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ORIGINAL ARTICLE

Time–cost tradeoff analysis considering funding variability and time uncertainty

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Abstract This paper presents a linear programming model for solution of the time–cost tradeoff problem. Although several analytical models have been developed for time–cost optimization (TCO), some of them mainly focused on projects where the contract duration is fixed. The optimization objective is therefore restricted to identify the minimum total cost only. Another group have primarily focused on project duration minimization. The model presented here considers scheduling characteristics that were ignored simultaneously in prior research. In the new formulation, variability of funding and uncertainty of project duration are considered simultaneously. A chance-constrained programming is used to incorporate the variability of funding, which is quantified by the coefficient of variation. The financial feasibility expressed as a stochastic constraint, which transformed into a deterministic equivalent at a pre-specified confidence level. Also, the project duration uncertainty incorporated into the model by applying PERT in scheduling and then the uncertainty is quantified by the coefficient of variation at a pre-specified confidence level. A system of objective function, which is minimizing direct cost and the group of constraints are solved by means of Lindo software. Two examples are conducted to demonstrate the model performance and its contributions. Four scenarios were adopted in solving the example problems to reflect the effect of each of funding variability and time uncertainty on project cost and duration. The results revealed that with 95% confidence level: 10% variability in funding versus neglecting it, would increase direct cost with 20% approximately for a pre-specified project deadline. Also, 10% variability in time versus neglecting it, would increase duration in range from 16.5% to 30% approximately, for a pre-specified direct cost. Also, considering 10% variability in funding and time would increase direct cost with more than 25% for a pre-specified project deadline. In parallel, an increase in project duration, more than 30% will occur for a pre-specified direct cost.

1. Introduction

When the contractor determines the optimal combination of time and cost, the cost will be used as a deterministic estimate to obtain funding to support the project. The fundamental goal of project financing is to ensure that, during construction there is enough funding available to compensate for the

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expenditures Yang [30]. Yet the planned funding, determined before the project begins, it is subject to variability as the project progresses.

In practice, it occurs frequently that available funding deviates from the original estimate during execution. Once the available funding is insufficient to support the costs, the project is forced to stop. If this occurs, the owner, contractors, and subcontractors would suffer from enormous economical losses. Thus, it is evident that funding variability is crucial in scheduling and hence should be incorporated into the deterministic time–cost tradeoff analysis. Dealing with this variability is one of the aims of the present paper.

Chance-constrained programming (CCP) offers a suitable means to incorporate funding variability into ordinary linear programming (LP) techniques Yang [30]. To reflect the uncertain nature, CCP models consider the objective function and constraints with stochastic coefficients in lieu of deterministic ones. The optimal solution is then determined by keeping the probability (risk) of violating certain constraints under a prescribed level α (for, example, 5%). In other words, the confidence level at which the constraint is satisfied has to be at least $(1 - \alpha)$ (for example, 95%). The proposed model handles the situation when the variability of the actual funding is anticipated and its impact on the time–cost tradeoff analysis needs proper assessment. On the other hand, to reflect the uncertain nature of project duration, Program Evaluation and Review Technique (PERT) will be applied. PERT present statistical information regarding the uncertainties associated with completing the different activities inherent in the project [6]. It will be used to determine the expected time of each activity for estimating the probability that the project completed at a prespecified deadline.

This paper presents a model for time–cost tradeoff that consider all possible precedence relationships, funding variability and project duration uncertainty in deterministic equivalent simultaneously. The paper is organized as follows. The first section is devoted to the review of previous studies. The second section explains chance-constrained programming as a mathematical optimization technique. The third section presents PERT calculations. Following it, is the proposed model formulation. An illustrative example is then presented to demonstrate the performance of the proposed model. Another example is highlighted to draw a conclusion. Analysis of the examples helps indicate the contributions of the proposed model. Conclusions are given in the last section.

2. Previous studies

Tradeoffs between project duration and direct cost are extensively discussed in the project scheduling literature because of its practical relevance. Since the early 1960s various analytical methods have been proposed for time cost optimization (TCO). These methods are heuristic methods, mathematical programming, and artificial intelligence.

A wide variety of heuristic procedure were used to solve the time–cost tradeoff problem [10,24,29,22]. In general, these procedure provided rule -of- thumb guidelines for crashing activities with least costs but cannot guarantee optimality [16].

Mathematical programming models constitute another group to deal with the time–cost tradeoff problem [14,11,13,23] presented linear programming (LP) models. Meyer and Shaffer [21] adopted integer programming (IP) to address discrete time–cost relationships. Butcher [3] applied a dynamic programming approach. Reda and Carr [26] presented mixed integer programming to solve time–cost tradeoff problem. Liu et al. [20] employed the LP/IP hybrid method to first locate the lower bound of the project time–cost relationship using LP and then find the exact solution by means of IP. Senouci and Adeli [27] presented a mathematical model, which handled resource-constrained scheduling, resource leveling, and project total cost minimization simultaneously. Yang [30] applied chance constrained programming to incorporate funding variability into ordinary linear programming (LP) techniques without considering uncertainty nature of project activities and hence project duration. Khalaf et al. [15] presented an approach of stretching non-critical activities to complete the project in shortest possible duration at least cost within available maximum budgeting by crashing all activities simultaneously in the project network. Then, Linear Programming (LP) technique was used to build a model to maximize the savings from stretching non-critical activities. The non-critical activities were stretched to their normal time until all slack in the different non-critical paths network was used up. The resultant savings from using of LP model were subtracted from the initial cost of crashing all activities to obtain the final cost of project.

Computational optimization techniques depending on artificial intelligence were also presented to solve time–cost tradeoff problems by means of genetic algorithms [9,18,12,19]. Senouci and Eldin [28] proposed a genetic algorithm model for resource scheduling considering all precedence relationships, resource leveling and resource-constrained scheduling, time–cost tradeoff problem. Zheng et al. [31] proposed a genetic algorithm-based multi objective approach for time–cost optimization to optimize total time and total cost simultaneously. Zheng and Ng [32] presented stochastic time–cost optimization model which incorporate fuzzy sets theory and non-replaceable front. Elbeltagi et al. [7] presented a solution for time–cost tradeoff problem by means of five evolutionary-based optimization algorithm. These are: Genetic Algorithm, Memtic Algorithm, Particle Swarm, Ant Colony, and Shuffled Forg Leaping. Recently, Abbasnia et al. [1] applied Fuzzy logic theory for time–cost tradeoff. They considered affecting uncertainties in total direct and indirect cost of a construction project and adopted genetic algorithm as an optimizer.

The common objective function of the previous approaches was to minimize cost (either direct or total project cost) subjected to precedence constraints between activities. Other constraints included resource leveling and constrained-resource scheduling [17,27,28]. Another constraint in addition to precedence constraints between activities is the funding variability Yang [30]. This paper presents a new mathematical model for TCO by considering multiple constraints: all precedence relationships, funding variability and uncertain nature of project duration simultaneously.

3. Chance-constrained programming

Chance-constrained programming was proposed in the 1950s [5,4] to provide a means to analyze the stochastic nature of ordinary mathematical optimization techniques. The applications of CCP abound for diverse fields such as structural analysis [8], and water reservoir management [25]. Yang [30] reported that a CCP formulation expressed in linear form is as follows:

$$\text{Maximize}\sum_{j=1}^{m} c_{ij}x_j \tag{1}$$

Subject to

$$\Pr\left(\sum_{i=1}^{n} a_{ij}x_j \leqslant b_i\right) \geqslant \alpha_i \quad j = 1,\ldots,m \tag{2}$$

$$x_j \geqslant 0 \tag{3}$$

where x_j = decision variable; i = constraint; c_{ij} = coefficient for the Jth variable in the ith constraint; a_{ij} = left hand side coefficient for the Jth variable in the ith constraint; b_i = right hand side coefficient for the ith constraint; and α_i = prescribed probability level (called confidence level). Eq. (2) reads as, "the probability to satisfy the constraint must be greater or equal to the prescribed α_i in any choice of x_j". Since α_i is a probability, it is always between 0 and 1.

Yang [30] demonstrated that in the CCP model, If a_{ij} being a random variable, the stochastic constraint can be converted to a deterministic form but the problem would then become nonlinear. When c_{ij} is a random variable, the objective is also a random variable for any given x_j. The case when b_i is randomly distributed will be adopted in this research. Eq. (2) may be rewritten as follows:

$$\Pr\left(\sum_{i=1}^{n} b_i \leqslant a_{ij}x_j\right) \leqslant (1-\alpha_i) \quad j = 1,\ldots,m \tag{4}$$

Estimating the mean and standard deviation for bi, the inequality inside the probability expression in Eq. (4) becomes:

$$(b_i - m_{b_i})/\sigma_{b_i} \leqslant \sum_{i=1}^{m}(a_{ij}x_j - m_{b_i})/\sigma_{b_i} \qquad j = 1,\ldots,m \tag{5}$$

where m_{b_i} = mean of b_i; and σ_{b_i} = standard deviation of b_i.

Assume b_i is distributed normally, the right hand side of inequality in Eq. (5) must follow the standard normal distribution with mean = 0 and standard deviation = 1. The left hand side of Eq. (5) may be rewritten in the following form:

$$(b_i - m_{b_i})/\sigma_{b_i} = Z_{\alpha_i} \tag{6}$$

where Z_{α_i} = inverse of the cumulative standardized normal distribution evaluated at probability α_i.

Thus, Eq. (5) may be rewritten in the following form (Eq. (7)). The original stochastic constaint can be converted to a deterministic equivalent [1] (see Eq. (8)).

$$Z_{\alpha_i} \leqslant \sum_{i=1}^{m}(a_{ij}x_j - m_{b_i})/\sigma_{b_i} \qquad j = 1,\ldots,m \tag{7}$$

$$\sum_{i=1}^{m}(a_{ij}x_j - m_{b_i})/\sigma_{b_i} \leqslant Z_{1-\alpha_i} \qquad j = 1,\ldots,m \tag{8}$$

where $Z_{1-\alpha_i}$ = inverse of the cumulative standardized normal distribution evaluated at probability $1-\alpha_i$.

$$\sum_{i=1}^{m} a_{ij}x_j \leqslant m_{b_i} + Z_{1-\alpha_i} \times \sigma_{b_i} \qquad j = 1,\ldots,m \tag{9}$$

The transformation from Eqs. (2), (4)–(9), however has the benefit of making the problem much more easier to solve.

4. Model formulation

To consider uncertain nature of project duration, the Program Evaluation and Review Technique (PERT) is applied. In this technique, the expected time (Te) for each activity is given by Eq. (10) [2] the relation between the most probable completion time (Tc), and a pre-specified deadline (Ts) is given by Eq. (11) [2]. Eq. (11) may be rewritten as in Eq. (12). The author suggests that if we choose the coefficient of variation (CV_t) and mean (Tc) instead of standard deviation (σ_t), Eq. (12) may be rewritten in the form of Eq. (13).

$$\text{Te} = (a + 4m + b)/6 \tag{10}$$

$$Z = \frac{[\text{Ts} - \text{Tc}]}{\sqrt{\Sigma\sigma^2}} \tag{11}$$

$$Z = (\text{Ts} - \text{Tc})/\sigma_t \tag{12}$$

$$\text{Ts} = Z^*(\text{CVt}^*\text{Tc}) + \text{Tc} \tag{13}$$

where a is the optimistic time; m is the most likely time; b is the pessimistic time; Z is inverse of the cumulative standardized normal distribution evaluated at probability level (α) or confidence level at which the project duration equal to or less than a pre-specified value; $\Sigma\sigma^2$ is the summation of critical activities' variance; and σ_t is the standard deviation of critical activities.

In the model formulation, funding variability and uncertain nature of project duration are considered. In this formulation, any relationship between activities is permissible. In other words, activity i is allowed finish–start, start–start, start–finish, or finish–finish precedence relationships with its preceding/succeeding activities. The activity time–cost relationship is assumed to be linear for simplicity.

4.1. Objective function and constraints

The objective function of the proposed model is to minimize the direct cost of the project

$$\text{Minimize}\sum_{i=1}^{I} C_i(\text{Te}_i) \tag{14}$$

where C_i = direct cost of activity i for a selected duration Te_i, I = number of project activities.

The objective function is subjected to four sets of constraints. The first set describes precedence relationships between activities. If activity j is a successor to activity i, then

a. Finish-to-Start (FS)

$$\text{ES}_i + \text{Te}_i + \text{LT}_{ij} \leqslant \text{ES}_j \tag{15}$$

where ES_i = early start of activity i; ES_j = early start of activity j; Te_i = expected duration of activity i; and LT_{ij} = lag/lead time between activities i and j.

b. Start-to-Start (SS)

$$\mathrm{ES}_i + \mathrm{LT}_{ij} \leqslant \mathrm{ES}_j \tag{16}$$

c. Start-to-Finish (SF)

$$\mathrm{ES}_i + \mathrm{LT}_{ij} \leqslant \mathrm{ES}_j + \mathrm{Te}_j \tag{17}$$

d. Finish-to-Finish (FF)

$$\mathrm{ES}_i + \mathrm{LT}_{ij} + \mathrm{Te}_i \leqslant \mathrm{ES}_j + \mathrm{Te}_j \tag{18}$$

The second set enforces the project financing feasibility or consider the available funding (AF) in a deterministic form, it is

$$\sum_{i=1}^{I} C_i \leqslant \mathrm{AF} \tag{19}$$

The available funding is usually a sum of funding from various sources, thus, it have a normal distribution Yang [30]. This assumption can be justified by the central limit theorem that states "the distribution of the sum of random variables will approach the normal distribution irrespective of the type (discrete or continuous), shape (skewed or symmetric), and number of individual distributions for contributing variables" [2]. Thus, the project financing constraint can be expressed as follows:

$$\Pr\left(\sum_{i=1}^{I} C_i \leqslant \mathrm{AF}\right) \geqslant \alpha \tag{20}$$

Adopting the same transformation process from Eqs. (4)–(9), the deterministic equivalent of Eq. (20) is given by Eq. (21). Yang [30] suggested that funding variability could be expressed using the coefficient of variation ($\mathrm{CV_{AF}}$) instead of the standard deviation. Accordingly, Eq. (21) leads to Eq. (22).

$$\sum_{i=1}^{I} C_i \leqslant m_{\mathrm{AF}} + Z_{1-\alpha_1}\sigma_{\mathrm{AF}} \tag{21}$$

where m_{AF} and σ_{AF} are the mean and standard deviation of available funding, respectively.

$$\sum_{i=1}^{I} C_i \leqslant m_{\mathrm{AF}} + Z_{1-\alpha_i} \times (\mathrm{CV_{AF}} \times m_{\mathrm{AF}}) \tag{22}$$

In Eq. (22), the scheduler has to determine two variables. First, he/she may choose a confidence level (α = 95%) (for example) at which the financial constraint must be satisfied. This makes $Z_{1-\alpha} = Z_{(1-95)\%} = Z_{5\%} = -1.65$. On the other hand, suppose the scheduler anticipates a significant variability would occur in obtaining the funding, therefore he assigns $\mathrm{CV_{AF}}$ to be 10% (for example). Eq. (22) becomes:

$$\sum_{i=1}^{I} C_i \leqslant m_{\mathrm{AF}} + (-1.65) \times (10\% \times m_{\mathrm{AF}}) \tag{23}$$

This implies that funding variability would reduce the right hand of Eq. (23). Consequently, the optimal project duration would be "longer" while the difference can be viewed as a contingency to account for funding variability.

The third set considers the uncertainty associated with project duration, it is

$$\mathrm{Ts} = Z^*(\mathrm{CVt}^*\mathrm{Tc}) + \mathrm{Tc} \tag{24}$$

In Eq. (24), again the scheduler has to determine two variables. First, he/she may choose a confidence level (α = 95%) (for example) at which the pre-specified project deadline is satisfied. This makes $Z_{(\alpha=95\%)} = 1.65$. On the other hand, suppose the scheduler anticipates a significant variability would occur in satisfying most probable project completion time (Tc), therefore he assigns CV_t to be 10% (for example). Eq. (24) becomes:

$$\mathrm{Ts} = 1.65^*(10\%^*\mathrm{Tc}) + \mathrm{Tc} \tag{25}$$

This implies that the more the variation anticipated in project duration, the longer the actual project duration (Ts). The difference between Ts and Tc can be viewed as a contingency to account for time uncertainty.

The fourth set of constraints defines the lower and upper bounds of activity durations as follows:

$$(\mathrm{Te}_i)_{\min} \leqslant (\mathrm{Te}_i) \leqslant (\mathrm{Te}_i)_{\max} \tag{26}$$

where $(\mathrm{Te}_i)_{\min}$ = lower bound of duration for activity i; and $(\mathrm{Te}_i)_{\max}$ = upper bound of duration for activity i.

The fifth set of constraints provides a condition for computation of most probable project duration (Tc) corresponding to 50% probability.

$$\mathrm{Tc} = \mathrm{Es_{fin}} + \mathrm{Te_{fin}} \tag{27}$$

where $\mathrm{Es_{fin}}$ = early start of last activity; $\mathrm{Te_{fin}}$ = duration of last activity.

4.2. Solving procedure

The optimization model is solved by means of commercial optimization software (Lindo). Four scenarios will be adopted in solving procedure to demonstrate the effect of each of funding variability and time uncertainty as follows:

1. In the first scenario, a traditional time–direct cost trade-off problem will be solved without considering both funding variability and time uncertainty. In the second, 10% of $\mathrm{CV_{FA}}$ for funding variability is assigned without considering time uncertainty. In the third, 10% of CV_t for project duration uncertainty is considered without considering funding variability. The fourth involved both 10% variability for funding and project duration uncertainty. Further, varying the coefficient of variation from 10% to 5% in scenarios 2, 3, 4 reflect the effect on time and direct cost. The confidence level adopted for scenarios 2, 3, and 4 is 95%.
2. Lindo software is used to optimize the objective function, which is minimizing the direct cost of the project Eq. (14), under the previously fifth sets of constraints. Then, a specific direct cost and a project duration associates with it are obtained. Each value represents a point on the direct cost/time curve.
3. The project duration resulted from step 2 will be crashed time unit by time unit, and the feasibility of the schedule is checked. If the schedule is nonfeasible the process of crashing will be terminated, otherwise the process will be continued.

5. First example

A numerical example is presented in order to illustrate the performance and capabilities of the proposed model. The data of the example was obtained from [6]. Table 1 shows the precedence relationships between activities for this example, which is a construction project consists of eight activities. The three estimates of each activity duration (a, m, and b) in normal conditions and the associated direct cost, the most likely value for activities duration in crash conditions and the associated direct cost are given also. For the purpose of the research the author added the optimistic and pessimistic values reasonably for activities duration in crash condition. An indirect cost of LE1000 weekly is assumed in this example.

The expected duration for each activity (Te) was calculated at normal and crash conditions and shown in Table 2. For example, activity C has expected time in normal conditions (Tec) = (2 + 4 * 4 + 6)/6 = 4. A simple linear regression was performed to produce a time–direct cost relationship for each activity depending on most likely value (m) to be used for traditional time–cost relationship (without considering time uncertainty). Another time–direct cost relationship for each activity was established depending on expected duration (Te) to be used when uncertainty in activities duration is considered. The results for direct cost–time relationship for both cases are shown in Table 2. It must be noted that an activity L with zero duration is assumed to terminate the network with one activity.

5.1. Complete formulations

In summary, the complete formulation for the example when considering time uncertainty:

$$\text{Minimize}\sum_{i=1}^{I} C_i \tag{28}$$

$$\sum_{i=1}^{I} C_i = 470,500 - 12,000\text{Te}_\text{A} - 3000\text{Te}_\text{B} - 10,000\text{Te}_\text{C} - 15,000\text{Te}_\text{D} - 4000\text{Te}_\text{F} - 16,000\text{Te}_\text{F} - 14,000\text{Te}_\text{G} - 13,500\text{Te}_\text{H} \tag{29}$$

Subject to:

1. Precedence constraints

$$\text{ESA} = 0 \tag{30}$$

$$\text{ESB} = 0 \tag{31}$$

$$\text{ESC} - \text{ESA} \geqslant \text{Te}_\text{A} \tag{32}$$

$$\text{ESD} - \text{ESB} \geqslant \text{Te}_\text{B} \tag{33}$$

$$\text{ESD} - \text{ESC} \geqslant \text{Te}_\text{C} \tag{34}$$

$$\text{ESE} - \text{ESA} \geqslant \text{Te}_\text{A} \tag{35}$$

$$\text{ESF} - \text{ESD} \geqslant \text{Te}_\text{D} \tag{36}$$

$$\text{ESG} - \text{ESE} \geqslant \text{Te}_\text{E} \tag{37}$$

$$\text{ESG} - \text{ESF} \geqslant \text{Te}_\text{F} \tag{38}$$

$$\text{ESH} - \text{ESD} \geqslant \text{Te}_\text{D} \tag{39}$$

$$\text{ESL} - \text{ESG} \geqslant \text{Te}_\text{G} \tag{40}$$

$$\text{ESL} - \text{ESH} \geqslant \text{Te}_\text{H} \tag{41}$$

where ES denotes early start of the activity.

2. Financial feasibility constraints

The financial feasibility constraint in deterministic equivalent is given in Eq. (42). In this example, confidence level at which the financial constraint must be satisfied is 95%, whereas coefficient of variation is 10%.

$$\sum_{i-1}^{I} C_i \leqslant m_\text{AF} + (-1.65) \times (\text{CV}_\text{AF} \times m_\text{AF}) \tag{42}$$

3. Uncertainty associated with project duration

The project duration associated to a given probability is given by Eq. (43). Confidence level (probability) at which the project duration will be estimated is 95%, whereas coefficient of variation is 10%.

$$\text{Ts} = 1.65^*(\text{CVt}^*\text{Tc}) + \text{Tc} \tag{43}$$

Table 1 Data of example problem.

Act.	Pred.	Normal conditions					Crash conditions				
		Time (weeks)				Direct cost (LE1000)	Time (weeks)				Direct cost (LE1000)
		a	*m*	*b*	Te		*a*	*m*	*B*	Te	
A	–	4	5	12	6	14	2	3	4	3	50
B	–	3	3	3	3	9	1	2	3	2	12
C	A	2	4	6	4	16	1	2	3	2	36
D	B, C	3	4	11	5	11	2	3	4	3	41
E	A	1	2	3	2	8	1	1	1	1	12
F	D	2	4	6	4	10	2	3	4	3	26
G	E, F	1	3	5	3	13	1	1	1	1	41
H	D	2	5	8	5	12	2	3	4	3	39

Table 2 Direct cost–time relationship neglecting versus considering time uncertainty.

Act.	Traditional relationship (relationship 1)	Relationship considering time uncertainty (relationship 2)
A	$CA = 104{,}000–18{,}000D_A$	$^{/}CA = 86{,}000–12{,}000Te_A$
B	$CB = 18{,}000–3000D_B$	$^{/}CB = 18{,}000–3000Te_B$
C	$CC = 56{,}000–10{,}000D_C$	$^{/}CC = 56{,}000–10{,}000Te_C$
D	$CD = 131{,}000–30{,}000D_D$	$^{/}CD = 86{,}000–15{,}000Te_D$
E	$CE = 16{,}000–4000D_E$	$^{/}CE = 16{,}000–4000Te_E$
F	$CF = 74{,}000–16{,}000D_F$	$^{/}CF = 74{,}000–16{,}000Te_F$
G	$CG = 55{,}000–14{,}000D_G$	$^{/}CG = 55{,}000–14{,}000Te_G$
H	$CH = 79{,}500–13{,}500D_H$	$^{/}CH = 79{,}500–13{,}500Te_H$

where C and $^{/}$C denotes the activity direct cost for relationship 1 and 2, respectively.

4. Upper and lower bounds of activity duration

$$6 \geqslant Te_A \geqslant 3 \quad (44)$$

$$3 \geqslant Te_B \geqslant 2 \quad (45)$$

$$4 \geqslant Te_C \geqslant 2 \quad (46)$$

$$5 \geqslant Te_D \geqslant 3 \quad (47)$$

$$2 \geqslant Te_E \geqslant 1 \quad (48)$$

$$4 \geqslant Te_F \geqslant 3 \quad (49)$$

$$3 \geqslant Te_G \geqslant 1 \quad (50)$$

$$5 \geqslant Te_H \geqslant 3 \quad (51)$$

The project duration is determined according to Eq (52).

$$Tc = ESL \quad (52)$$

5.2. Solving scenarios

To demonstrate the effect of each of funding variability and time uncertainty, four scenarios will be adopted in solving the example problem as previously given. In scenario 1 (traditional time direct cost relationship), a project duration of 20 weeks and a direct cost of LE93000 were obtained. In scenario (2b), 10% of CVFA was assigned without considering time uncertainty, a project duration of 20 weeks and direct cost of LE111377 were obtained. In scenario (3b), 10% of CV_t was assigned without considering funding variability, a project duration (Ts) of 25.6 weeks and a direct cost of LE93000 were obtained. In scenario (4b), 10% for both CV_{FA} and CVt, was assigned, a project duration of 25.6 weeks and a direct cost of LE111377 were obtained. In each scenario, the project was crashed week by week for obtaining intermediate points. It was found that the minimum feasible project duration is 14 weeks. For the purpose of comparison, an interval for project duration from 14 to 20 weeks was adopted when uncertainty in project duration is neglected, whereas this interval ranged from 14 to 25.6 weeks when uncertainty is considered. Table 3 shows the results, while Fig 1 represents it graphically. It must be noted that any duration corresponding to a prespecified direct cost or the vice versa could be obtained from interpolation in Table 3 or Fig. 1.

5.3. Analysis of results

Comparison between scenarios 1, 2, 3, and 4 in Table 3 and Fig. 1 reveals the following results:

Table 3 Time–direct cost tradeoff results.

Project dur. (weeks)	Scenario 1	Scenario 2		Scenario 3		Scenario 4	
	Traditional time cost relationship	Funding variability		Time uncertainty		Funding variability and time uncertainty	
		A 5%	b 10%	a 5%	b 10%	A 5%	b 10%
	Cost (LE)	Cost (LE)	Cost (LE)	Cost (LE)	Cost (LE)	Cost (LE)	Cost (LE)
25.6	–	–	–	–	93,000	–	111,377
25	–	–	–	–	98,408	–	117,854
24	–	–	–	–	106,991	–	128,133
23	–	–	–	100,529	116,090	109,568	139,030
22	–	–	–	109,767	126,391	119,637	151,366
21	–	–	–	120,206	136,691	131,014	163,702
20	93,000	101,362	111,377	131,291	146,992	143,097	176,038
19	103,000	112,262	123,353	142,377	158,674	155,179	190,029
18	113,000	123,161	135,329	154,206	170,691	168,072	204,420
17	127,000	138,420	152,096	167,139	183,116	182,167	219,301
16	141,000	153,679	168,862	180,291	195,992	196,502	234,720
15	159,000	173,297	190,419	194,148	210,672	211,605	252,302
14	177,000	192,916	211,976	208,976	235,994	227,767	282,627

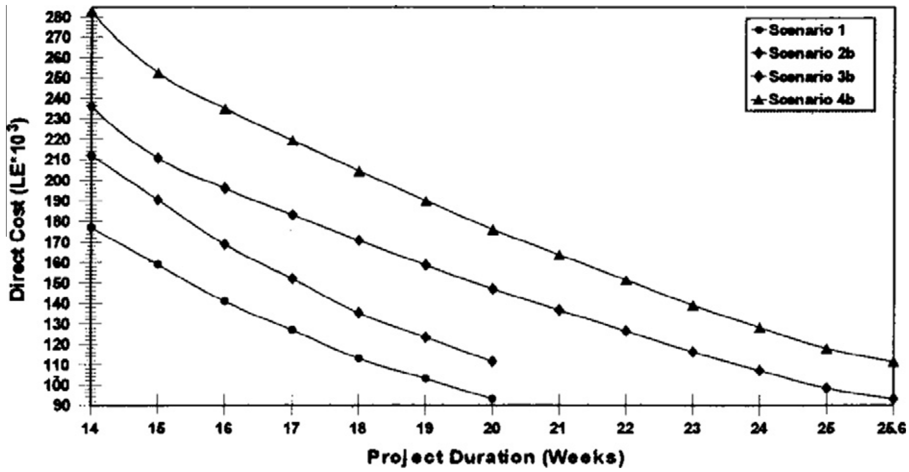

Figure 1 Time–direct cost curves for four adopted scenarios.

1. Assume a direct cost is LE176038 (for example). The project can be completed within 14.1, 15.7, 17.6, or 20 weeks for traditional time–direct cost relationship (scenario 1), considering 10% funding variability (scenario 2b), considering 10% time uncertainty (scenario 3b) or considering 10% for funding variability and time uncertainty (scenario 4b), respectively. Thus, the percentages increase in duration is 11.3%, 24.8%, and 41.8% related to scenario 1, respectively. This implies that time uncertainty has significant effect on time–direct cost relationship when considered separately, and the largest effect occurs when both funding variability and time uncertainty are considered simultaneously.
2. If the project deadline is 19 weeks (for example), the necessary direct cost to achieve it would be 103,000, 123,353, 158,674 or LE190029 for scenarios 1, 2b, 3b, or 4b respectively. Thus, the percentages increase related to scenario 1 are 19.8%, 54%, and 84.5%, respectively. This reveals that for the same pre-specified project deadline, funding variability have significant effect on direct cost, while time uncertainty have larger effect on direct cost than funding variability. The largest effect occurs when both funding variability and time uncertainty are considered simultaneously.
3. For comparing scenarios 2a and 2b corresponding to 5% and 10% funding variability respectively, assume a direct cost is LE152096 (for example), the project completed within 17 weeks for 10% variation instead of 16.02 week for 5% variation. In parallel assume a pre-specified project deadline, for instance, 15 weeks, the necessary direct cost to achieve it would be LE190419 instead of LE173297. This implies that increasing variation in funding from 5% to 10% will increase duration by 6% for the same direct cost and increase direct cost by 10% approximately, for the same duration.
4. For comparing scenarios 3a and 3b, i.e. when increasing variation in time from 5% to 10% assume a direct cost is LE158674 (for example), the project completed within 19 weeks instead of 17.6 weeks. In parallel assume a pre-specified project deadline, for instance, 18 weeks, the necessary direct cost to achieve it would be LE170691 instead of LE154206. This implies that increasing variation in time from 5% to 10% will increase duration by 8% for the same direct cost and increase direct cost by 11% approximately, for the same durations.
5. For comparing scenarios 4a and 4b corresponding to 5% and 10% variability in funding and time uncertainty simultaneously, assume a direct cost is LE190029 (for example), the project completed within 19 weeks instead of 16.5 weeks. In parallel, assume a pre-specified project deadline, for instance, 18 days, the necessary direct cost to achieve it would be LE204420 instead of LE168072. This implies that increasing variation from 5% to 10% will increase duration by 15% for the same direct cost and increase direct cost by 22% approximately, for the same duration.

The time–direct cost curve generated by the proposed model can be used to obtain the total cost curve by considering the indirect costs. Fig. 2 plots five curves: direct cost and total cost for 10% variability in funding and time (curves 1 and 2), direct cost and total cost for traditional relationship (curves 3 and 4) and indirect cost (curve 5). The calculation of the total cost is shown in Table 4. The optimal project duration is 25.6 weeks with a minimum total cost LE137377 if variability in funding and time is considered while these values are LE113000 and 20 weeks in traditional time–direct cost relationship.

6. Second example

The purpose of this example is to use the results obtained from it and from the first example to draw a conclusion about the percentages increase in project direct cost or duration due to the effect of both funding and time variability considered separately or simultaneously. This example is a construction project consists of 12 activities, with different types of precedence relationships; FS, SS, SF, and FF from [28]. Adopting traditional time cost relationship in solving the example, results in a direct cost ranged from LE48140 at 21 days to LE51145 at 17 days. Applying 10% variability in funding and time, results in a direct cost ranged from LE57656 at 25 days to LE61095 at 20 days. Analysis of the results obtained from this example are among the remarks given in the next section.

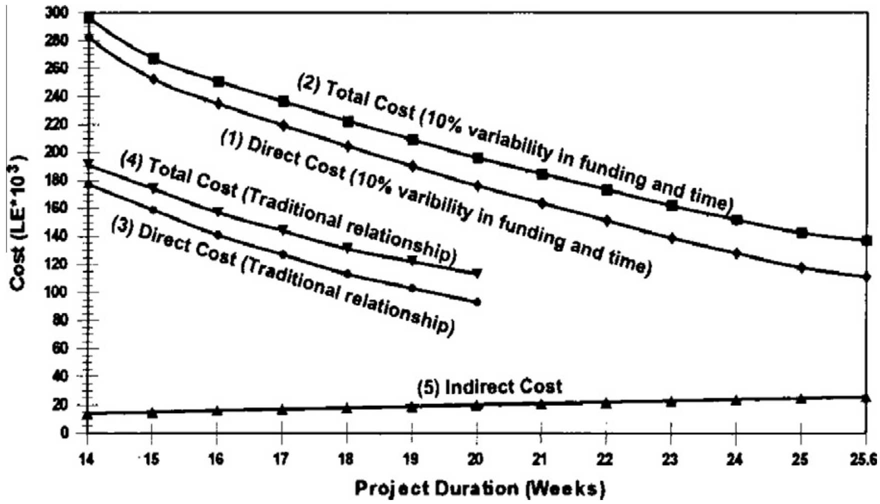

Figure 2 Time–cost curves (direct cost, indirect cost, and total cost).

Table 4 Total project cost calculation.

Project dur. (weeks)	Direct cost (LE)		Indirect cost (LE)	Total cost (LE)	
	(Traditional time cost relationship) Scenario (1)	Funding and time variability (10%) Scenario (4b)		(Traditional time cost relationship	Funding and time variability (10%)
25.6		111,377	26,000	–	137,377
25	–	117,854	25,000	–	142,854
24	–	128,133	24,000	–	152,133
23	–	139,030	23,000	–	162,030
22	–	151,366	22,000	–	173,366
21	–	163,702	21,000	–	184,702
20	93,000	176,038	20,000	113,000	196,038
19	103,000	190,029	19,000	122,000	209,029
18	113,000	204,420	18,000	131,000	222,420
17	127,000	219,301	17,000	144,000	236,301
16	141,000	234,720	16,000	157,000	250,720
15	159,000	252,302	15,000	174,000	267,302
14	177,000	282,627	14,000	191,000	296,627

7. Remarks on the results of the two examples

Depending on the results obtained from examples 1 and 2, and relating percentages increase in project duration or cost to traditional direct cost–time relationship and adopting 95% confidence level it can be concluded that:

1. For 10% variability in funding, an increase 19.8% in direct cost will occur for a pre-specified project duration. This is because direct cost would be less than or equal to 0.835 mean funding) (see Eq. (23)) and in turn mean funding is larger than or equal to 1.198 cost.
2. For 10% variability in project duration, an increase in project duration ranges from 16.5% to 30% approximately, will occur for a pre-specified direct cost. This increase in duration consists of two parts, the first is due to the relation between Ts and Tc (Ts = 1.165Tc, see Eq. (25)). The second one is due to the difference between Te for each activity and the most likely value (m), which in turn lead to a difference between Tc and project duration for traditional relationship.
3. Considering 10% variability in funding and time versus neglecting them would increase direct cost with more than 25% for a pre-specified project deadline. In parallel, an increase in project duration, more than 30% will occur for a pre-specified direct cost at the same conditions of confidence level and variability.

8. Conclusions and recommendations for future work

In this paper a time–cost tradeoff model was presented. Two important aspects were considered simultaneously, these are funding variability and time uncertainty. The presented linear programming model optimizes the direct cost. The system of objective function and constraints was solved by means of a classic optimization technique. Two example problems were presented to demonstrate how the model performs and its contributions. In solving the two examples, four scenarios were adopted to quantify the effect of considering funding variability and time uncertainty separately and then simultaneously at a specified degree of confidence 95%, and coefficient of variation 10%, for example. Also, coefficient of variation was varied from 5% to 10%. A time–cost curve for each scenario was plotted. Depending on the results obtained from the two examples, one can draw a conclusion that: for 10% variability in funding an increase, approximately 20% in direct cost for a pre-specified project duration will occur. On the other hand, for 10% variability in project duration, an increase in project duration ranges from 16.5% to 30% approximately, will occur for a pre-specified direct cost. Considering 10% variability in funding and time versus neglecting them would increase direct cost with more than 25% for a pre-specified project deadline. In parallel, an increase in project duration, more than 30% will occur for a pre-specified direct cost at the same conditions of confidence level and variability.

In future research, it is recommended that, various degrees of confidence levels for example 90%, 85%, 80%, and various values for coefficient of variation, for example 15%, 20%, and 25% may be adopted for both funding variability and time uncertainty to quantify the effect on both direct cost and project duration. Then, direct cost–time curve may be drawn for each case, such that for a pre-specified project deadline the scheduler may obtain the corresponding direct cost. In parallel, for a pre-specified direct cost he may obtain the corresponding project duration.

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