



ELSEVIER

SCIENCE @ DIRECT®

PHYSICS LETTERS B

Physics Letters B 566 (2003) 137–141

[www.elsevier.com/locate/npe](http://www.elsevier.com/locate/npe)

# The wavelength of neutrino and neutral kaon oscillations

H. Burkhardt<sup>a,1</sup>, J. Lowe<sup>a</sup>, G.J. Stephenson Jr.<sup>a</sup>, T. Goldman<sup>b</sup>

<sup>a</sup> *Physics and Astronomy Department, University of New Mexico, Albuquerque, NM 87131, USA*

<sup>b</sup> *Theoretical Division, Los Alamos National Laboratory, Los Alamos, NM 87545, USA*

Received 7 March 2003; received in revised form 1 May 2003; accepted 23 May 2003

Editor: H. Georgi

## Abstract

Neutral kaons, and probably also neutrinos, exhibit oscillations between flavor eigenstates, as a result of being produced in a superposition of mass eigenstates. Several recent papers have addressed the question of the energies and momenta of the components of these states, and their effect on the coherence of the states and on the wavelength of the oscillations. We point out that the mass eigenstates need have neither equal momentum nor equal energy, but can nevertheless be coherent, and that a correct treatment of the kinematics recovers the usual result for the wavelength of the flavor oscillations.

© 2003 Published by Elsevier B.V. Open access under [CC BY license](http://creativecommons.org/licenses/by/2.0/).

PACS: 13.20.Eb; 13.25.Es; 14.60.Pq

Keywords: Neutrinos; Kaons; Oscillations

## 1. Introduction

When neutral particles are produced in a flavor eigenstate that is not also a mass eigenstate, the resultant system is, in general, a superposition of mass eigenstates. If so, the system may oscillate between the different flavor eigenstates. This situation has been familiar for many years for the case of neutral kaons; if these are produced in one of the strangeness eigenstates,  $K^0$  ( $S = 1$ ) or  $\bar{K}^0$  ( $S = -1$ ), the system is a superposition of the mass eigenstates,  $K_L$  and  $K_S$ , and oscillates between the two strangeness eigenstates. Re-

cently, strong evidence has been found that neutrinos show a similar behaviour [1–4].

The standard quantum-mechanical treatment of kaon oscillations [5] has been known for many years and results in an expression relating the wavelength of the strangeness oscillations to the mass difference between the mass eigenstates,  $\delta m = m_L - m_S$ . In the last decade, several papers have appeared which question this treatment, sometimes resulting in a different relation between the wavelength and  $\delta m$ . Srivastava et al. [6] derived a relation that differs by at least a factor of 2 from the standard result. The origin of this factor was studied by Lowe et al. [7] and by Burkhardt et al. [8] who found an error in Ref. [6], and demonstrated that the standard result is recovered when this error is corrected.

*E-mail address:* [lowe@panda3.phys.unm.edu](mailto:lowe@panda3.phys.unm.edu) (J. Lowe).

<sup>1</sup> Also at Shell Centre for Mathematical Education, University, Nottingham NG7 2RD, England.

Other treatments [9–16] have studied some consequences of differing assumptions about the system and, in particular, the energies and momenta of the mass eigenstates. Since the  $K_L$  and  $K_S$  states that make up the oscillating  $K^0$ – $\bar{K}^0$  system have different masses, they cannot have *both* the same momentum *and* the same energy. For example, Lipkin and collaborators [9,13] have studied the consequences of assuming either equal momentum or equal energy for the  $K_L$  and  $K_S$ . In several of the above papers these two kinematic assumptions are examined, and some papers predict a wavelength for the oscillations which differs by a factor of exactly 2 from the standard treatment. A recent paper by Okun et al. [16] gives a concise summary of the situation.

However, we are not free to choose the energy and momentum of the mass eigenstates. Usually, the neutral particles are produced either in a reaction (e.g.,  $\pi^- p \rightarrow \Lambda K^0$ ) for the case of kaons, or a decay (e.g.,  $\pi \rightarrow \mu \nu_\mu$ ) for neutrinos. In either case, the mass eigenstates have neither the same energy nor the same momentum for a given center-of-mass energy (mass in the rest frame of the source). The energies and momenta are determined by the kinematics. This was pointed out initially by Boehm and Vogel [17], Goldman [18], Srivastava et al. [6], Dolgov [15] and also by the present authors [7,8], who showed that when this is taken into account correctly, a consistent treatment of the kinematics follows and the standard result for the wavelength of oscillations is recovered.

However, Lipkin et al. [9,13] and Stodolsky [12] claim that the two mass eigenstates must have the same energy. In particular, Lipkin [13] states that mass eigenstates with different energy cannot interfere to produce the oscillations. In Section 2 we show that this argument is incorrect and that the  $K_L$  and  $K_S$  states can indeed interfere to give strangeness oscillations. In Section 3, we show that the mass eigenstates with kinematically correct energies and momenta produce oscillations with the same wavelength as in the standard treatment, without any additional factors.

## 2. Coherence of the mass eigenstates

In this section, we examine the coherence of two interfering wave functions. Suppose the wave functions are plane waves,  $\psi_1$  and  $\psi_2$ . These might

be, for example, two parts of the wave function in a 2-slit optical experiment, or an electron diffraction experiment. If so, they will have the same energy and momentum, but for generality, we keep both energies and momenta distinct for now.

The wave function at the point of interference,  $(x, t)$ , is

$$\begin{aligned} \psi &= \psi_1 + \psi_2 \\ &= \sqrt{\frac{1}{2}} [\exp\{i(p_1 x - E_1 t)\} \\ &\quad + \exp\{i(p_2 x - E_2 t + \phi)\}], \end{aligned} \quad (1)$$

where  $\phi$  is some phase angle introduced by the geometry (for example, the path difference between the two slits to the interference point or some difference induced at the presumed common source of the two components). We assume only that  $\phi$  is fixed and does not vary, e.g., randomly for different times due, for example, to fluctuations in the background medium (e.g., due to index of refraction fluctuations). It is in this sense that we may refer to the two waves as “coherent” [19]. The probability density is

$$|\psi|^2 = 1 + \cos[(p_1 - p_2)x - (E_1 - E_2)t + \phi]. \quad (2)$$

In general, the second term oscillates in time with a time period characteristic of the energy scale of the system. For the optical case, this is  $\sim 10^{-15}$  s, and in particle-physics experiments, the characteristic time is much shorter. In either case, this is well below the time resolution of any normal detector, so the second term averages to zero in the measurement, and  $\psi_1$  and  $\psi_2$  are therefore incoherent in the sense of Lipkin et al. [9,13]. There may be other reasons for the cross term in Eq. (2) to vanish (e.g., different spin wave functions or different internal states of particles associated with  $\psi_1$  and  $\psi_2$ ), but in the absence of any such reason, the rapid time dependence of the energy term is the only reason why  $\psi_1$  and  $\psi_2$  are orthogonal when  $E_1$  and  $E_2$  are different. It is this energy term that seems to be the basis for statement often made (e.g., in [13]) that the kaon mass eigenstates are incoherent, and will not interfere, unless they have the same energy.

However, we must examine Eq. (2) in the particular case where  $\psi_1$  and  $\psi_2$  are  $K_L$  and  $K_S$  states. Here,  $E_1 - E_2$  is of order  $\delta m$ , which is  $\sim 3 \times 10^{-6}$  eV. The associated time scale is  $\sim 2 \times 10^{-10}$  s, which is readily measurable. Thus, even in this plane-wave case (that

is, in the absence of packeting), the response of a detector to  $|\psi|^2$  at a fixed point  $x$  will oscillate measurably in time as given by Eq. (2). Furthermore, in actual experiments, the kaons do not appear in continuous plane waves, but as wave packets. Since kaon velocities in experiments are usually an appreciable fraction of  $c$ , the distance scale is many cm, again readily measurable. Of course, if the measurement averages over time or distance scales large compared with these values, then the cross term vanishes and the states become incoherent.

Thus, although the fact that the kinematics of the kaon (or neutrino) production process preclude equal energy for the two interfering states, the states may nevertheless be coherent, in the sense that we refer to above, and interference may be observable in certain situations.

### 3. Kinematics

Here, we calculate the kinematics for a specific case. For definiteness, we choose the  $K^0-\bar{K}^0$  example rather than neutrinos because:

- (i) There are just two states rather than three or more;
- (ii) Accurate numerical values are known for the masses and the mass difference;
- (iii) For neutrinos, it has been argued that a full treatment requires the inclusion of the detector in the system [13]. However, for kaons, the  $\Delta S = \Delta Q$  rule implies that the  $K^0$  and  $\bar{K}^0$  components can be identified from the kaon decay, without the need for a specific detector, thus simplifying the problem.

Neutrinos or kaons are produced either by a reaction such as

$$\pi p \rightarrow \Lambda K^0$$

or by a decay, for example,

$$\pi \rightarrow \mu\nu\mu.$$

For the first of these, the center-of-mass energies and momenta of the mass eigenstates are given by

$$p_i^2 = \frac{(s - m_i^2 - m_A^2)^2 - 4m_i^2 m_A^2}{4s},$$

$$E_i = \frac{s + m_i^2 - m_A^2}{2\sqrt{s}}, \tag{3}$$

where  $i = S$  or  $L$ . Similar expressions hold for the neutrinos from pion decay. The quantity  $\sqrt{s}$  is the total center-of-mass energy for a reaction or the mass of the decaying particle for a decay. Although there may be a spread in the value of  $\sqrt{s}$  for the overall system wave packet, our analysis proceeds component by component, i.e., at a precise value of  $\sqrt{s}$  (within the constraints of the uncertainty principle). Thus  $p_S \neq p_L$  and  $E_S \neq E_L$ . In the following, to make the equations more readable, we ignore CP violation and we omit the widths of the kaon states.

Since the above reaction produces a pure  $K^0$  state, the wave function at the reaction point, where  $x = t = 0$ , is

$$|K^0\rangle = \sqrt{\frac{1}{2}}(|K_L\rangle + |K_S\rangle). \tag{4}$$

This state develops in time as

$$\psi(x, t) = \sqrt{\frac{1}{2}} \{ \exp[i(p_L x - E_L t)] |K_L\rangle + \exp[i(p_S x - E_S t)] |K_S\rangle \}. \tag{5}$$

Since Eq. (3) give  $p_i$  and  $E_i$  in the center-of-mass frame, the  $x$  and  $t$  in Eq. (5) should also be in this frame. However, Eq. (5) and all following equations involve only invariants, so may be reinterpreted in the lab frame. Note that the two components in Eq. (5) are coherent in the sense that we defined above: no fluctuation of the relative phase occurs at the source. Under the assumption of propagation in a vacuum, there is also no medium to induce phase fluctuations coupled to the medium. At  $(x, t)$ , the probability amplitude for detecting a  $K^0$  is

$$\begin{aligned} \langle K^0 | \psi(x, t) \rangle &= \frac{1}{2} \{ \exp[i(p_L x - E_L t)] + \exp[i(p_S x - E_S t)] \} \end{aligned} \tag{6}$$

using  $\langle K^0 | K_L \rangle = \langle K^0 | K_S \rangle = \sqrt{1/2}$ . This is just as Eq. (1) above (with  $\phi = 0$ ), giving the probability density as

$$\begin{aligned} |\langle K^0 | \psi(x, t) \rangle|^2 &= \frac{1}{2} \{ 1 + \cos[(p_L - p_S)x - (E_L - E_S)t] \}. \end{aligned} \tag{7}$$

Eq. (7) describes a plane-wave situation, with a unique value of  $\sqrt{s}$ , and with  $x$  and  $t$  as independent variables. In a realistic case, it would be used to form wave packets for all particles, with a spread of values of  $\sqrt{s}$ . The size of such wave packets might be determined, for example, by the time and position resolution of a detector in the incident beam or by the time structure of the accelerator beam. In any case, the packet cannot be larger than the  $K_S$  lifetime,  $\tau_S \sim 0.9 \times 10^{-10}$  s, which gives a special extent of typically about 2 cm. Thus the outgoing kaon moves in a packet of this size, centered at the classical position. So the observation of the kaon at position  $x$  must be made at a time when the packet is present, i.e., at time that is equal to  $x/\beta$  or which differs from it by no more than the half-width of this wave packet. We therefore replace  $t$  in Eq. (7) with the time defined in this way using beta calculated from the average  $K_L$  and  $K_S$  parameters:

$$\beta = \frac{p_L + p_S}{E_L + E_S}, \quad (8)$$

which lies between the velocities  $p_L/E_L$  and  $p_S/E_S$ . It is crucial that the observation is made at a *single* space–time point; no meaning can be attached to the interference of wave functions at different values of  $x$  or  $t$  (see [8,16]). In any realistic case, the separation of the centres of the  $K_L$  and  $K_S$  wave packets is much smaller than their size, so in practice, there is no loss of coherence due to separation of the  $K_L$  and  $K_S$  packets.

Thus Eq. (7) becomes

$$\begin{aligned} & |\langle K^0 | \psi(x, t) \rangle|^2 \\ &= \frac{1}{2} \left[ 1 + \cos \left\{ (p_L - p_S)x - \frac{(E_L - E_S)}{\beta} x \right\} \right]. \end{aligned} \quad (9)$$

Using

$$\begin{aligned} \frac{1}{\beta} &= \frac{E_L^2 - E_S^2}{(p_L + p_S)(E_L - E_S)} \\ &= \frac{p_L - p_S}{E_L - E_S} + \frac{2m\delta m}{(E_L - E_S)(p_L + p_S)}, \end{aligned} \quad (10)$$

Eq. (9) becomes

$$|\langle K^0 | \psi(x, t) \rangle|^2 = \frac{1}{2} \left[ 1 + \cos \frac{m\delta m}{p} x \right], \quad (11)$$

where  $m$  and  $p$  are the mean neutral kaon mass and momentum. Thus the wave number of strangeness oscillations in space is  $k = m\delta m/p$ . This is often

expressed as an oscillation in time, in which case, the angular frequency is  $\omega = m\delta m/E$ . Both of these results are in agreement with the standard results [5], without any assumption about equality of momenta or energies.

#### 4. Discussion

We conclude that a treatment of kaon and neutrino oscillations, with the kinematics treated in full, does indeed give the correct relation between the mass difference and the oscillation wavelength. Most of the recent literature is in agreement with this standard result for the wavelength, though sometimes using incorrect kinematics. If CP violation and the finite kaon lifetimes are incorporated in the above algebra, a more realistic result is obtained, with more cumbersome-looking equations, but the wavelength of the oscillations remains the same. The full equations are given in [7,8].

Lipkin [13] has suggested that the detector should also be included in the wave function of the system, since interaction with it depends on the details of the neutrino or kaon wave function. By choosing the kaon system here, we avoid this problem since the particles resulting from  $K^0$  or  $\bar{K}^0$  semileptonic decay identify the strangeness eigenstate, so no kaon detector, as such, is required. However, inclusion of a detector would not change our conclusions; at the zeros of the  $K^0$  oscillation pattern, there are only  $\bar{K}^0$  mesons and no  $K^0$  mesons, so no detector could detect a  $K^0$  at such a point.

If the mass eigenstates did in fact have equal momenta or equal energies, then this would imply a failure of 4-momentum conservation in the kaon or neutrino production process. Such a failure would be evident well outside the oscillation region; the  $K_L$ , which is the only state left in the asymptotic region, would, in principle, have an energy or momentum inconsistent with 4-momentum conservation.

In a recent preprint by Field [20], the kinematics and other aspects of the neutrino production are treated quite differently, giving a wide range of correction factors to the standard result for the wavelength. However, there is an error in his derivation of the pion decay rates (Eq. (7) of Ref. [20]). When this is corrected, the motivation for his later modifications

to the standard treatment, with their bizarre physical consequences, is removed.

## Acknowledgements

This research is supported in part by the Department of Energy under contracts W-7405-ENG-36 and DE-FG03-94ER40821, and by the NSF under contract PHY-0099385.

## References

- [1] LSND Collaboration, A. Aguilar, et al., Phys. Rev. Lett. 81 (1998) 1774;  
A. Aguilar, et al., Phys. Rev. D 64 (2001) 112007.
- [2] M. Ambrosio, et al., Phys. Lett. B 517 (2001) 59.
- [3] SNO Collaboration, Q.R. Ahmad, et al., Phys. Rev. Lett. 89 (2002) 011301;  
Q.R. Ahmad, et al., Phys. Rev. Lett. 89 (2002) 011302.
- [4] K. Eguchi, et al., Phys. Rev. Lett. 90 (2003) 021802.
- [5] See, e.g., the treatment in: W.E. Burcham, M. Jobes, Nuclear and Particle Physics, Longmans, Harlow, UK, 1995.
- [6] Y.N. Srivastava, A. Widom, E. Sassaroli, Phys. Lett. B 344 (1995) 436;
- Y.N. Srivastava, A. Widom, E. Sassaroli, Z. Phys. C 66 (1995) 601;
- Y.N. Srivastava, A. Widom, hep-ph/9707268;
- Y.N. Srivastava, A. Widom, hep-ph/9509261;
- Y.N. Srivastava, A. Widom, hep-ph/9612290.
- [7] J. Lowe, B. Bassalleck, H. Burkhardt, T. Goldman, A. Rusek, G.J. Stephenson Jr., Phys. Lett. B 384 (1996) 288.
- [8] H. Burkhardt, J. Lowe, G.J. Stephenson Jr., T. Goldman, Phys. Rev. D 59 (1999) 054018.
- [9] H. Lipkin, Phys. Lett. B 348 (1995) 604;  
Y. Grossman, H. Lipkin, Phys. Rev. D 55 (1997) 2760.
- [10] B. Kayser, Phys. Rev. D 24 (1981) 110.
- [11] B. Kayser, in: Proceedings of the Moriond Workshop on Electroweak Interactions and Unified Theories, Les Arcs, France, 1995, unpublished.
- [12] L. Stodolsky, Phys. Rev. D 58 (1998) 036006, hep-ph/9802387.
- [13] H.J. Lipkin, hep-ph/0212093.
- [14] A.D. Dolgov, A.Yu. Morozov, L.B. Okun, M.G. Schepkin, Nucl. Phys. B 502 (1997) 3.
- [15] A.D. Dolgov, hep-ph/0004032;  
A.D. Dolgov, Phys. Rep. 370 (2002) 333.
- [16] L.B. Okun, M.G. Schepkin, I.S. Tsukerman, hep-ph/0211241.
- [17] F. Boehm, P. Vogel, Physics of Massive Neutrinos, Cambridge Univ. Press, Cambridge, 1987, p. 93.
- [18] T. Goldman, hep-ph/9604357.
- [19] W.H. Zurek, private communication.
- [20] J.H. Field, hep-ph/0301231.