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Dynamical superfield theory of free massive superspin-1 multiplet

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Abstract

We construct an $N = 1$ supersymmetric Lagrangian model for the massive superspin-1 superfield. The model is described by a dynamical spinor superfield and an auxiliary chiral scalar superfield. On-shell this model leads to a spin-$\frac{1}{2}$, spin-$\frac{3}{2}$ and two spin-1 propagating component fields. The superfield action is given and its structure in the fermionic component sector is presented. We prove that the most general theory is characterized by a one parameter family of actions. The massless limit is shown to correspond to the dynamics of both the gravitino and superhelicity-$\frac{1}{2}$ multiplets.

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1. Construction of a supersymmetric Lagrangian formulation for free arbitrarily high spin massive fields is still an unsolved problem in field theory. Although free non-supersymmetric Lagrangian models for any integer or half-integer spin massive field have been presented long ago [1], the off-shell supersymmetrization of these theories is a non-trivial problem. For this purpose the results of Ref. [1], given in the conventional field theory formalism, are practically useless. This is to be expected since these theories have a complicated auxiliary field structure, which would lead to complicated off-shell supersymmetry transformations. Therefore, we must develop a new, quite independent approach. The realization of supersymmetric theories is usually carried out in the framework of the superfield formalism. Superfield methods were the basis for solving the analogous problem for supersymmetric massless theories [2] (see also [3,4]) both for Minkowski and for AdS spaces.

In the recent paper [5] we have constructed a superfield Lagrangian model for the $N = 1$ massive multiplet with superspin-$\frac{3}{2}$ having the highest spin-2. This model is a natural off-shell supersymmetric generalization of the known Fierz–Pauli theory [6] and describes the dynamics of massive component fields with the spins 2, $\frac{3}{2}$ and 1. We also found that such a superfield formulation demands taking into account an auxiliary general vector superfield.[1]

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[1] See the on-shell formulation of the dynamical superspin-$\frac{3}{2}$ multiplet in a recent preprint [7].

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The purpose of this Letter is to continue the work [5] and develop a superfield Lagrangian formulation describing the dynamics of a massive multiplet with superspin-1 having the highest component field of spin-3/2. Such a model looks like a supersymmetric generalization of the massive Rarita–Schwinger theory [8]. It is worth noting from the beginning that the structure of the models with integer superspin superfields should differ from theories with half integer superspin. This is analogous to the difference between conventional field models with integer and half integer spin fields [1].

To derive a superfield action for the massive superspin-1 model we will refer to the procedure outlined in Ref. [5]. We begin with a superfield that carries the massive irreducible representation of the Poincaré supergroup. The corresponding representations are characterized by the mass m and superspin Y. On-shell, they contain propagating component fields of spins $(Y - 1/2, Y, Y, Y + 1/2)$ [4]. Using a suitable superfield, we construct the most general quadratic superfield action. This action reproduces the conditions that describe the irreducible representations in the space of superfields [4] as a consequence of the equations of motion. This procedure fails if we work with only the superfields corresponding to the given irreducible representation (we call such superfields physical). The solution to this problem is to couple the physical superfield to an auxiliary superfield within the action. In the case of superspin-1, the role of the physical superfield is played by a complex spinor superfield V_α and the auxiliary superfield is a chiral scalar superfield Φ.

We note activity and recent progress in higher spin field theories. It was noted in Ref. [9] that massive irreducible representations in Minkowski and (A)dS spaces can, in principle, be obtained from massless theories by dimensional reduction. This suggests a derivation of massive models from massless models in higher dimensions. Higher spin massless models have been formulated in arbitrary dimensional constant curvature spaces [10].[2] This may open a possibility for constructing massive models in constant curvature spaces. However, applying the above described procedure to supersymmetric theories cannot be straightforward since supersymmetry has not been universally formulated for all dimensions. Alternatively, supersymmetric massive higher spin field theories may also be derived from superstring theory since any string model contains an infinite tower of higher spin massive modes (see, e.g., [11]). However, there is no guarantee that such an approach leads to field dynamics corresponding to irreducible representations of the Poincaré supergroup. Finally, there was some progress in the study of massive field dynamics in constant curvature space [12–15]. The corresponding supersymmetric formulation is unknown.

This Letter is organized as follows. We discuss the conditions necessary for an arbitrary superfield to form an irreducible massive superspin-1 representation. Then, we construct the superfield action that leads to these conditions as a consequence of the equations of motion. To clarify the structure of the superfield action obtained, we analyze the form of this action in the fermionic sector and show that it reproduces the conventional Rarita–Schwinger and Dirac on-shell equations. We give a brief discussion of the massless limit and find that it is equivalent to standard superfield actions for the gravitino [3] and the superhelicity-$\frac{1}{2}$ supermultiplets [4].

2. We seek a multiplet which contains a massive spin-3/2 field at the $\theta\bar{\theta}$ level. We would also like this theory to be comparable with the known gravitino multiplet in the massless limit. Since the gravitino multiplet is described by a spinor superfield [3], we choose an arbitrary spinor superfield V_α as the physical superfield. We must find the appropriate conditions for this field to form an irreducible representation of massive supersymmetry. To find these conditions we will use the theory of projectors developed in [18] and subsequently modified for massive superfields (see, for example, [4]). First, this field must satisfy the Dirac equation[3]

$$i\partial_{\underline{a}}\overline{V}^{\dot\alpha} + mV_\alpha = 0. \tag{1}$$

[2] Also it is worth pointing out a new development of massless higher spin theories [16,17].

[3] In this equation and throughout this presentation, we use *superspace* [19] conventions. Here an underlined vector index simultaneously denotes the usual Minkowski 4-vector index as well as a pair of undotted and dotted Weyl spinor indices.

Next, we seek the conditions that diagonalize the superspin Casimir operator. The superspin operator, as described in [4], acting on this field is given by

$$\mathbf{C}V_\alpha = m^4\left(\frac{3}{4}I + \frac{3}{4}\mathcal{P}_{(0)} + \mathbf{B}\right)V_\alpha, \tag{2}$$

where $\mathcal{P}_{(0)}$ is the linear subspace projection operator and $\mathbf{B}$ is given by

$$\mathbf{B} = \frac{1}{4m^2}\left(M_{\alpha\beta}\mathbf{P}^\beta{}_{\dot\alpha} - \overline{M}_{\dot\alpha\dot\beta}\mathbf{P}_\alpha{}^{\dot\beta}\right)\left[D^\alpha, \overline{D}^{\dot\alpha}\right]. \tag{3}$$

Here, $\mathbf{P}_{\underline{a}} = -i\partial_{\underline{a}}$, the momentum operator, and $M_{\alpha\beta}$ is the Lorentz generator written in the spinor representation $SL(2,C)$. If V_α is in the chiral subspace, it would satisfy a superspin-1/2 representation since $\mathcal{P}_{(0)}V_\alpha = 0$ and $\mathbf{B}V_\alpha = 0$. If V_α is in the linear subspace, i.e.,

$$D^2V_\alpha = 0, \qquad \overline{D}^2V_\alpha = 0 \tag{4}$$

then the combination $\mathbf{B}V_\alpha$ becomes

$$\mathbf{B}V_\alpha = \frac{1}{8m^2}D_\alpha\overline{D}^2D_\beta V^\beta + \frac{1}{2}V_\alpha. \tag{5}$$

If V_α satisfies the following conditions:

$$D^\alpha V_\alpha = 0, \qquad \overline{D}_{\dot\alpha}\overline{V}^{\dot\alpha} = 0 \tag{6}$$

then $\mathbf{B}V_\alpha = \frac{1}{2}V_\alpha$. Thus $\mathbf{C}V_\alpha = m^4 2V_\alpha = m^4 1(1+1)V_\alpha$, and V_α satisfies a superspin-1 representation. At the component level this representation contains spin-$(\frac{1}{2}, 1, 1, \frac{3}{2})$ fields. The conditions (4) restricts V_α to the linear subspace, while the conditions (6) select out the superspin-1 state of that subspace. Note, that because this field has only one species of spinor index, we do not need the supplementary condition $\partial_{\underline{a}}V^\alpha = 0$ which is usually applied to massive representations.

Thus, we have shown that an irreducible superspin-1 representation can be obtained using the general superfield V_α. At the $\theta\bar\theta$ level, this representation contains a spin-$\frac{3}{2}$ component field. The superfield V_α must satisfy the Dirac equation (1), and be in the linear subspace (4). We also require the conditions (6) to select the superspin-1 state of the linear subspace.

3. Now that we know the conditions required to make V_α a superspin-1 irreducible representation, we search for an action that reproduces these conditions as a consequence of the equations of motion. We proceed by writing the most general action quadratic in V_α. Then, we show that this action cannot produce the required on-shell equations: (1), (4), (6). Finally, we show that the superspin-1 representation can be obtained by coupling V_α to a chiral scalar superfield Φ.

To begin constructing the action, we set the mass dimension of V_α so that the spin-3/2 component field has the canonical mass dimension. The most general action quadratic in V_α and constructed from spinorial covariant derivatives is

$$\begin{aligned}\mathcal{S}[V_\alpha] = \int d^8z\big\{&\alpha_1 V^\alpha D_\alpha\overline{D}_{\dot\alpha}\overline{V}^{\dot\alpha} + \alpha_2 V^\alpha\overline{D}_{\dot\alpha}D_\alpha\overline{V}^{\dot\alpha}\\ &+ m\left(V^\alpha V_\alpha + \overline{V}_{\dot\alpha}\overline{V}^{\dot\alpha}\right)\\ &+ \beta V^\alpha D^2V_\alpha\\ &+ \beta^*\overline{V}_{\dot\alpha}\overline{D}^2\overline{V}^{\dot\alpha}\\ &+ \gamma V^\alpha\overline{D}^2V_\alpha + \gamma^*\overline{V}_{\dot\alpha}D^2\overline{V}^{\dot\alpha}\big\}.\end{aligned} \tag{7}$$

Here α_1 and α_2 are real. The equation of motion for the superfield V_α is

$$\begin{aligned}E_\alpha := \frac{\delta\mathcal{S}}{\delta V^\alpha} &= \alpha_1 D_\alpha\overline{D}_{\dot\alpha}\overline{V}^{\dot\alpha} + \alpha_2\overline{D}_{\dot\alpha}D_\alpha\overline{V}^{\dot\alpha} + 2mV_\alpha\\ &\quad + 2\beta D^2V_\alpha + 2\gamma\overline{D}^2V_\alpha = 0.\end{aligned} \tag{8}$$

Taking $\overline{D}^2E_\alpha = 0$ and setting $\beta = \alpha_1 = 0$, implies that $\overline{D}^2V_\alpha = 0$. The equation of motion now takes the form

$$E_\alpha = \alpha_2\overline{D}_{\dot\alpha}D_\alpha\overline{V}^{\dot\alpha} + 2mV_\alpha = 0. \tag{9}$$

Next, taking $D^\alpha E_\alpha = 0$ yields:

$$-\frac{\alpha_2}{2}D^2\overline{D}_{\dot\alpha}\overline{V}^{\dot\alpha} + 2mD^\alpha V_\alpha = 0 \tag{10}$$

and we are forced to set $\alpha_2 = 0$ if we want $D^\alpha V_\alpha = 0$. With this choice, (9) now reads $2mV_\alpha = 0$, thus eliminating the entire superfield V_α on-shell. This procedure fails whether we choose to set $\overline{D}^2E_\alpha = 0$ first, as in this derivation, or $D^\alpha E_\alpha = 0$ first. This means that the most general action cannot produce the proper on-shell equations to make V_α an irreducible representation. This does not come as a surprise. In Ref. [3], it was shown that the most general *massless* action cannot give rise to a single irreducible representation of supersymmetry [18]. Using the same arguments as in [3], one can see that the addition of mass terms alone cannot ensure that the action describes one irreducible representation. We need a different mechanism to remove the unwanted subspaces and make V_α the proper irreducible representation.

We are forced to couple V_α to an auxiliary field to alleviate the situation. If the auxiliary field vanishes on-shell, the consistency of the equation of motion will then imply a differential constraint on V_α. Once V_α is constrained, the above action can reproduce the correct on-shell conditions. We choose a chiral scalar superfield Φ with the hope of removing the first term in Eq. (10). This term arises from the variation of a coupling term of the form $\Phi \overline{D}^{\dot\alpha} \overline{V}_{\dot\alpha}$. The most general form of the auxilliary sector of the action is the following:

$$\begin{aligned}\mathcal{S}_{\text{aux}}[V_\alpha, \Phi] &= \int d^8 z \left\{ -\frac{1}{2} \Phi D^\alpha V_\alpha - \frac{1}{2} \overline{\Phi} \overline{D}_{\dot\alpha} \overline{V}^{\dot\alpha} + \gamma_1 \Phi \overline{\Phi} \right\} \\ &\quad + \frac{m}{2} \gamma_2 \int d^6 z\, \Phi\Phi + \frac{m}{2} \gamma_2^* \int d^6 \bar{z}\, \overline{\Phi}\overline{\Phi}.\end{aligned} \tag{11}$$

The equations of motion become

$$\frac{\delta(\mathcal{S} + \mathcal{S}_{\text{aux}})}{\delta V^\alpha} = 0 \quad \Rightarrow \quad E_\alpha + \frac{1}{2} D_\alpha \Phi = 0 \tag{12}$$

and for the auxiliary field

$$\begin{aligned}&\frac{\delta \mathcal{S}_{\text{aux}}}{\delta \Phi} = 0 \quad \Rightarrow \\ &\frac{1}{8} \overline{D}^2 D^\alpha V_\alpha - \frac{1}{4} \gamma_1 \overline{D}^2 \overline{\Phi} + m\gamma_2 \Phi = 0.\end{aligned} \tag{13}$$

Note that if $\Phi = 0$ we would have $\overline{D}^2 D^\alpha V_\alpha = 0$, the desired differential constraint. Contracting $\overline{D}^2 D^\alpha$ on (12) we have

$$\gamma_3 \overline{D}^2 D^2 \overline{D}_{\dot\alpha} \overline{V}^{\dot\alpha} + 2m \overline{D}^2 D^\alpha V_\alpha + 8\Box\Phi = 0, \tag{14}$$

where $\gamma_3 = \alpha_1 - \frac{1}{2}\alpha_2$. Using the equation of motion for Φ we have

$$\begin{aligned}&\{1 + 4\gamma_1\gamma_3\} 8\Box\Phi + \{\gamma_1 - 2\gamma_2^*\gamma_3\} 4m \overline{D}^2 \overline{\Phi} \\ &\quad - 16m^2 \gamma_2 \Phi = 0.\end{aligned} \tag{15}$$

The following choices of coefficients will constrain Φ to vanish

$$\gamma_1 = -\frac{1}{4\gamma_3}, \qquad \gamma_2 = -\frac{1}{8\gamma_3{}^2}. \tag{16}$$

From (13), the vanishing of Φ implies the following condition on V_α

$$\overline{D}^2 D_\alpha V^\alpha = 0, \qquad D^2 \overline{D}_{\dot\alpha} \overline{V}^{\dot\alpha} = 0. \tag{17}$$

The equation of motion for V_α, now takes the same form as Eq. (8). Multiplying by $\overline{D}^2$ and setting $\alpha_1 = \beta = 0$ we have $\overline{D}^2 V_\alpha = 0$, and $D^2 \overline{V}_{\dot\alpha} = 0$. Then the equation of motion becomes (9) exactly. Contracting D^α on Eq. (9) now yields $2m D^\alpha V_\alpha = 0$. With this result, V_α is fully irreducible in the superspace. The equation of motion now takes the form of the Dirac equation

$$-2i\alpha_2 \partial_{\underline{a}} \overline{V}^{\dot\alpha} + 2m V_\alpha = 0. \tag{18}$$

Thus, with $\alpha_2 = -1$, V_α will satisfy (1), (4), and (6) on-shell. We can now write the full action

$$\begin{aligned}\mathcal{S}[V_\alpha, \Phi] &= \int d^8 z \left\{ -V^\alpha \overline{D}_{\dot\alpha} D_\alpha \overline{V}^{\dot\alpha} + m\left(V^\alpha V_\alpha + \overline{V}_{\dot\alpha} \overline{V}^{\dot\alpha}\right) \right. \\ &\qquad + \gamma V^\alpha \overline{D}^2 V_\alpha + \gamma^* \overline{V}_{\dot\alpha} D^2 \overline{V}^{\dot\alpha} - \frac{1}{2} \Phi \overline{\Phi} \\ &\qquad \left. - \frac{1}{2} \Phi D^\alpha V_\alpha - \frac{1}{2} \overline{\Phi} \overline{D}_{\dot\alpha} \overline{V}^{\dot\alpha} \right\} \\ &\quad - \frac{m}{4} \int d^6 z\, \Phi\Phi - \frac{m}{4} \int d^6 \bar{z}\, \overline{\Phi}\overline{\Phi}.\end{aligned} \tag{19}$$

We would like to point out that all coefficients have been determined except for γ. The γ terms are interesting because they are purely auxiliary. We illustrate this by formally integrating over the superspace, i.e.,

$$\begin{aligned}&\int d^8 z \left\{ \gamma V^\alpha \overline{D}^2 V_\alpha \right\} \\ &\quad = \frac{1}{8} \int d^4 x \left\{ \gamma D^2 \overline{D}^2 V^\alpha \big| \overline{D}^2 V_\alpha \big| \right. \\ &\qquad \left. - \frac{\gamma}{2} D^\beta \overline{D}^2 V^\alpha \big| D_\beta \overline{D}^2 V_\alpha \big| \right\}.\end{aligned} \tag{20}$$

On-shell (20) vanishes since $\overline{D}^2 V_\alpha = D^2 V_\alpha = 0$. We can also exhibit the irrelevance of these terms by considering the following field redefinition:

$$V_\alpha \to V_\alpha + \frac{a}{m} \overline{D}^2 V_\alpha, \tag{21}$$

where a is an arbitrary complex number. With this redefinition we can change γ to any arbitrary number, without influencing the on-shell results of the Lagrangian. Although these terms are purely auxiliary in the massive theory, they will play an important role in understanding the massless limit.

We have given a two real parameter family of actions, governed by one complex parameter. These actions lead to the on-shell equations which describe an irreducible superspin-1 multiplet. The off-shell degrees of freedom include the physical spinor superfield V_α and an auxiliary chiral scalar field Φ. On-shell $\Phi = 0$, and V_α becomes an irreducible representation of the Poincaré supergroup having superspin-1.

4. Here we give the fermionic component action and show how the equations of motion reproduce the correct fermionic massive representations. We will see that there are two propagating fermions of spin-$\frac{1}{2}$ and spin-$\frac{3}{2}$. The spin-$\frac{3}{2}$ field has no divergence and satisfies the Rarita–Schwinger equation, while the spin-$\frac{1}{2}$ field satisfies the Dirac equation.

Using the following component definitions:

$$\begin{aligned} &V_\alpha| = \lambda_\alpha, \qquad D_\alpha \Phi| = \phi_\alpha, \\ &-\frac{1}{4}D^2 V_\alpha\Big| = \eta_\alpha, \qquad -\frac{1}{4}\overline{D}^2 V_\alpha\Big| = \chi_\alpha, \\ &D_\alpha \overline{D}_{\dot\alpha} V_\beta\Big| = \psi_{\alpha\dot\alpha\beta}, \qquad \frac{1}{16}D^2\overline{D}^2 V_\alpha\Big| = \Lambda_\alpha, \end{aligned} \tag{22}$$

we formally integrate over the fermionic coordinates. The fermionic sector of the action becomes

$$\begin{aligned} \mathcal{S}_f = \int d^4x \bigg\{ & \psi^{\beta\underline{a}}\left(\frac{i}{4}\partial_{\underline{b}}\bar\psi^{\dot\beta}{}_{\underline{a}} + \frac{m}{4}\psi_{\beta\underline{a}} - \frac{i}{4}\partial_{\underline{a}}\phi_\beta\right) \\ & + \frac{i}{8}\phi^\alpha\partial_{\underline{a}}\bar\phi^{\dot\alpha} + \frac{m}{8}\phi^\alpha\phi_\alpha + \frac{1}{2}\phi^\alpha\Lambda_\alpha \\ & + \Lambda^\alpha\bar\psi_{\dot\alpha\alpha}{}^\alpha + 2m\Lambda^\alpha\lambda_\alpha \\ & - 8\gamma\chi^\alpha\Lambda_\alpha + i\chi^\alpha\partial_{\underline{a}}\bar\chi^{\dot\alpha} \\ & + 2m\chi^\alpha\eta_\alpha + \text{h.c.}\bigg\}. \end{aligned} \tag{23}$$

It is trivial to see that on-shell $\Lambda_\alpha = \chi_\alpha = \eta_\alpha = 0$. By taking the divergence of the $\bar\psi$ equation of motion, such that the $\bar\phi$ term becomes $i\Box\bar\phi^{\dot\alpha}$, and substituting the ϕ equation of motion, we see that $\phi_\alpha = 0$. This leaves the following equations of motion:

$$i\partial_{\underline{b}}\bar\psi^{\dot\beta}{}_{\underline{a}} + m\psi_{\beta\underline{a}} = 0, \tag{24}$$

$$\partial^{\underline{b}}\psi_{\alpha\underline{b}} = 0, \tag{25}$$

$$\bar\psi_{\underline{a}}{}^{\dot\alpha} + 2m\lambda_\alpha = 0. \tag{26}$$

The trace of (24) − (25) yields the Dirac equation on $\psi^\alpha{}_{\dot\alpha\alpha}$:

$$i\partial_{\underline{a}}\psi^{\beta\dot\alpha}{}_\beta + m\bar\psi^{\dot\beta}{}_{\alpha\dot\beta} = 0. \tag{27}$$

Thus, (26) implies that λ_α also satisfies the Dirac equation. Using these facts, one can show that the trace of (24) + (25) leads to

$$\begin{aligned} &\partial^{\underline{b}}\bar\psi_{(\dot\alpha\dot\beta)\beta} + 2i\Box\bar\lambda_{\dot\alpha} = 0 \quad \Rightarrow \\ &\partial^{\underline{b}}\left(\bar\psi_{(\dot\alpha\dot\beta)\beta} - \frac{2}{3}i\partial_{\beta(\dot\alpha}\bar\lambda_{\dot\beta)}\right) = 0. \end{aligned} \tag{28}$$

Taking this symmetric divergenceless field as the gravitino

$$\widehat{\overline{\boldsymbol{\Psi}}}_{\dot\alpha\dot\beta\beta} := \bar\psi_{(\dot\alpha\dot\beta)\beta} - \frac{2}{3}i\partial_{\beta(\dot\alpha}\bar\lambda_{\dot\beta)} \tag{29}$$

we can now show that the symmetric part of (24) implies the Rarita–Schwinger equation on $\widehat{\boldsymbol{\Psi}}$:

$$\begin{aligned} &-\frac{i}{4}\partial^{\dot\beta}{}_{(\beta}\bar\psi_{\alpha)(\dot\alpha\dot\beta)} + \frac{i}{4}\partial_{\dot\alpha(\beta}\bar\psi^{\dot\gamma}{}_{\alpha)\dot\gamma} + \frac{m}{2}\psi_{(\alpha\beta)\dot\alpha} \\ &\quad = -\frac{i}{4}\partial^{\dot\beta}{}_{(\beta}\widehat{\overline{\boldsymbol{\Psi}}}_{\alpha)\dot\alpha\dot\beta} + \frac{m}{2}\widehat{\boldsymbol{\Psi}}_{\alpha\beta\dot\alpha} \\ &\qquad + \left(-\frac{1}{6} + \frac{1}{2} - \frac{1}{3}\right)\partial_{\dot\alpha(\beta}\partial_{\alpha)\dot\beta}\bar\lambda^{\dot\beta} = 0. \end{aligned} \tag{30}$$

Here, we have used (26) and the Dirac equation on $\bar\lambda_{\dot\beta}$ extensively. Thus, the gravitino satisfies the Rarita–Schwinger equation

$$\begin{aligned} &-\frac{i}{2}\partial^{\dot\beta}{}_{(\beta}\widehat{\overline{\boldsymbol{\Psi}}}_{\alpha)\dot\alpha\dot\beta} + m\widehat{\boldsymbol{\Psi}}_{\alpha\beta\dot\alpha} \\ &\quad = \frac{1}{2}\epsilon_{\underline{abcd}}\partial^{[c}\widehat{\overline{\boldsymbol{\Psi}}}{}^{d]\dot\beta} + m\widehat{\boldsymbol{\Psi}}_{\alpha\beta\dot\alpha} = 0. \end{aligned} \tag{31}$$

In summary, the fermionic sector of (19) describes two propagating fermions. $\widehat{\boldsymbol{\Psi}}_{(\alpha\beta)\dot\beta}$ satisfies (31) and (28), thus, forming a spin-$\frac{3}{2}$ representation, and $\psi^{\alpha\dot\beta}{}_\alpha$ forms a spin-$\frac{1}{2}$ representation. All other fermionic components of V_α vanish except, for λ_α which is proportional to $\psi^{\alpha\dot\beta}{}_\alpha$.

5. The analysis of the massless limit of the action (19) is quite interesting. As it turns out, this limit describes a reducible representation of the massless Poincaré supergroup for arbitrary values of γ. But, in the special case when $\gamma = \frac{1}{4}$, the representation is irreducible and corresponds to the standard gravitino multiplet.

The irreducible representation corresponding to the gravitino multiplet is given by the chiral field strength $W_{\alpha\beta} := \overline{D}^2 D_{(\alpha} V_{\beta)}$ [3] and it's equation of motion:

$$D^\alpha W_{\alpha\beta} = 0. \tag{32}$$

It is invariant under the gauge transformations:

$$\begin{aligned} &\delta V_\alpha = \Lambda_\alpha + i D_\alpha K, \\ &K = \overline{K}, \qquad \overline{D}_{\dot\alpha} \Lambda_\alpha = 0. \end{aligned} \tag{33}$$

We will show that for arbitrary values of γ this multiplet is contained in the massless limit of (19). The gravitino multiplet is described in detail in [3,20,21]. The connection between the different formalisms is stated rather concisely in [4].

Setting $m = 0$ leads us to the following action:

$$\begin{aligned} &\mathcal{S}_{m=0}[V_\alpha, \Phi] \\ &\quad = \int d^8 z \Big\{ -V^\alpha \overline{D}_{\dot\alpha} D_\alpha \overline{V}^{\dot\alpha} + |\gamma| V^\alpha \overline{D}^2 V_\alpha \\ &\qquad + |\gamma| \overline{V}_{\dot\alpha} D^2 \overline{V}^{\dot\alpha} - \frac{1}{2}\Phi\overline{\Phi} - \frac{1}{2}\Phi D^\alpha V_\alpha \\ &\qquad - \frac{1}{2}\overline{\Phi}\overline{D}_{\dot\alpha}\overline{V}^{\dot\alpha} \Big\}. \end{aligned} \tag{34}$$

Here, we have absorbed the phase of $\gamma = |\gamma| e^{i\phi}$ by making the following field redefinitions:

$$V_\alpha \to e^{-\frac{i}{2}\phi} V_\alpha, \qquad \Phi \to e^{\frac{i}{2}\phi}\Phi. \tag{35}$$

This is only possible now because $m = 0$.

To see what gauge symmetries are present we look for the massless representations implied by the equations of motion. The equations of motion for V_α and Φ are:

$$E'_\alpha := -\overline{D}_{\dot\alpha} D_\alpha \overline{V}^{\dot\alpha} + 2|\gamma| \overline{D}^2 V_\alpha + \frac{1}{2} D_\alpha \Phi = 0, \tag{36}$$

$$\frac{1}{8} D^2 \Phi + \frac{1}{8} D^2 \overline{D}_{\dot\alpha} \overline{V}^{\dot\alpha} = 0. \tag{37}$$

Taking the divergence of (36), i.e., $\partial_{\dot\alpha}{}^\alpha E'_\alpha$, and substituting for Φ using (37) we find

$$\begin{aligned} \partial_{\dot\beta}{}^\alpha E'_\alpha &= -\frac{i}{8} \overline{D}^{\dot\alpha} \overline{W}_{(\dot\alpha\dot\beta)} + \frac{i}{2}\left(4|\gamma|^2 - \frac{1}{4}\right) \overline{D}^2 D^2 \overline{V}_{\dot\beta} \\ &= 0. \end{aligned} \tag{38}$$

Note that if $|\gamma| = \frac{1}{4}$ we have the equation of motion for $W_{\alpha\beta}$, (32). At this point we turn our attention to the following projection of (36):

$$D^\alpha \frac{\overline{D}^2 D^2}{16\Box} E'_\alpha = \frac{1}{2} \overline{D}_{\dot\alpha} D^2 \overline{V}^{\dot\alpha} + 2|\gamma| D^\alpha \overline{D}^2 V_\alpha = 0. \tag{39}$$

This equation is of the form, $4|\gamma| H = -\overline{H}$, which requires that either $|\gamma| = \frac{1}{4}$ or $D^\alpha \overline{D}^2 V_\alpha = 0$. Since $D^\alpha \overline{D}^2 V_\alpha = 0 \Rightarrow D^2 \overline{D}^2 V_\alpha = 0$, we see that (38) is equivalent to the equation of motion of $W_{\alpha\beta}$, (32), for any value of $|\gamma|$, meaning that $W_{\alpha\beta}$ propagates for arbitrary γ.

When $|\gamma| = \frac{1}{4}$ the action (34) is equivalent to the descriptions of the gravitino multiplet given in the literature.[4] This was shown in detail in [4]. In the case where $|\gamma| \neq \frac{1}{4}$, Eq. (39) implies the equation of motion for a superhelicity-$\frac{1}{2}$ representation. That is, $D^\alpha \overline{D}^2 V_\alpha = 0$ means that the chiral field strength $\Omega_\alpha = \overline{D}^2 V_\alpha$ is an on-shell representation of the massless Poincaré supergroup. The gauge transformations that leave both $W_{\alpha\beta}$ and Ω_α invariant are:

$$\begin{aligned} &\delta V_\alpha = \Lambda_\alpha + i D_\alpha K, \\ &\overline{D}_{\dot\alpha} K = 0, \qquad \overline{D}_{\dot\alpha} \Lambda_\alpha = 0. \end{aligned} \tag{40}$$

In comparison to the gauge transformations that leave only $W_{\alpha\beta}$ invariant, (33), there is less gauge freedom here since K is a chiral field.

We have shown that the $m = 0$ limit of the superspin-1 model (19) generically forms a reducible representation of the massless Poincaré supergroup. This massless model (34) contains superhelicity-1 and superhelicity-$\frac{1}{2}$ irreducible representations. Furthermore, if we set $\gamma = \frac{1}{4}$, this model contains only the superhelicity-1 state, and is equivalent to the gravitino multiplet.

6. To summarize, we have presented a new $4D$, $N = 1$ supersymmetric model which describes propagating spin-3/2, spin-1 and spin-1/2 massive fields. The model is completely formulated in terms of a spinor superfield V_α and chiral scalar superfield Φ.

[4] We point out that there exists another off-shell formulation of the gravitino multiplet in the literature (see second paper in [2] and [4]). Such a formulation is given in terms of a constrained complex linear transverse vector superfield or equivalently in terms of spin-tensor superfield $\Psi_{\alpha\beta\dot\alpha}$.

The superfield V_α is propagating and carries the superspin-1 massive irreducible representation of the Poincaré supergroup. The chiral superfield Φ is auxiliary and its role is to ensure the existence of a Lagrangian formulation that is compatible with the conditions defining the massive irreducible representation of superspin-1. The corresponding superfield action is given by Eq. (19).

Eq. (19) actually represents a two parametric family of actions that all lead to the same on-shell dynamics. In the massless limit, this two parametric family of actions becomes a one parametric family of actions (34). These massless models describe propagating helicity-3/2, helicity-1/2 and two helicity-1 fields.

In terms of massive theories with arbitrarily high integer superspin, the superspin-1 theory is the simplest. In the same sense the model constructed in Ref. [5] is simplest Lagrangian for half-integer higher superspin massive fields. We believe that these two models can be considered as the basis for constructing Lagrangian superfield models with arbitrary integer and half-integer superspins.

Note added

Near the completion of our work, we noted the appearance of a work by Engquist, Sezgin and Sundell [22]. Their work seems closely related to both the topic of this Letter as well as to some of the research that has appeared in [2] which presented results on AdS geometry, SUSY and higher spin multiplets.

"The art of doing mathematics consists in finding that special case which contains all the germs of generality."

—D. Hilbert

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