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Identification of Measurement Points for Calibration of Water Distribution Network Models

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Abstract

Much importance is given to determining the input data for water distribution system networks, particularly with regard to urban networks, because the design and the management of WDS are based on a verification model. Good calibration of models is required to obtain realistic results. This is possible by the use of a certain number of measurements: flow in pipes and pressure in nodes. The purpose of this paper is to analyze a new model able to provide guidance on the choice of measurement points to obtain the site data. All analyses are carried out firstly on literature networks and then on a real network using a new approach based on sensitivity matrices.

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1. Introduction

For the management of WDS is important that the results obtained by the models used for the analyses reflect reality and this is possible by calibration. In the calibration procedures the roughness is calculated using pressure in nodes and flow in pipes as the input parameters. Three different types of approaches to calibration are mentioned in literature: 1) heuristic models, 2) explicit models and 3) implicit models. In heuristic or trial-and-error models [1][2], unknown parameters are updated at each iteration using heads and flows obtained by solving the set of mass balance and energy equations. Explicit models [3] are based on solving an extended set of mass balance and energy equations; initial equation and other equations are derived from the available head and flow measurements. In the last few years, a

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particular attention has been devoted to implicit models, which take into account the measured data by using optimization coupled with a hydraulic solver. Different models and applications of the implicit calibration method were reported by [4][5][6][7][8][9][10][11][12][13][14][15][16][17][18][19]. Kapelan et al. [20] and Veltri et al. [21] proposed further models based on a probabilistic approach by considering the parameters to be estimated as random variables.

To proceed with calibration, a certain number of measurements of pressure at nodes and flow along pipes is always required. These have to be obtained on site and under various operating conditions to provide the most information needed for the calibration model [22]. To do this it is necessary to identify the optimal points where measurements are more sensitive to the variation of roughness and/or flow, so an optimal Sampling Design (SD) is required. The sampling design is used to determine: 1) the magnitude to observe (pressure or flow); 2) when to observe it; 3) where to observe it and 4) under what conditions [23]. Some authors proposed different types of SD, which can be classified under three different categories: D-optimality criteria, A-optimality criteria and V-optimality criteria.

The D-optimality and A-optimality criteria are based on the analysis of the Jacobian matrix: particularly the A-optimality, which minimizes the average parameter variance by minimizing the inverse matrix, whereas D-optimality maximizes the determinant of the same matrix. According to D-optimality criteria, first [5] and then [24] proposed three different sampling design models: 1) Max-Sum; 2) Min-Max and 3) Weighted-Sum. The last type of criterion, V-optimal, is concerned with prediction uncertainty: first [25] and then [26][23][27] used genetic algorithms (GA) single or multi-objective, to solve the SD problem.

In this paper, a new D-optimality based method is proposed to solve the problem of Sampling Design. The choice of measurement points for roughness or demand calibration, according to different operating conditions of the system at a relatively low computational cost, is the first step in order to obtain accurate results.

2. Sensitivity matrices and methodology

In particular, each element of the matrix represents the variation in pressure, head or flow rate, versus the variation of the demand, Q_i, supplied to the i-th node or the roughness coefficient, ε_j, of the j-th pipe.

Each element of the sensitivity matrix for pressure P_i, at nodes and flow rates, q_j, in pipes is as follows:

$$\partial p_{j,i} = \partial p_i / \partial \varepsilon_j \quad \textit{and} \quad \partial q_{j,j} = \partial q_j / \partial \varepsilon_j \tag{1}$$

for roughness coefficient variation matrix and

$$\partial p_{j,i} = \partial p_i / \partial Q_i \quad \textit{and} \quad \partial q_{j,j} = \partial q_j / \partial Q_i \tag{2}$$

for demand rate variation matrix, where $\partial p_{i,i}$ and $\partial q_{i,j}$ are respectively the variation of the load in the i-th node and the variation of the flow circulating in the j-th pipe when varying the coefficient ε of the j-th pipeline; and $p_{i,i}$ and $q_{i,j}$ are respectively the variation of the load in the i-th node and the variation of the flow circulating in the j-th pipe when varying the base demand at the i-th node. Therefore, if *n* and *l* are respectively the number of network nodes and that of pipes, two matrices can be derived for each case, $S_{p,\varepsilon}$, $S_{p,Q}$ and $S_{q,\varepsilon}$, $S_{q,Q}$ and $S_{q,Q}$, that will have the form:

$$S_{p,\varepsilon} = \begin{bmatrix} \partial p_1 / \partial \varepsilon_1 & \cdots & \partial p_1 / \partial \varepsilon_l \\ \vdots & \ddots & \vdots \\ \partial p_n / \partial \varepsilon_1 & \cdots & \partial p_n / \partial \varepsilon_l \end{bmatrix} \quad \textit{and} \quad S_{q,\varepsilon} = \begin{bmatrix} \partial q_1 / \partial \varepsilon_1 & \cdots & \partial q_1 / \partial \varepsilon_l \\ \vdots & \ddots & \vdots \\ \partial q_l / \partial \varepsilon_1 & \cdots & \partial q_l / \partial \varepsilon_l \end{bmatrix} \tag{3}$$

and

$$S_{p,Q} = \begin{bmatrix} \partial p_1/\partial Q_1 & \cdots & \partial p_1/\partial Q_n \\ \vdots & \ddots & \vdots \\ \partial p_n/\partial Q_1 & \cdots & \partial p_n/\partial Q_n \end{bmatrix} \quad \textit{and} \quad S_{q,Q} = \begin{bmatrix} \partial q_1/\partial Q_1 & \cdots & \partial q_1/\partial Q_n \\ \vdots & \ddots & \vdots \\ \partial q_1/\partial Q_1 & \cdots & \partial q_1/\partial Q_n \end{bmatrix} \tag{4}$$

having respectively size (n x l) and (l x l) in the roughness coefficients variation and (n x n) and (l x n) in the base demand variation. The matrices can be obtained in discrete terms (3) and (4), so each element of the roughness coefficient variation is obtained as follows:

$$a_{kj} = \frac{\partial p_i}{\partial \varepsilon_k} \cong \frac{p_{kj} - p_j}{\Delta \varepsilon_k} \tag{3}$$

where a_{kj} is the discrete element of the load matrix, p_{kj} is the value of load at the j-th node after the variation, p_j is the value of load at the j-th node used as prior and $\Delta\varepsilon_k$ is the amount of variation of the roughness coefficient;

$$b_{kj} = \frac{\partial q_i}{\partial \varepsilon_k} \cong \frac{q_{kj} - q_j}{\Delta \varepsilon_k} \tag{4}$$

where b_{kj} is the discrete element of the flow matrix, q_{kj} is the value of the flow at the j-th pipe after the variation, q_j is the value of flow at the j-th pipe used as prior and $\Delta\varepsilon_k$ is again the amount of variation of the roughness coefficient.

Similarly, the discrete elements (5) and (6) of the demand variation matrices are as follows:

$$a_{kj} = \frac{\partial p_i}{\partial Q_k^{'}} \cong \frac{p_{kj} - p_j}{\Delta Q_k^{'}} \tag{5}$$

where a_{kj} is the discrete element of the load matrix, H_{kj} is the value of load at the j-th node after the variation, H_j is the value of load at the j-th node used as prior and $\Delta Q'_k$ is the amount of variation of base demand at the k-th node;

$$b_{kj} = \frac{\partial q_i}{\partial Q_k^{'}} \cong \frac{q_{kj} - q_j}{\Delta Q_k^{'}} \tag{6}$$

where b_{kj} is the discrete element of the flow matrix, q_{kj} is the value of the flow at the j-th pipe after the variation, q_j is the value of flow at the j-th pipe used as prior and $\Delta Q'_k$ is the amount of variation of base demand at the k-th node. Each element of the matrices is subjected to uncertainties due to initial conditions assumed for roughness of pipes and demand at nodes; uncertainty is a consequence of the type of simulation analysis, when using both Demand Driven Analysis (DDA) or Pressure Driven Analysis (PDA).

2.1. Evaluation of results

Each element of matrices indicates the variation of pressure or flow in each node or pipe, respectively, under a variation of roughness in pipes or demand at nodes. Each row of the matrices represents the variation of pressure or flow for the i-th node or j-th pipe versus the variation of roughness in pipes or demand at nodes respectively. So as for the Max-Sum Model, in the method here proposed the more sensitive nodes and pipes are those with the highest sum per line, Σr_{Sp} and Σr_{Sq} respectively. In addition, the proposed method allows an analysis for columns that define nodes and pipes that affect the hydraulic behavior of the WDS by the highest sum per column, Σc_{Sp} and Σc_{Sq}. This is possible because each element of a column defines the variation of the pressure in the node or of the flow in the pipe versus a variation of demand at the same node or roughness in the same pipe.

3. Literature case studies

As already pointed out the objective of this study is to propose a new method to determine the most sensitive pipes and nodes in a water distribution network and where to make measurements, to solve the problem of model calibration. Two literature networks were used for the analyses: the network proposed by [1] and that proposed by [28].

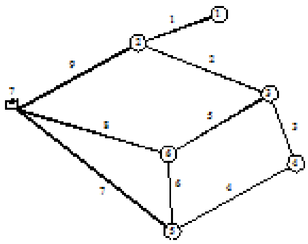

Fig. 1 - Walski's network (1983)

The Walski's network (Fig. 1) consists of one tank, nine pipes, six demand nodes, three loops and one branch. The Greco and Di Cristo's network (Fig. 2) consists of four loops, two nodes of power, eleven nodes of delivery and sixteen pipelines. For each network matrices of pressure and flow were obtained, for both roughness variation in pipe and a demand variation at nodes, by discrete elements, steady-state condition and a random change of 10% of the roughness coefficient ε used as a prior.

3.1. Results

This method produces a list of sensitive nodes and pipes and a list of nodes and pipes that affect the hydraulic behaviour of the network ranked from best to worst, i.e., highest to lowest sensitivity. Best nodes and pipes can be used as measuring points for field data. The following tables report the results of analysis using a variation of roughness in pipes. Table 1 shows the results obtained for the Walski's network for a roughness variation based analysis, where the most sensitive node is NODE 1 (Table 1A) and the most sensitive pipe is PIPE 8 (Table 1B). Both nodes and PIPE 8 (Table 1C) affects the hydraulic behaviour of the network the most.

Table 1 – Walski's network results: A) Sensitive nodes; B) Sensitive pipes; C) Pipes that affect the network

A)	Node	Σr_{Sp}	B)	Pipes	Σr_{Sp}	C)	Pipe	Σc_{Sp}
	1	0.161		8	1.546		8	0.331
	3	0.121		2	1.241		9	0.203
	4	0.119		9	1.222		7	0.096
	5	0.118		5	1.073		1	0.049
	6	0.116		6	1.021		2	0.028
	2	0.115		7	0.898		5	0.023
				3	0.785		4	0.014
				4	0.785		6	0.006

		1	0.000	3	0.004

Table 2 – Walski's network results: A) Sensitive nodes; B) Sensitive pipes; C) Nodes that affect the network

A)	Node	Σr_{Sp}	B)	Pipes	Σr_{Sp}	C)	Pipe	Σc_{Sp}
	1	1.305		8	16.866		6	1.782
	4	1.107		9	11.772		3	1.647
	3	1.089		2	10.881		2	1.143
	5	0.972		5	8.46		1	0.738
	2	0.963		3	6.714		4	0.558
	6	0.945		4	6.453		5	0.513
				6	5.823			
				7	5.391			
				1	2.268			

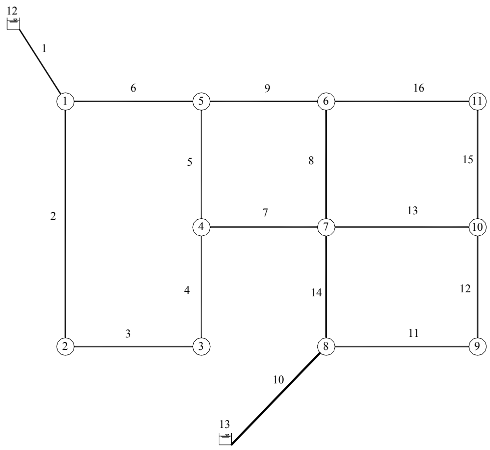

Fig. 2 - Greco's network (1999)

Table 2 shows the results obtained for the Walski's network for a demand variation based analysis, where the most sensitive node is NODE 1 (Table 2A) and the most sensitive pipe is PIPE 8 (Table 2B) and the NODE 6 (Table 2C) affects the hydraulic behaviour of the network the most. For Greco and Di Cristo's network an analysis based on variation of 10% of the roughness coefficient ε with different base demand value was also carried out. The results are in Table 3, which shows for different values of demand (Q, 0.5xQ and 0.7xQ), the most sensitive nodes.

Table 3 –Greco and Di Cristo's network results: A) Sensitive nodes; B) Pipes affecting the network

A)	Q		0,5 Q		0,7 Q		B)	Q		0,5 Q		0,7 Q	
	Node	Σr_{Sp}	Node	Σr_{Sp}	Node	Σr_{Sp}		Pipe	Σc_{Sp}	Pipe	Σc_{Sp}	Pipe	Σc_{Sp}
	5	0.562	5	1.440	5	2.781		1	1.616	1	4.176	1	8.073
	2	0.554	2	1.395	2	2.709		6	1.183	6	2.997	6	5.544
	3	0.554	11	1.233	7	2.673		2	0.557	2	1.809	2	3.204
	11	0.546	4	1.233	6	2.673		11	0.501	10	0.927	11	2.322
	4	0.545	3	1.233	3	2.655		12	0.450	11	0.837	10	2.034

6	0.544	6	1.188	4	2.628	10	0.447	9	0.576	12	1.600
7	0.544	10	1.179	11	2.556	14	0.202	12	0.261	14	0.936
10	0.532	7	1.170	10	2.475	9	0.148	14	0.234	9	0.927
9	0.374	1	1.080	1	1.746	15	0.039	3	0.180	3	0.252
1	0.323	9	0.747	9	1.737	3	0.032	16	0.054	15	0.153
8	0.140	8	0.288	8	0.621	13	0.030	5	0.054	13	0.081
						5	0.013	7	0.045	5	0.063
						8	0.005	15	0.027	8	0.036
						4	0.001	8	0.009	16	0.018
						7	0.001	4	0.000	4	0.009
						16	0.000	13	0.000	7	0.000

Table 4 – Greco and Di Cristo's network results: A) Sensitive pipes; B) Pipes affecting the network

A)	Pipe	ΣrSp	B)	Pipe	ΣcSp
	6	0.993		6	2.723
	9	0.967		2	1.922
	1	0.925		1	1.883
	10	0.925		12	0.989
	11	0.804		11	0.946
	12	0.804		10	0.626
	8	0.765		14	0.540
	7	0.705		9	0.475
	3	0.671		15	0.261
	2	0.671		13	0.260
	4	0.671		5	0.137
	16	0.572		3	0.094
	15	0.572		8	0.025
	13	0.431		16	0.012
	14	0.283		7	0.003
	5	0.147		4	0.002

In particular, NODE 5 (Table 3A) and PIPE 1 affect the hydraulic behaviour of the network for a Demand Driven Analysis (Table 3B). The results for a Pressure Driven Analysis are in Table 4. The most sensitive pipe is PIPE 6 (Table 4A and B).

4. Real case study

Similar analyses were performed for the real network (Fig. 3) of the town of San Mango d'Aquino (CZ, Italy).

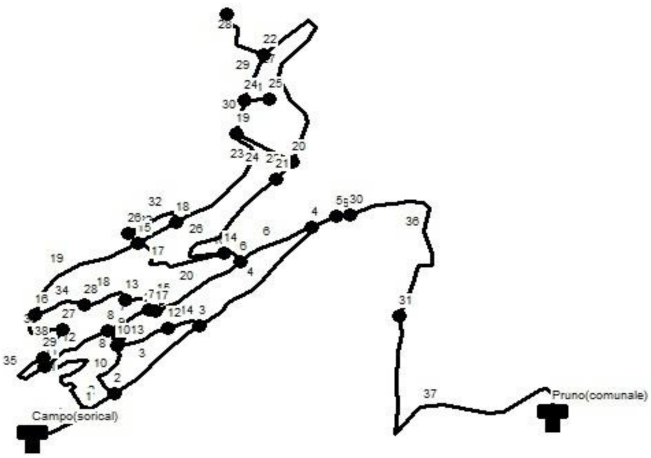

Fig. 3. The network of San Mango d'Aquino (CZ, Italy)

The network (Fig. 3) consist of two sources/tanks, six loops, thirty-one demand nodes, thirty-nine pipes and one valve (flow control). For this network an analysis based on variation of 10% of the roughness coefficient ε with different base demand value was carried out. The results are in tables 5,6 and 7.

Table 5. Results with 10% ε reduction: A) Sensitive nodes; B) Pipes affecting the network

A)	Node	Σr_{Sp}	Node	Σr_{Sp}	Node	Σr_{Sp}	B)	Pipe	Σc_{Sp}	Pipe	Σc_{Sp}	Pipe	Σc_{Sp}
	30	1,746	25	0,495	8	0,450		1	9,387	35	0,108	18	0,009
	31	1,296	13	0,495	18	0,441		39	3,888	17	0,090	12	0
	21	0,648	4	0,495	12	0,441		37	0,945	16	0,063	14	0
	19	0,639	7	0,495	27	0,441		11	0,585	33	0,045	20	0
	20	0,639	24	0,495	14	0,432		23	0,342	19	0,045	21	0
	22	0,612	6	0,468	15	0,432		36	0,333	38	0,036	24	0
	23	0,594	5	0,468	3	0,432		6	0,288	13	0,027	25	0
	29	0,549	26	0,459	9	0,423		2	0,243	15	0,027	27	0
	11	0,504	10	0,459	2	0,405		26	0,189	8	0,027	28	0
	17	0,504	16	0,459	1	0,360		3	0,162	4	0,027	29	0
	28	0,504						22	0,135	30	0,027	31	0
								10	0,120	9	0,018	32	0
								7	0,108	5	0,009	34	0

Table 6 - Results: A) Sensitive pipes; B) Pipes affecting the network

A)	Pipe	Σr_{Sp}	Pipe	Σr_{Sp}	Pipe	Σr_{Sp}	B)	Pipe	Σc_{Sp}	Pipe	Σc_{Sp}	Pipe	Σc_{Sp}
	4	0,405	12	0,207	34	0,117		39	2,025	15	0,135	27	0,045
	2	0,279	13	0,207	10	0,108		6	0,540	16	0,099	20	0,009
	3	0,270	21	0,171	23	0,054		11	0,513	8	0,090	24	0,009
	6	0,243	20	0,171	24	0,054		7	0,207	33	0,090	25	0,009
	1	0,234	8	0,162	25	0,054		13	0,198	38	0,090	1	0

36	0,234	9	0,162	26	0,054	35	0,180	37	0,072	5	0
39	0,234	35	0,153	22	0,054	17	0,180	9	0,072	21	0
5	0,234	38	0,153	30	0,018	19	0,171	22	0,072	28	0
37	0,234	33	0,153	29	0,018	26	0,171	18	0,063	29	0
7	0,225	19	0,144	27	0,018	2	0,162	36	0,054	30	0
11	0,225	16	0,144	28	0	23	0,153	4	0,054	31	0
17	0,216	18	0,117	31	0	10	0,144	14	0,054	32	0
14	0,207	15	0,117	32	0	3	0,135	12	0,054	34	0

Table 5 and 6 show the most sensitive nodes, NODE 30 (Table 5A) and the most sensitive pipes, PIPE 4 (Table 6A). PIPE 1 (Table 5B) is the most affecting the network hydraulic behaviour when a Pressure Driven Analysis is carried out, whereas PIPE 39 (Table 6B) is the most affecting the network hydraulic behaviour when a Demand Driven Analysis is carried out. As for the Greco and Di Cristo's network, an analysis with a different amount of base demand was performed and results are:

Table 7 - Results: A) Sensitive nodes (with a 0.5xQ on left, 0.7xQ at centre and 1.3xQ on right); B) Pipes affect the network (with a 0.5xQ on left, 0.7xQ o centre and 1.3xQ on right)

A)	Node	Σr_{SH}	Node	Σr_{SH}	Node	Σr_{SH}	B)	Pipe	Σc_{SH}	Pipe	Σc_{SH}	Pipe	Σc_{SH}
	30	1,134	30	1,170	30	1,260		39	1,629	39	3,042	1	7,128
	31	0,675	31	0,702	31	0,792		37	0,765	1	1,485	39	5,292
	24	0,189	19	0,414	19	0,774		36	0,288	37	0,873	11	1,152
	4	0,117	25	0,369	21	0,774		4	0,117	36	0,333	37	1,044
	5	0,108	20	0,351	24	0,729		6	0,090	11	0,297	2	0,594

5. Conclusions

In this paper a new method to solve the problem of sampling design with sensitivity analysis (D-optimality criteria based) of water networks is presented. In particular a new method of reading sensitivity matrices is introduced to find pipes and nodes that affect the hydraulic behaviour of the entire system. The method was verified on case studies including both literature networks and a real one, in order to determine which are the most sensitive nodes and pipes and which are nodes and pipes affecting the system versus variation of roughness in pipes and base demand in nodes. The network analyses were carried out with the Epanet model [29] for the literature networks proposed by [1] [28] and the real network of the town of San Mango d'Aquino (CZ, Italy). To check if the methodology obtains stable results about sensitive nodes and pipes, the sensitivity matrices were obtained with the results of networks analysis carried out under different initial conditions. The good quality of the results shows that the method is a good way in order to characterize the measuring points without a high computational cost.

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