbf{m}}) = m_i \epsilon_{ijk} [H_{jk} + M_{jk} + K_{jk} t_3] t_3. \tag{32}
$$

Notice that even though these potential terms do seem to break the global SO(3) rotation invariance of the vector fields $\vec{\mathbf{t}}$ and $\vec{\mathbf{m}}$, we have manifest SO(3) covariance.

We shall now continue with somewhat more speculative comments on the possible phase structures of the Yang–Mills theory in the infrared limit, how a Lagrangian such as (1) could emerge. We start by noting that the Lagrangian (20) is quite reminiscent of a Lagrangian to which the Coleman–Weinberg–Savvidy [9,10] arguments apply. Indeed, one can show [11] that logarithmic corrections at the one-loop level lead to a dimensional transmutation with ρ acquiring a nontrivial ground state expectation value

$$
\langle \rho^2 \rangle = \Lambda^2 \neq 0. \tag{33}
$$

Due to the last two terms in (25) this would imply both the U(1) invariant vector B_μ and the vector $\vec{\mathbf{t}}$ become massive.

We proceed by considering the properties of the Lagrangian (25)–(26) in a naive derivative expansion where we treat each of the variables subsequently as a "slow" variable and then study the response of the remaining "fast" variables in this background. That is, we envision a Born–Oppenheimer type approximation to become applicable.

We first take the electric variable $\vec{\mathbf{t}}$ to be a fast variable in the background of the slow magnetic variables. For this we note that nontrivial average values $\langle\vec{\mathbf{u}}\rangle$ and $\langle\vec{\mathbf{v}}\rangle$ would imply that the underlying symmetries become broken. Since these symmetries relate to rotation symmetry in the Euclidean four-space which cannot become broken, it is reasonable to set

$$\langle\vec{\mathbf{u}}\rangle = \langle\vec{\mathbf{v}}\rangle = 0.$$

The terms linear in $\vec{\mathbf{u}}$ and $\vec{\mathbf{v}}$ then vanish to the leading order and we conclude that in the first approximation when we also replace ρ by its expectation value (33), the Lagrangian (25)–(26) simplifies into

$$L_{\mathrm{YM}} \approx \frac{1}{4}(H_{\mu\nu} + \vec{\mathbf{t}} \cdot \partial_\mu \vec{\mathbf{t}} \times \partial_\nu \vec{\mathbf{t}})^2 + \Lambda^2 (\partial_\mu \vec{\mathbf{t}})^2 + \Lambda^2 B_\mu^2 + \frac{1}{8}\Lambda^4 t_3^2 + \frac{1}{2}\Lambda^2 (t_+ S_+ + t_- S_-). \tag{34}$$

Here we identify the model (1) in interaction with a massive vector field. Note that due to the potential term the global SO(3) symmetry of $\vec{\mathbf{t}}$ becomes transformed into a covariance w.r.t. the background. Note also that this potential term is a combination of the t_3 mass term together with the analog of an external magnetic field coupling to $t_\pm$. As such, the potential term is present whenever the Coleman–Weinberg–Savvidy argument is applicable and the background $\mathbf{e}_\mu$ is not identically constant, as

$$S_+ = \langle \partial_\mu \mathbf{e} \cdot \partial_\mu \mathbf{e} \rangle. \tag{35}$$

We note that the result (34) strongly suggests that in the infrared limit the electric phase of the SU(2) Yang–Mills theory describes the dynamics of massive knotted solitons [2].

The model (1) also emerges from a similar Born–Oppenheimer limit for the dual magnetic variables, when we consider them as fast variables in the background of slowly varying electric variables. For this we note that a nontrivial expectation value in $\langle\vec{\mathbf{t}}\rangle$ implies that the underlying global symmetry becomes broken. But we expect that

$$\langle\vec{\mathbf{t}}\rangle = 0$$

From $\vec{\mathbf{t}} \cdot \vec{\mathbf{t}} = 1$ we then conclude that

$$\langle\vec{\mathbf{t}}_1^2\rangle = \langle\vec{\mathbf{t}}_2^2\rangle = \langle\vec{\mathbf{t}}_3^2\rangle = \frac{1}{3}.$$

When we average the Lagrangian over $\vec{\mathbf{t}}$ we find to the leading order

$$L_{\mathrm{YM}} \approx \frac{1}{4}K_{\mu\nu}^2 + \frac{1}{12}H_{\mu\nu}^2 + \Lambda^2 B_\mu^2 + \frac{1}{2}\Lambda^2 \left(|\partial_\mu \vec{\mathbf{u}}|^2 + |\partial_\mu \vec{\mathbf{v}}|^2\right) + \frac{1}{6}\Lambda^2 K_{\mu\nu} G_{\mu\nu}, \tag{36}$$

and when we specify this to static configurations we find for the Hamiltonian

$$H = \Lambda^2 (\partial_i \vec{\mathbf{m}})^2 + \frac{1}{2}(\vec{\mathbf{m}} \cdot \partial_i \vec{\mathbf{m}} \times \partial_j \vec{\mathbf{m}})^2 + \frac{1}{12}H_{ij}^2 + \Lambda^2 B_i^2 + \frac{1}{6}\Lambda^2 m_i \epsilon_{ijk} \langle K_{jk} \rangle. \tag{37}$$

Note that the explicit global SO(3) symmetry in m_i becomes broken when the background analog of the external magnetic field $\epsilon_{ijk} K_{jk}$ is nontrivial: again, we have a potential term that removes massless states in $\vec{\mathbf{m}}$ from the spectrum. A comparison with (34) also reveals a manifest duality between the electric and magnetic variables. In particular, we find that both sets of variables lead to a description that naturally contains massive knotted solitons in the spectrum [2].

In conclusion, we have introduced an explicit realization of the electric and magnetic variables in SU(2) Yang–Mills theory and found that both variables relate to an effective action of the form (1). This symmetric appearance of the electric and magnetic variables both at the level of the field decomposition, and at the level of the effective action then suggest a duality structure. In particular, our results support our earlier proposal that the nonperturbative spectrum of the Yang–Mills theory describes stable knots which are made out of the confining string.

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