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Complexity and information

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1. Introduction

Science, and its attendant technologies, are in the midst of a serious crisis. The diagnostic of this crisis is our inability to control the trajectories of organisms or social structures as we can control, say, the trajectory of a rocket. We have long believed that the same kinds of laws, and the same modes of understanding and explanation, are of universal currency, and should in principle cover all situations of interest; hence our difficulties in the biological and social realms are regarded as transient phenomena arising from insufficient information and insufficient cleverness.

We have come to use a single word, *complexity*, to cover these difficulties. Whenever a system does unexpected or unpredicted things, when it exhibits new qualities which we may call emergent novelties, when our 'common-sense' attempts at control turn out to exacerbate previous difficulties or create unanticipated new ones, we tend to invoke this word as an explanatory principle.

If we believe that our present modes of system description and explanation are indeed universal, then this notion of complexity connotes one more system property, much like mass or temperature. It is part of a system phenotype, and raises no new issues of principle. It is synonymous with complication, and can be measured by such things as the dimension of a state space, or the length or cost of a program; it is a number whose magnitude tells us how 'complex' a system is.

We can, however, take an alternative and more radical view. Namely, we can imagine that the myriad difficulties we presently associate with complexity represent not simply a technical matter within a universal paradigm, but rather indicate a failure of that paradigm. That is, we can imagine that complexity confronts us with a conceptual, and not a technical problem. This is the view we shall adopt in what follows.

To have a specific context before us, we shall consider the relation of biology (which everyone agrees is a science of complex systems) and physics (which, I will argue, is presently the science of simple systems). In these days of 'molecular biology', the dominant view is that of reductionism; that organic behavior is no different in principle than the behavior of the solar system, or of a gas, only more complicated. Nevertheless, the fact is that there is no significant inferential chain from anything in physics to anything in biology. We will argue that the fault lies in our physics; in the tacit epistemological assumptions underlying even the most general physical laws. As we shall see, these assumptions restrict us from the outset to a very special class of material systems (which may be called simple systems, or mechanisms). We shall briefly indicate what happens to physics itself when these restrictions are removed.

What is true of biology is also true of human sciences. There are many profound homologies between biological and social organizations, which allow biological concepts to be transported, *mutatis mutandis*, to the human realm. However, we shall not discuss these in detail here.

2. The modelling relation

All science rests on two basic presumptions. The first of them is that the events we perceive in the external world are not totally whimsical, but exhibit regularities; satisfy laws or rules. The second is that these regularities can be perceived and articulated by the human mind. In short, we must believe that the order which is manifest in sequences of events in the external world (which we may describe as *causal order*) can be reflected in a corresponding relation between propositions which describe these events (which we may call an *inferential* or *logical order*). The causal order may be probed directly by observation and experiment; the inferential order between propositions belongs to logic and mathematics; the establishment of explicit relations between the two kinds of order is the essential task of theoretical science.

When we have established a congruence between causal order in the external world, and inferential order in a corresponding logical or mathematical system, we have created a *model* of the former in the latter. This *modelling relation* is at the heart of all theoretical science, and will be the basic point of departure for all that follows. Thus, it is well to spend a few moments discussing it in more detail.

Let us consider the schematic diagram represented in Fig. 1. Here, the box on the left represents some circumscribed collection of behaviors or events in the external world, which we designate as a *natural system* N. The box on the right represents some collection of propositions and inferential rules (production rules) which collectively constitute a *formal system* F. To establish a relation between these two disparate kinds of things, we must fix a way of naming attributes of N in the propositions of F. That is, we must establish a dictionary, or *encoding*, between events and propositions, utilizing procedures of measurement and observation. The nature of the relation we establish between N and F depends entirely on the nature of our

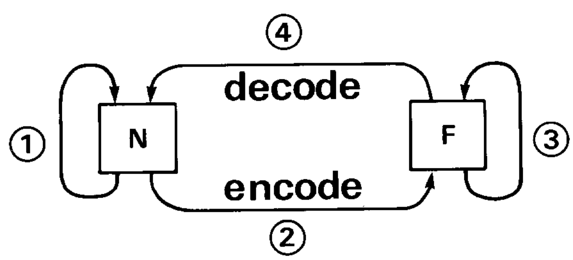

Fig. 1.

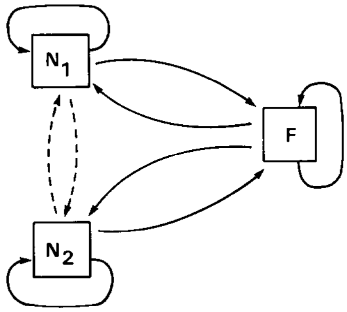

Fig. 2.

encoding, and of the inverse *decoding* operation which converts propositions in F back to assertions about N.

We say that a *modelling relation* exists between F and N when the inferential relations in F exactly mirror the causal relations in N. That is, we always get the same answer, whether we simply sit as observers and watch the causal order unfold in N (the arrow ①), or whether we encode, use the inferential structure in F to derive consequences or theorems in F, and then decode these theorems into predictions about N. We then have commutativity in the sense that

① = ② + ③ + ④,

and we can say that F is a *model* of N (or alternatively, that N is a *realization* of F).

This deceptively simple diagram has numerous important implications. We can only touch on a few of them here. For instance, suppose that two different natural systems N_1, N_2 stand in a modelling relation to the same formal system F, as shown in Fig. 2. We can then generally establish a direct relation (indicated by the dotted arrows) between the two natural systems N_1, N_2. This relation is like a modelling relation, except that it involves two natural systems, instead of a natural system and a formal one. This relation is one of *analogy*, in the sense of analog computation; the dotted arrows then represent transformations which allow data pertaining to one of them to be transformed into corresponding data about the other.

The dual of the situation, shown in Fig. 3, arises when we have the same natural system N encoded into two different formal systems F_1, F_2. It turns out that in this case, we *cannot* generally establish a mathematical transformation between F_1 and F_2. It thus follows that the same natural system N may have many distinct models.

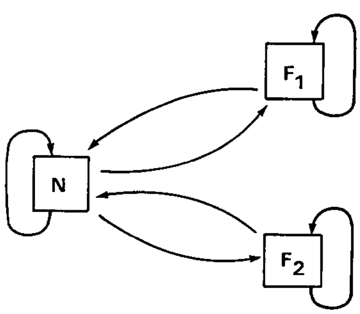

Fig. 3.

It is this last fact which shall be central to our discussion of system complexity. Quite generally, we may pose the following question: given a natural system N, what kinds of mathematical or formal systems can sit on the right-hand side of Fig. 1, and satisfy the commutativity relation? These will constitute some kind of family (category) of mathematical objects. Further, given such a category of formal images or models of N, what relations exist between them? In particular, is there a 'biggest' (*free*) image of N in this category, which in some sense maps effectively onto all the others?

All of our traditional scientific epistemology, which has been drawn essentially from the structure of Newtonian mechanics and has persisted essentially unchanged since then, asserts that

(a) the category of images of any natural system N is generally a category of dynamical systems, and

(b) among these images is a unique largest one.

We shall call any natural system which satisfies these two properties a *simple system*, or *mechanism*. The paradigm of Newtonian mechanics, which has become the general paradigm of theoretical science, basically then asserts that *every system is simple*. It also asserts the universality of reductionism as a strategy of system analysis; this is simply the search for the unique free image posited in (b).

Any system N which does not satisfy (a) or (b) is thus not a simple system; we will call it a *complex system*. If complex systems in this sense exist, then there is some fundamental deficiency already tacit in what we have called the Newtonian paradigm. In fact, there are several, and they reside in the choice of encoding and decoding arrows in Fig. 1. We shall not pause to critically review this here (cf. [2]); rather, we shall exhibit some alternate encodings, which immediately take us outside that paradigm.

3. Information

We return now to the relation between physics and biology. The basic difficulty in establishing an effective relation between them has been one of encoding and decoding, as we might expect from the above considerations. The technical vocabulary of physics is dominated by such concepts as energy; potential; force; work and the like. These terms do not appear in theoretical biology; instead, we find a vocabulary rich in informational words; code, translation, recognition, specificity, memory, learning, computation, and so on. The reductionistic tradition tells us that these distinct vocabularies are a manifestation of the immaturity of biology as a science, and that the informational terms currently dominating biological discourse are only a *facon de parler*, to be replaced as soon as feasible by the technical terms of physics.

As will soon become evident, our use of the term 'information' has very little to do with the 'Information Theory' of Shannon [3]. This theory was developed for dealing with a special class of questions in communication engineering, and is of a purely syntactic nature. Rather, we shall take the view that 'information' pertains to whatever can be the answer to a question, and is thus of an essentially semantic nature.

The relation of interrogation to 'information' is in itself interesting, because interrogation sits outside of formal logic. Questions belong only to informal discourse, and do not enter into any of the mathematical formalisms which we use to image natural systems. This is perhaps one of the main reasons why 'information' has proved so hard to quantify in traditional terms.

Nevertheless, interrogations have played an important role even in theoretical physics. The concept of the *virtual displacement* involves a quintessential question: if we vary some feature of a given situation, what happens to some other feature? In a sense, we may regard what follows as an extrapolation of the concept of the virtual displacement to more general dynamical contexts.

It was Poincare who first drew attention to dynamical systems as direct images of natural systems. He regarded them as generalizations of the Newtonian equations of motion, considered as vector fields on phase spaces. Newton's Laws relate such vector fields directly to impressed forces. But Poincaré's approach, augmented and applied by many others, was to regard general dynamical systems as immediate descriptions of the natural world, without necessarily detouring through impressed forces and standard mechanics. This was in fact a profound innovation, but one so familiar to us that we now take it for granted.

Let us see then what happens when we apply the notion of the virtual displacement to a general dynamical system of the form

$$\mathrm{d}x_i/\mathrm{d}t = f_i(x_i, \ldots, x_n), \quad i = 1, \ldots, n. \tag{1}$$

(We note that, buried in the functions f_i are a family of structural parameters, which play the central role in determining the character of the system; for the moment, we shall confine ourselves to virtual changes of the state variables x_i.)

It was Higgins [1] who first pointed out a relation between the stability properties of the dynamical system (1) and the informational qualities of virtual displacements, by drawing attention to the quantities

$$u_{ij}(x_1, \ldots, x_n) = \frac{\partial}{\partial x_j}\left(\frac{\mathrm{d}x_i}{\mathrm{d}t}\right). \tag{2}$$

He showed on the one hand that the *signs* (not the specific magnitudes) of the u_{ij} in a state were decisive for stability, thus relating stability to generalized forces, and on the other hand, showed that the u_{ij} had a specific informational interpretation. Namely, if a function u_{ij} is positive in a state, it means that a (virtual) increase in x_j increases the rate at which x_i is changing; alternatively, a (virtual) decrease in x_n decreases that rate. Thus it is natural to say that x_j is an *activator* of x_i in these circumstances. Conversely, if u_{ij} is negative in a state, it means that x_j is an *inhibitor* of x_i (a virtual increase in x_j decreases the rate of change in x_i, etc.). Now clearly, activation and inhibition are informational concepts, tied by Higgins to stability and thus indirectly to forces. Moreover, in many contexts, it is more natural to describe a system through their activation-inhibition structure (i.e. the functions u_{ij}) than through a system of rate equations directly; e.g. in neural networks and in ecosystems, to give but two examples.

Thus the question immediately arises: can we reconstruct a system of rate equations (1) from a posited activation-inhibition pattern $\{u_{ij}\}$? Comparing (2) and (1), the answer is evident: from a given activation-inhibition pattern $\{u_{ij}\}$, construct the differential forms

$$\omega_i = \sum_{j=1}^{n} u_{ij}\, \mathrm{d}x_j. \tag{3}$$

If these forms are exact, then there are functions f_i such that

$$\omega_i = \mathrm{d}f_i. \tag{4}$$

Putting

$$f_i = \mathrm{d}x_i/\mathrm{d}t \tag{5}$$

gives us back our set of rate equations (at least, up to some inessential scale factors).

But the exactness relations (4) are extremely nongeneric. Indeed, to postulate that all activation-inhibition patterns arising in nature must satisfy them is an incredibly strong restriction on the capability of systems for informational interactions. We can see this in another way by looking at the standard necessary conditions that the forms (3) be exact; namely,

$$\frac{\partial}{\partial x_k} u_{ij} = \frac{\partial}{\partial x_j} u_{ik}, \quad \forall i,j,k. \tag{6}$$

Now the quantities

$$u_{ijk}(x_1,\dots,x_n) = \frac{\partial}{\partial x_k}\left(\frac{\partial}{\partial x_j}\left(\frac{\mathrm{d}x_i}{\mathrm{d}t}\right)\right) \tag{7}$$

appearing in (6) themselves have an informational interpretation; if such a quantity is positive, it means that a (virtual) increase in x_k potentiates the effect of x_j on the rate of change of x_i; it is thus natural to call x_k an *agonist* of x_j. Conversely, if u_{ijk} is negative, a (virtual) change in x_k attenuates this effect; hence x_k is then an *antagonist* of x_j. The exactness conditions (6) thus mandate that the agonist-antagonist relation and the activator-inhibitor relation are completely symmetric; again, an excessively strong condition to impose on informational interactions.

Indeed, if we start from a set of rate equations (1), then the activation-inhibition pattern, the agonist-antagonist pattern $\{u_{ijk}\}$, and indeed all the higher-order patterns we can obtain by iterating the procedure, are completely determined; indeed, any one of them determines all of the others by a simple process of differentiation or integration. But if any of the differential forms (3), or their higher-level analogs, are *inexact*, then

(a) in general, there are no rate equations (1) which can describe the system, and

(b) all the informational levels $\{u_{ij}\}$, $\{u_{ijk}\},\dots$ become logically independent and must be posited separately.

Thus, we see that the mathematical language of networks of informational levels which we have sketched is far more general than is the language of dynamics. Thus, if we could show that there are natural systems N which admit such descriptions, then these systems N could not be simple.

4. Complex systems

Let us begin by contrasting the characteristics of conventional dynamical systems with those of the more general informational networks which we have sketched in the preceding section.

Dynamical systems are dual structures. On the one hand, they possess a *state space*, which we suppose can be fixed once and for all; on this is superimposed a set of dynamical laws, which is also assumed fixed. This mathematical dualism is tacitly imputed back to the external world, where in mechanics it takes the form of a dualism between phases and forces; between system and environment, or inside and outside. Once the state space is given, and the forces specified, the behavior of the system is completely determined; there can be no surprises, nothing counter-intuitive, no emergence, and no error in the system. This is the essence of mechanical behavior, and this is why we have chosen to call natural systems describable in this way *mechanisms*.

The formalism of informational networks is vastly different in these regards. There is no 'state space' which can be fixed once and for all, and there are no global dynamical laws. That is, the dualism between phases and forces on which the mechanical paradigm is based completely disappears. As we have noted, that paradigm, and its attendant duality, has persisted essentially unchanged from Newtonian times, despite all the upheavals and revolutions in science since then. Even in quantum mechanics, for example, which occasioned the most far-reaching conceptual changes in physics, the main issue involved the redefinition of the concept of state itself, and how the state of a system was to be related to what is actually observed; there was never any thought of abandoning either the concept of state, or the duality between states and dynamical laws.

Nevertheless, there is a deep relation between the dynamical systems which describe simple systems, and the web of informational networks which can define complex ones. It is important to describe this relation, though we cannot enter into formal details here. The informational networks we have described, when appropriately characterized in mathematical terms, form a category, just as the dynamical systems do. The category of dynamical systems is, in fact, a very small subcategory of the category of informational networks. In the bigger category, we may impose a metric structure, and with it, an idea of *approximation* of one structure by another. Intuitively, an informational network, though it does not arise from a dynamical system, can nevertheless be *approximated* by one which does arise from a dynamical system, locally and temporarily. In other words, our complex systems can be approximated by simple ones, just as a planar map may approximate the surface of a sphere. As the curvature of the sphere becomes more and more important, we must use more and more distinct planar approximations; similarly, as the complexity of a system becomes more and more manifest, we must shift from one approximating dynamical system to another. And just as we may think of a sphere as the limit of its planar approximations, without itself being a plane, so a complex system can be regarded roughly as the limit of its approximating dynamical systems, without itself being a dynamical system.

In these facts we find an understanding of why it is that we have been able to proceed so far with the hypothesis that all systems are simple, and how we must modify that hypothesis if we are to proceed further. The 'counter-intuitivity' of complex systems, the manifestations of what are called emergence and system error, depending on the context, are all interpretable as the deviation of behavior between a complex system and a particular approximating simple system; a sign that the approximation relating the one to the other is breaking down.

Thus we see another fundamental distinction between simple systems and complex ones. Namely, the category of mathematical images of simple systems are all dynamical systems, and all approximate to the largest image, which again is a dynamical system. Complex systems possess a multitude of partial mathematical images which are dynamical systems, but these cannot be combined into a largest image of the same form.

One final distinction between simple systems and complex ones may be briefly remarked upon here. In simple systems, as is well known, there is no room for any kind of behavior which may be termed *anticipatory*. An anticipatory system may be defined as one in which present change of state is, at least in part, determined by future state or future input. Indeed, any form of dependence of the present upon the future is associated with the Aristotelian category of final causation, and dismissed as teleology. Indeed, it can be rigorously shown that anticipation of any kind is incompatible with the concept of state, and hence is excluded from the outset in the mechanical paradigm. Thus, indeed, simple systems cannot anticipate. But in the complex systems we have considered, there is no state set; thus these arguments against anticipation do not apply. Hence in these systems there is at least the possibility of anticipatory behavior; in fact we have shown (cf. [2]) that biological systems are replete with modes of anticipatory control, arising at every level of organization. Thus, in the context of complexity, we can hope to approach a theory of anticipatory systems in a completely rigorous, non-mystical way.

Now we can return to the question: are there any natural systems N which are not mechanisms? That is, are there any natural systems which possess mathematical images which are not dynamical systems? On several grounds, I feel that this question must be answered in the affirmative. One of them has already been mentioned: namely, that organisms are capable of anticipatory behavior. Indeed, our question can be posed rather differently and more provocatively: are there any natural systems which are simple? I would answer: perhaps not. If not, then the revolutions we have already seen in physics are as nothing compared with those which must come.

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