

## Editorial

Rewrite techniques are being increasingly recognized as a powerful tool in automated reasoning. In recent years significant progress has been made not only in rewrite-based equational theorem proving, but also in applications of rewriting to inductive and first-order theorem proving. For instance, *reduction orderings*, a key concept in term rewriting, have been used to develop improved versions, both in terms of efficiency and completeness, of *resolution/paramodulation* type deduction methods. This special issue is devoted to applications of rewrite techniques to theorem proving.

Two papers, by Pais & Peterson and by Rusinowitch, respectively, present refutationally complete inference systems for first-order logic with equality. Both inference systems refine the resolution/paramodulation calculus and incorporate inference rules for simplification by rewriting. They coincide with an *ordered completion* method when restricted to equational logic. Their main difference is in the respective proof techniques used to establish refutational completeness. Pais & Peterson use an adaptation of model theoretic *forcing*, while Rusinowitch reasons with *transfinite semantic trees*. (A third general proof technique to originate from the study of rewriting are *proof orderings*, which are outlined in another paper in this volume.) We feel that the discovery of these powerful proof techniques, all of which are based on the notion of reduction ordering, is an important contribution of rewriting to the theoretical foundation of automated deduction.

Several papers discuss aspects of completion. Ganzinger describes a *conditional completion* method that extends the standard Knuth-Bendix completion procedure to conditional equations (i.e., Horn clauses in which the only atoms are equations). This conditional completion procedure is formulated as an equational inference system and has been designed to handle both reductive and non-reductive equations. Proof orderings are used to establish suitable proof normalization results for conditional equational logic, which guarantee the correctness of the completion procedure.

The standard completion method has also been applied to the problem of proving *inductive theorems* of equational theories. The key concept in this context is a notion of *consistency* of a set of equations with respect to a given inductive theory. Kapur, Narendran & Zhang present an *inductive completion* procedure which is based on a new method of using test sets for checking consistency. The authors also discuss experiments with an implementation of their method and compare it to other approaches.

Kounalis & Rusinowitch suggest a notion of *saturated set* for Horn clauses and give sufficient conditions under which the use of saturated sets as rewrite rules provides a decision procedure for the word problem in the underlying Horn theory. They also apply their ideas to inductive theorem proving in Horn logic.

Another completion-based approach, to first-order theorem proving, uses a convergent rewrite system for Boolean expressions to convert first-order formulas into a normal form based on conjunction and exclusive disjunction. Socher discusses various aspects of the relation of the deductive component of this completion method to resolution.

Finally, in the paper by Plaisted & Potter rewriting is combined with Prolog-based theorem proving. The authors suggest several rewrite techniques for such a framework

and discuss experiments with their prover on theorems of Von Neumann–Bernays–Goedel set theory.

For researchers interested in other aspects of rewriting and its applications, we recommend the survey, *Rewrite Systems* by N. Dershowitz and J.-P. Jouannaud (to appear in the *Handbook of Theoretical Computer Science*, vol. B, ed. J. van Leeuwen, North-Holland, 1991), which also provides further references.

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