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Dynamic analysis of large structures with uncertain parameters based on coupling component mode synthesis and perturbation method

D. Sarsri [a], L. Azrar [b,c,*]

[a] LTI, National School for Applied Sciences of Tangier, Abdelmalek Essaadi University, Tangier, Morocco
[b] MMC, Department of Mathematics, Faculty of Sciences and Techniques of Tangier, Abdelmalek Essaâdi University, Tangier, Morocco
[c] Department of Mechanical Engineering, Faculty of Engineering, King Abdulaziz University, Jeddah, Saudi Arabia

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Abstract This paper presents a methodological approach to compute the stochastic eigenmodes of large FE models with parameter uncertainties based on coupling of second order perturbation method and component mode synthesis methods. Various component mode synthesis methods are used to optimally reduce the size of the model. The statistical first two moments of dynamic response of the reduced system are obtained by the second order perturbation method. Numerical results illustrating the accuracy and efficiency of the proposed coupled methodological procedures for large FE models with uncertain parameters are presented.

1. Introduction

Dynamic analyses of complex industrial structures by finite element method lead to large finite element models. As the reduction of the order of the model, the system can be condensed by component mode synthesis. Component mode synthesis (CMS) consists in performing the dynamics analysis of structures by a decomposition of the structure into substructures, and these substructures are separately condensed and then coupled. Substructuring techniques differ from the chosen Ritz representation basis for substructure motion; the latter include the vibration normal modes, the rigid body modes, the static modes, the attachment modes, etc. They also differ in terms of the assembling procedures, by elimination or by transformation. Craig and Bampton method [1] uses a basic of fixed-interface eigenmodes and constrained modes; assembly is performed on the junction degree of freedom. The free interface method uses a basic of free-interface eigenmodes and attachment modes. MacNeal [2] includes the static effects of higher normal modes not retained in the component

* Corresponding author at: Mathematical Modeling and Control, Department of Mathematics, Faculty of Sciences and Techniques of Tangier, Abdelmalek Essaâdi University, Tangier 90 000, Morocco. Tel.: + 212 6 62 88 71 48.
E-mail addresses: dsarsri@ensat.ac.ma (D. Sarsri), l.azrar@uae.ma, azrarlahcen@yahoo.fr (L. Azrar).
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representation. Rubin [3] extended MacNeal's method to include the inertial effects of higher normal modes by using a second-order Maclaurin-series, both methods are based on junction force assembling. The junction dofs (jdofs) are therefore missing in the final condensed problem. A method proposed by Bouhaddi and Lambard [4] allows assembling the substructures with the junction degree of freedom.

In most of the CMS method, the resulting condensed problem is conditioned by the number of junction dof. In some cases, the size of coupled system is still large due to the great number of the degree of freedom at the interface. Further reduction of these dofs must often be considered. Bourquin [5] has proposed a method based on the use of the interface modes for Craig–Bampton method and Tran [6] extended this method to various free and hybrid method.

Recently Weng et al. [7] have proposed a substructuring method to calculate the eigensolutions and eigensensitivities for the model updating purposes.

CMS methods are commonly accomplished assuming deterministic behavior of loads and model parameters. However, in many cases the uncertainties associated with model parameters such as geometry, material properties, constitutive law, boundary conditions, and excitation, have to be considered giving arise to stochastic structures.

The analysis of dynamic response of stochastic FE system can be done in the frequency domain using the eigenmodes and frequency transfer functions or in the time domain by a direct integration of the equations of motion, using numerical procedures [8]. The analysis of these stochastic structures commonly seeks the first two moments of the response once the first two moments of the random fields modeling the structural uncertainties are known.

A direct simulation of Monte Carlo [9] is often used and considered as a reference for calculations. Nevertheless, it is in general quite inefficient due to much large number of samples required to guarantee accurate statistical results.

An alternative approach is based on the expansion of the response in terms of a series of polynomials that are orthogonal with respect to mean value operations [10,11]. More precisely, the Karhunen–Loeve expansion is used to discretize the stochastic variables into a denumerable set of random variables, thus providing a denumerable function space in which the problem is cast. The polynomial chaos expansion is then used to represent the solution in this space and the expansion coefficients are evaluated via a Galerkin procedure in the Hilbert space of random variables.

Recent review papers by Stefanou [12] and by Schueller and Pradlwarter [13] summarized the assessment of the past and current status of the procedure for stochastic structural analysis.

The perturbation method based on the Taylor series developments of the response around the average values of the random variables was initiated by Hien and Kleiber [8] to calculate the first two moments of eigenmodes. An improved perturbation method, proposed by Muscolino et al. [14], takes into account the mean and correlation information on uncertain parameters to analyze the dynamic response of structures with mechanical uncertainties under deterministic input. Although, in perturbation method, the variables must have a weak dispersion.

CMS methods are commonly accomplished assuming deterministic behavior of loads and model parameters. Perturbation methods and CMS are used by Hinke et al. [15] to replace numerically expensive operations, such as solving an eigenvalue problem. Sarsri et al. [16] used the CMS coupled with polynomial chaos expansions at first and second orders to compute the frequency transfer functions of stochastic structures.

In this paper various methods of component mode synthesis to reduce the dimensions of the model are used. The first two moments of eigenmodes of structure using a perturbation method are computed. For the needed derivative of various condensed matrices, assembly by transformation is used.

This paper is organized as follows: in Section 2, used CMS methods with fixed and free interfaces method are presented. A procedure of reduction of degree of freedom at the interface in fixed interface and free interface methods is described in Section 3. Various methods of component mode synthesis are used to calculate the first two moments of stochastic eigenvalues and eigenvectors using second order perturbation method in Section 4. Numerical examples are presented to illustrate the efficiency for the proposed technique as well as its accuracy over the whole structure.

2. Component mode synthesis

2.1. Reduced equation of motion

Component mode synthesis (CMS) techniques are well used for static and dynamic in the analysis of large and complex structures. CMS techniques have an advantage of enhancing computational efficiency by reducing the number of degrees of freedom of a structure. An overview of the used CMS is given bellow.

Let us consider a structure, which is decomposed into n_s substructures $\mathrm{SS}^{(k)}$ $(k = 1, \ldots, n_s)$ which do not overlap. For each substructure k the displacement vector $\mathbf{y}^{(k)}$ is partitioned into a vector $\mathbf{y}_j^{(k)}$, called interface dof and $\mathbf{y}_i^{(k)}$ which is the vector of internal dof. The force vector $\mathbf{f}^{(k)}$ is composed into vectors $\mathbf{f}_j^{(k)}$ and $\mathbf{f}_e^{(k)}$, called interface force and external applied force.

In the component mode synthesis methods, the physical displacements of the substructure $\mathrm{SS}^{(k)}$ are expressed as a linear combination of the substructure modes. After some algebraic transformations, a set of Ritz vectors $\mathbf{Q}$ is obtained and the displacements of $\mathrm{SS}^{(k)}$ are expressed as [6]:

$$\mathbf{y}^{(k)} = \mathbf{Q}^{(k)} \begin{Bmatrix} \mathbf{y}_j^{(k)} \\ \boldsymbol{\mu}^{(k)} \end{Bmatrix} = \mathbf{Q}^{(k)} \boldsymbol{\eta}^{(k)} \tag{1}$$

where $\boldsymbol{\mu}^{(k)}$ are the generalized coordinates. Details about the used component mode synthesis methods and related matrices $\mathbf{Q}$ are given in [10] and summarized in Appendix A.

Using Eq. (1) the kinetic energy and the strain energy of each substructure become

$$\begin{aligned} T^{(k)} &= \frac{1}{2}\, {}^T\dot{\boldsymbol{\eta}}^{(k)} \mathbf{M}_c^{(k)} \dot{\boldsymbol{\eta}}^{(k)} \\ U^{(k)} &= \frac{1}{2}\, {}^T\boldsymbol{\eta}^{(k)} \mathbf{K}_c^{(k)} \boldsymbol{\eta}^{(k)} \end{aligned} \tag{2}$$

where $\mathbf{M}_c^{(k)}$ and $\mathbf{K}_c^{(k)}$ are the condensed matrices of the substructure (k) given by

$$\mathbf{M}_c^{(k)} = {}^T\mathbf{Q}^{(k)}\mathbf{M}^{(k)}\mathbf{Q}^{(k)}$$
$$\mathbf{K}_c^{(k)} = {}^T\mathbf{Q}^{(k)}\mathbf{K}^{(k)}\mathbf{Q}^{(k)} \tag{3}$$

The work of the applied external forces is

$$\tau^{(k)} = {}^T\boldsymbol{\eta}^{(k)}\,\mathbf{f}_c^{(k)} \tag{4}$$

where

$$\mathbf{f}_c^{(k)} = {}^T\mathbf{Q}^{(k)}(\mathbf{f}_e + \mathbf{f}_j)^{(k)} \tag{5}$$

For the assembled structure with n substructures, the kinetic energy, the strain energy and the work of the applied external forces are given by

$$T = \frac{1}{2}\sum_{k=1}^{n} {}^T\dot{\boldsymbol{\eta}}^{(k)}\mathbf{M}_c^{(k)}\dot{\boldsymbol{\eta}}^{(k)}$$
$$U = \frac{1}{2}\sum_{k=1}^{n} {}^T\boldsymbol{\eta}^{(k)}\mathbf{K}_c^{(k)}\boldsymbol{\eta}^{(k)}$$
$$\tau = \sum_{k=1}^{n} {}^T\boldsymbol{\eta}^{(k)}\mathbf{f}_c^{(k)} \tag{6}$$

In order to assemble the components, the force and displacement continuity at the interface are used. That is to say for n substructures coupled at a common boundary one has

– Displacement continuity:

$$\mathbf{y}_j^1 = \mathbf{y}_j^2 = \cdots = \mathbf{y}_j^n = \mathbf{y}_j \tag{7}$$

– Equilibrium of coupling forces:

$$\sum_{k=1}^{n}\mathbf{f}_j^k = 0 \tag{8}$$

The conservation of interface dof allows assembling these matrices as in the ordinary finite element methods. Let us denote by $\mathbf{y}_c$ the vector of independent displacements of the assembled structure:

$$\mathbf{y}_c = \begin{Bmatrix} \boldsymbol{\mu}^{(1)} \\ \vdots \\ \boldsymbol{\mu}^{(n)} \\ \mathbf{y}_j \end{Bmatrix} \tag{9}$$

The compatibility of interface displacements of the assembled structure is obtained by writing for each substructure $S^{(k)}$ the following relation:

$$\boldsymbol{\eta}^{(k)} = \boldsymbol{\beta}^{(k)}\mathbf{y}_c \tag{10}$$

where $\boldsymbol{\beta}^{(k)}$ is the matrix of localization or of geometrical connectivity of the $SS^{(k)}$ substructure. It makes possible to locate the dof of each substructure $SS^{(k)}$ in the global dof of the assembled structure. They are the Boolean matrices whose elements are 0 or 1.

The free, fixed interface component mode synthesis methods will be used in this paper and the corresponding matrices are explicitly given in Appendix A.

A transformation matrix can be defined for each substructure $SS^{(k)}$ by

$$\mathbf{Z}^{(k)} = \mathbf{Q}^{(k)}\boldsymbol{\beta}^{(k)} \tag{11}$$

where $\mathbf{Q}^{(k)}$ is given by the considered CMS method.

The kinetic energy, the strain energy and the work of the external forces are then given by

$$T = \frac{1}{2}{}^T\dot{\boldsymbol{\eta}}\mathbf{M}_c\dot{\boldsymbol{\eta}}$$
$$U = \frac{1}{2}{}^T\boldsymbol{\eta}\mathbf{K}_c\boldsymbol{\eta}$$
$$\tau = {}^T\boldsymbol{\eta}\mathbf{f}_c \tag{12}$$

where

$$\mathbf{M}_c = \sum_{k=1}^{n} {}^T\mathbf{Z}^{(k)}\mathbf{M}^{(k)}\mathbf{Z}^{(k)}$$
$$\mathbf{K}_c = \sum_{k=1}^{n} {}^T\mathbf{Z}^{(k)}\mathbf{K}^{(k)}\mathbf{Z}^{(k)}$$
$$\mathbf{f}_c = \sum_{k=1}^{n} {}^T\mathbf{Z}^{(k)}(\mathbf{f}_j^{(k)} + \mathbf{f}_e^{(k)}) \tag{13}$$

Using the interface dof compatibility of displacements, it can easily be shown that

$$\sum_{k=1}^{n} {}^T\mathbf{Z}^{(k)}\mathbf{f}_j^{(k)} = 0 \tag{14}$$

Thus, the work of the applied forces becomes

$$\mathbf{f}_c = \sum_{k=1}^{n} {}^T\mathbf{Z}^{(k)}\mathbf{f}_e^{(k)} \tag{15}$$

For dynamic systems with viscous damping it is necessary to add a force of viscous dissipation $f^* = -\mathbf{C}\dot{\mathbf{y}}$. With the CMS concept this force can be rewritten as

$$\mathbf{f}_c^* = -\mathbf{C}_c\dot{\boldsymbol{\eta}} \tag{16}$$

where

$$\mathbf{C}_c = \sum_{k=1}^{n} {}^T\mathbf{Z}^{(k)}\mathbf{C}^{(k)}\mathbf{Z}^{(k)} \tag{17}$$

Finally, the reduced equation of motion can be written as follows:

$$\mathbf{M}_c\ddot{\mathbf{y}}_c + \mathbf{C}_c\dot{\mathbf{y}}_c + \mathbf{K}_c\mathbf{y}_c = \mathbf{f}_c \tag{18}$$

The correspondent undamped eigenvalue problem is

$$(\mathbf{K}_c - \lambda\mathbf{M}_c)\phi_c = 0 \tag{19}$$

where ϕ_c are the eigenvectors of the assembled structure.

Noted that the two problems may still be large due to the interface dofs. The size of these systems can be reduced.

2.2. Reduction of interface degrees of freedom

In most of the CMS methods, the coupling of the substructures is performed through the interface displacements, especially when the size of the coupled system is still large due to great number of degrees of freedom at the interface. In order to reduce the number of interface coordinates and therefore the size of the coupled system, a procedure based on the interface modes is used [6].

The interface modes matrix $\boldsymbol{\varphi}$ is defined as the first eigenmodes of the reduced eigenproblem:

$$(\mathbf{K}_{cj} - \lambda_j\mathbf{M}_{cj})\varphi_j = 0 \tag{20}$$

This results from the Guyan condensation [17] of the whole structure to the interface. The displacements of the interface dof are expressed as

$$\mathbf{y}_j = \boldsymbol{\varphi}\boldsymbol{\mu}_\mathbf{j} \tag{21}$$

For the assembled structure, the vector of independent displacement is rewritten as

$$\boldsymbol{\eta} = \begin{Bmatrix} \boldsymbol{\mu}^{(1)} \\ \vdots \\ \boldsymbol{\mu}^{(n)} \\ \mathbf{y}_j \end{Bmatrix} = \begin{bmatrix} \mathbf{I}^{(1)} & & \\ & \ddots & \\ & & \mathbf{I}^{(n)} \\ & & \boldsymbol{\varphi} \end{bmatrix} \begin{Bmatrix} \boldsymbol{\mu}^{(1)} \\ \vdots \\ \boldsymbol{\mu}^{(n)} \\ \boldsymbol{\mu}_\mathbf{j} \end{Bmatrix} = \mathbf{T}\bar{\boldsymbol{\eta}} \tag{22}$$

In this case, the transformation matrix becomes

$$\mathbf{Z}^{(k)} = \mathbf{Q}^{(k)}\boldsymbol{\beta}^{(k)}\mathbf{T} \tag{23}$$

Based in this double reduction, the resulting system is small enough to be handled. For CPU time reduction the perturbation method will be applied to the resulted frequency and time dependant systems.

3. Stochastic perturbation method

The basic idea of the stochastic perturbation method is to expand all random variables and matrices via Taylor series about their spatial expectation using a small parameter. Let us assume that for each substructure the mass $M^{(k)}$ and the stiffness $K^{(k)}$ matrices are related to a vector of the random variables α_i ($i = 1,\ldots,I$). Thus, the condensed mass $\mathbf{M}_c$ and stiffness $\mathbf{K}_c$ matrices are related to same vector of the random variables.

The first two moments of eigenmodes (average and variance), will be calculated by using the second order perturbation method.

One defines the vector of the average parameters $\bar{\alpha}_i$, and the quantity $d\alpha_i = \alpha_i - \bar{\alpha}_i$. All the matrices and the vector in Eqs. (18) and (20) are random, and are expanded through second order Taylor series as follows:

$$\begin{aligned}
\mathbf{M}_c &= \mathbf{M}_c^0 + \mathbf{M}_c^n\, d\alpha_n + \mathbf{M}_c^{np}\, d\alpha_n\, d\alpha_p \\
\mathbf{K}_c &= \mathbf{K}_c^0 + \mathbf{K}_c^n\, d\alpha_n + \mathbf{K}_c^{np}\, d\alpha_n\, d\alpha_p \\
\lambda_i &= \lambda_i^0 + \lambda_i^n\, d\alpha_n + \lambda_i^{np}\, d\alpha_n\, d\alpha_p \\
\phi_i^c &= \phi_i^0 + \phi_i^n\, d\alpha_n + \phi_i^{np}\, d\alpha_n\, d\alpha_p
\end{aligned} \tag{24}$$

where $[\cdot]^0$, $[\cdot]^n$ and $[\cdot]^{np}$ are deterministic matrices corresponding to the zero, the first and the second order partial derivatives with respect to the random parameter α_i and given by

$$A^0 = A(\alpha)|_{\bar{\alpha}} \quad A^n = \left.\frac{\partial A(\alpha)}{\partial \alpha_n}\right|_{\bar{\alpha}} \quad A^{np} = \frac{1}{2}\left.\frac{\partial^2 A(\alpha)}{\partial \alpha_n \partial \alpha_p}\right|_{\bar{\alpha}} \tag{25}$$

Indicial notations are used, with indices n, p running over the sequence $1,2,\ldots,I$ as well as the repeated indices summation.

For structures with small uncertainties, one can assume that the transformation matrix $\mathbf{Z}$ is deterministic. The zero, first and second-order derivatives of the condensed matrices $\mathbf{M}_c$ and $\mathbf{K}_c$ are given by:

$$\begin{aligned}
\mathbf{M}_c^0 &= \sum_{k=1}^{N} {}^T Z^{(k)}(\bar{\alpha})\mathbf{M}^{(k)}(\bar{\alpha})Z^{(k)}(\bar{\alpha}) \\
\mathbf{M}_c^n &= \sum_{k=1}^{N} {}^T Z^{(k)}(\bar{\alpha})\left.\frac{\partial \mathbf{M}^{(k)}}{\partial \alpha_n}\right|_{\bar{\alpha}}(\bar{\alpha})Z^{(k)}(\bar{\alpha}) \\
\mathbf{M}_c^{np} &= \sum_{k=1}^{N} {}^T Z^{(k)}(\bar{\alpha})\left.\frac{\partial^2 \mathbf{M}^{(k)}}{\partial \alpha_n \partial \alpha_p}\right|_{\bar{\alpha}}(\bar{\alpha})Z^{(k)}(\bar{\alpha}) \\
\mathbf{K}_c^0 &= \sum_{k=1}^{N} {}^T Z^{(k)}(\bar{\alpha})\mathbf{K}^{(k)}(\bar{\alpha})Z^{(k)}(\bar{\alpha}) \\
\mathbf{K}_c^n &= \sum_{k=1}^{N} {}^T Z^{(k)}(\bar{\alpha})\left.\frac{\partial \mathbf{K}^{(k)}}{\partial \alpha_n}\right|_{\bar{\alpha}}(\bar{\alpha})Z^{(k)}(\bar{\alpha}) \\
\mathbf{K}_c^{np} &= \sum_{k=1}^{N} {}^T Z^{(k)}(\bar{\alpha})\left.\frac{\partial^2 \mathbf{K}^{(k)}}{\partial \alpha_n \partial \alpha_p}\right|_{\bar{\alpha}}(\bar{\alpha})Z^{(k)}(\bar{\alpha})
\end{aligned} \tag{26}$$

These partial derivatives with respect to the random variables will be used to predict the stochastic eigenmodes and frequencies as well as the stochastic responses in frequency and time domains.

Substituting the developments (24) into the reduced Eq. (19), and equating terms of same order obtain for each mode i the following equations:

Zero order equation:

$$(\mathbf{K}_c^0 - \lambda_i^0\mathbf{M}_c^0)\phi_{ci}^0 = 0 \tag{27}$$

First order equation:

$$(\mathbf{K}_c^0 - \lambda_i^0\mathbf{M}_c^0)\phi_{ci}^n\, d\alpha_n = (\mathbf{K}_c^n - \lambda_i^n\mathbf{M}_c^0 - \lambda_i^0\mathbf{M}_c^n)\phi_{ci}^0\, d\alpha_n \tag{28}$$

Second order equation:

$$\begin{aligned}
(\mathbf{K}_c^0 - \lambda_i^0\mathbf{M}_c^0)\phi_{ci}^{np}\, d\alpha_n\, d\alpha_p = -((&\mathbf{K}_c^{np} - \lambda_i^{np}\mathbf{M}_c^0 - 2\lambda_i^n\mathbf{M}_c^p \\
&- \lambda_i^0\mathbf{M}_c^{np})\phi_{ci}^0 + (\mathbf{K}_c^n - \lambda_i^n\mathbf{M}_c^0 \\
&- \lambda_i^0\mathbf{M}_c^n)\phi_{ci}^p)d\alpha_n\, d\alpha_p
\end{aligned} \tag{29}$$

The computational detail of the first two moments of the eigenmodes is given in Appendix B.

The zero, the first and second order partial derivatives of eigenvectors corresponding to the substructure $\mathrm{SS}^{(k)}$ are then given by

$$\begin{aligned}
\phi^{0(k)} &= \mathbf{Z}^{(k)}\phi_c^0 \\
\phi^{n(k)} &= \mathbf{Z}^{(k)}\phi_c^n \\
\phi^{np(k)} &= \mathbf{Z}^{(k)}\phi_c^{np}
\end{aligned} \tag{30}$$

where $\mathbf{Z}^{(k)}$ is the transformation matrix corresponding to the substructure $\mathrm{SS}^{(k)}$ and the vector ϕ_c is the eigenvector corresponding to the reduced equation.

The means and the covariance of the eigenvalues and the eigenvectors are given by the following relationships:

$$\begin{aligned}
E[\lambda_i] &= \lambda_i^0 + \frac{1}{2}\lambda_i^{(2)} \\
\mathrm{cov}(\lambda_i\lambda_j) &= \sum_{n,p}\lambda_i^n\lambda_j^p\mathrm{cov}(\alpha_n,\alpha_p) \\
E[\phi_i] &= \phi_i^0 + \frac{1}{2}\phi_i^{(2)} \\
\mathrm{cov}(\phi_i,\phi_j) &= \sum_n\sum_p\sum_{i=1}\sum_{j=1}\phi_i^n\phi_j^p\mathrm{cov}(\alpha_n,\alpha_p)
\end{aligned} \tag{31}$$

4. Numerical examples

In order to demonstrate the efficiency of this methodological approach, some benchmark tests are elaborated for beam and assembled plates with linear and nonlinear stochastic parameters.

4.1. Stochastic beam

Let us consider the transverse vibration of an Euler beam discretized by 100 simple FE. Each node has 2 dofs in-plane rotation and a transverse displacement. The beam is of length L and of circular cross-section with radius r. In order to use the presented CMS methods, the beam is assumed to be composed of two substructures $SS^{(1)}$ and $SS^{(2)}$ as presented in Fig. 1. The first substructure consists of 60 finite elements and the second substructure consists of 40 ones. The beam is assumed to be clamped at both ends and the assembled structure has a total of 198 dofs. The substructure $SS^{(1)}$ has 120 dofs in which 2 are the interface dofs and the substructure $SS^{(2)}$ has 80 dofs in which 2 are the interface dofs. Let E and ρ denote element Young modulus and mass density.

The pulsation range of interest is chosen to be 0–2000 rd/s. For the Craig–Bampton method (CB) and the free interface method (FI), the substructure modes whose pulsations are smaller than a cutout pulsation defined by $\omega_{cp} = 2 \cdot \omega_u$ are selected. For (CB) method, the size of the reduced system is 17, 9 normal modes are retained for the substructure $SS^{(1)}$, 6 modes for $SS^{(2)}$ and 2 interface dofs. For (FI) method, 10 normal modes for the substructures $SS^{(1)}$, 7 modes for $SS^{(2)}$, and 2 interface dofs are retained. The size of reduced system is thus 19.

The modal parameters calculated by the present component mode synthesis (CMS) method are compared with those directly calculated using the whole structure. Tables 1 and 2 give the eigenmodes errors based on the following error criteria:

$$\varepsilon_{\lambda} = 100 \times \frac{|\lambda_c - \lambda_{\text{exact}}|}{\lambda_{\text{exact}}}$$
$$\varepsilon_{\phi} = 100 \times \frac{\|\phi_c - \phi_{\text{exact}}\|}{\|\phi_{\text{exact}}\|} \tag{32}$$

where λ_{exact} and ϕ_{exact} are obtained by solving the whole FE discretized system.

It is clearly shown that the eigenmodes of the entire structure are accurately obtained using the Craig–Bampton (CB) and the free interface (FI) methods and the (FI) method is more accurate.

For stochastic case, let us note that some random parameters such as Young modulus and mass density intervene linearly, and others such the radius intervenes nonlinearly in the stiffness and mass matrices. This nonlinear effect is harder

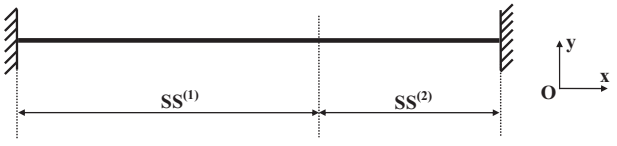

Figure 1 Example 1: Sub structured clamped beam.

Table 1 Comparison of beam eigenmodes obtained by the whole structure and fixed interface method for deterministic case.

Mode	Whole structure (rd/s)	Fixed interface method (rd/s)	Error eigenvalues (%)	Error eigenvectors (%)
1	81.7611	81.7618	0.0008	0.0537
2	225.3777	225.3852	0.0033	0.1670
3	441.8304	441.8583	0.0063	0.3297
4	730.3683	730.8264	0.0627	1.2793
5	1091.045	1091.403	0.0329	1.2124
6	1523.856	1527.933	0.2675	4.1079
7	2028.805	2037.082	0.4080	5.8452

Table 2 Comparison of beam eigenmodes obtained by the whole structure and free interface method for deterministic case.

Mode	Whole structure (rd/s)	Free interface method (rd/s)	Error eigenvalues (%)	Error eigenvectors (%)
1	81.7611	81.7611	0.0000	0.0002
2	225.3777	225.3777	0.0000	0.0023
3	441.8304	441.8305	0.0000	0.0111
4	730.3683	730.3696	0.0002	0.0629
5	1091.045	1091.048	0.0003	0.1039
6	1523.856	1523.924	0.0044	0.4691
7	2028.805	2029.058	0.0125	0.8887

to be analyzed. For stochastic case, the radius parameter is supposed to be a random variable and defined as follows:

$$r = r_0\left(1 + \frac{\sigma_r}{r_0}\xi_r\right) \tag{33}$$

where ξ_r is a zero mean value Gaussian random variable, $r_0 = 0.01$ m is the mean value and σ_r is the standard deviation of this parameter. In this nonlinear case, the perturbation method combined with the fixed interface method (CB) and the free interface method (FI) is developed. The following data are considered:

$$L = 1 \text{ m}; \quad E = 21 \times 10^{10} \text{ N/m}^2; \quad \rho = 7800 \text{ kg/m}^3$$

The mean and variance of the eigenmodes have been calculated by the proposed approach. The obtained results are compared with those obtained by direct Monte Carlo simulation 500 samples using the whole structure (WS, MCS) for $\sigma_r = 2\%$ based on the following error criteria:

Relative errors on the mean and variance of eigenvalue:

$$\varepsilon_m = 100 \times \frac{|\text{mean}(\lambda_c) - \text{mean}(\lambda_{\text{exact}})|}{\text{mean}(\lambda_{\text{exact}})}$$
$$\varepsilon_v = 100 \times \frac{|\text{var}(\lambda_c) - \text{var}(\lambda_{\text{exact}})|}{\text{var}(\lambda_{\text{exact}})} \tag{34}$$

Relative errors on the mean and variance of eigenvectors:

$$\varepsilon_{m\phi} = 100 \times \frac{\|\text{mean}(\phi_c) - \text{mean}(\phi_{\text{exact}})\|}{\|\text{mean}(\phi_{\text{exact}})\|}$$
$$\varepsilon_{v\phi} = 100 \times \frac{|\text{var}(\phi_c) - \text{var}(\phi_{\text{exact}})|}{|\text{var}(\phi_{\text{exact}})|} \tag{35}$$

The obtained results are plotted in Figs. 2–5 and an agreement between these results is clearly observed. It is clearly observed that the coupling Free Interface and perturbation method lead to better results. The CPU time, needed by the proposed approaches is presented in Table 3 for the considered beam. It is clearly shown that the proposed methods using the whole structure and the condensed approaches lead to impressive CPU time reductions.

4.2. Assembled plates

In order to use the CMS methods with reduction of the interface dof, let us consider an assembled plate as presented in Fig. 6. This structure fixed at two ends is used to test the accuracy of the proposed approach to predict the first two moments of eigenvalues and eigenvectors. The plate and its finite element discretization are shown in Fig. 6. The considered finite element mesh of the whole structure has 576 quadrilateral thin plate elements and 3834 degrees of freedom (6 dofs/node). The two substructures SS1, SS2 and the geometrical

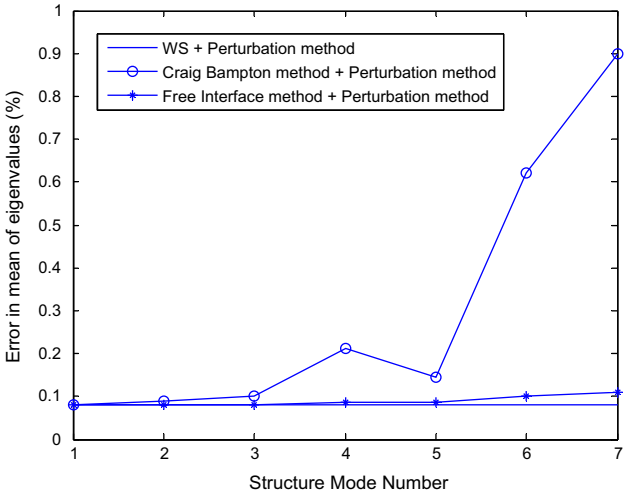

Figure 2 Percent error in mean of eigenvalues, MCS with 500 samples, perturbation method with the whole structure WS and with CB and FI methods. $\sigma_r = 2\%$.

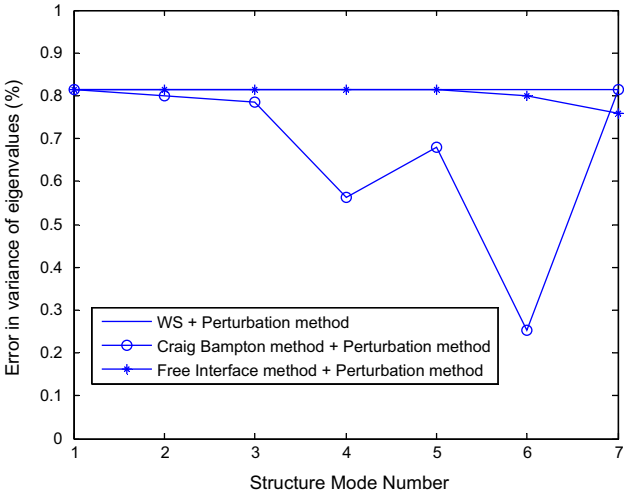

Figure 3 Percent error in variance of eigenvalues, MCS with 500 samples, perturbation method with the whole structure WS and with CB and FI methods. $\sigma_r = 2\%$.

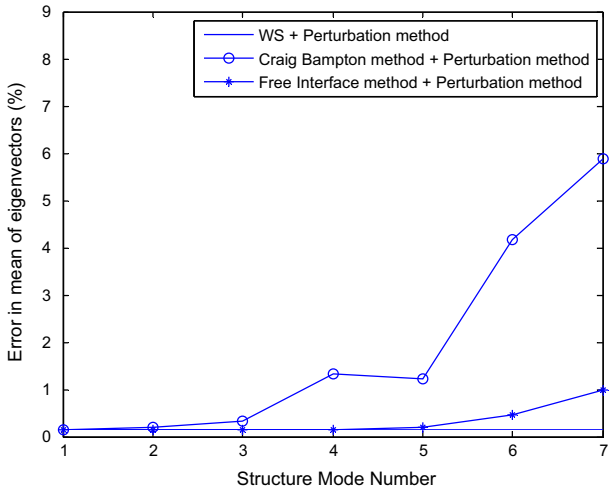

Figure 4 Percent error in mean of eigenvectors, MCS with 500 samples, perturbation method with the whole structure WS and with CB and FI methods. $\sigma_r = 2\%$.

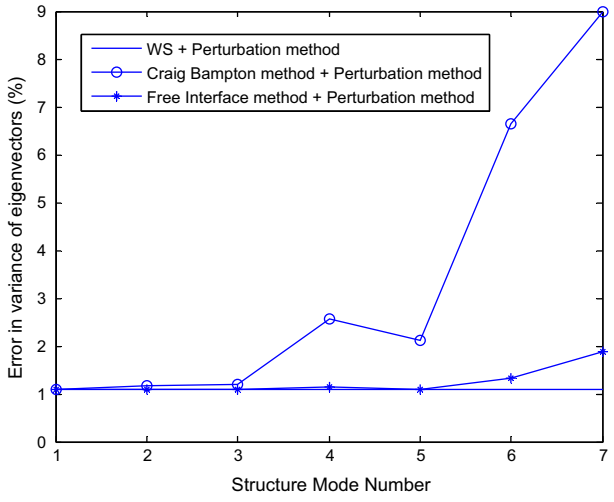

Figure 5 Percent error in variance of eigenvectors, MCS with 500 samples, perturbation method with the whole structure WS and with CB and FI methods. $\sigma_r = 2\%$.

Table 3 CPU time (s) comparison for stochastic eigenmodes of the considered beam, perturbation method with whole structure and component mode synthesis methods.

Monte Carlo simulation with whole structure	Perturbation method with whole structure	Perturbation method with Craig Bampton method	Perturbation method with free interface method
291.741850	1.327213	0.373967	0.369395

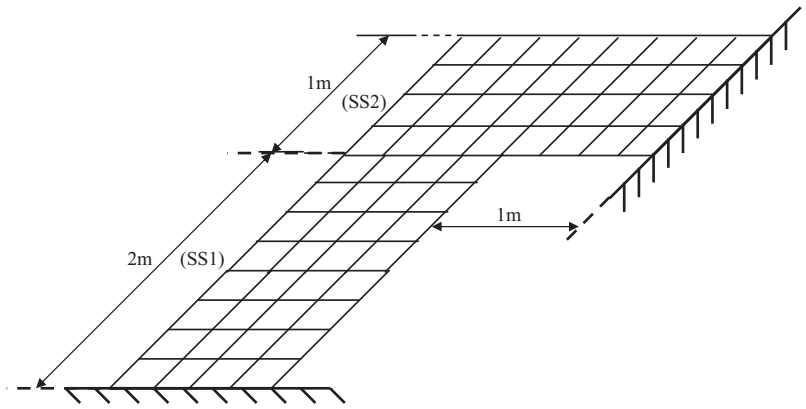

Figure 6 Assembled plates, boundary conditions, loads and discretization, 3834 dofs.

dimensions of the assembly are also given in Fig. 6. Each substructure has 1950 degrees of freedom in which 78 are the junction dofs. In this study the geometrical parameters, the thickness ($e = 0.02$), and the Poisson's ratio ($\mu = 0.3$) are assumed to be deterministic. The masse density ρ and the Young's modulus E are assumed independent Gaussian random variables.

$$E = E_0\left(1 + \frac{\sigma_E}{E_0}\xi_E\right)$$
$$\rho = \rho_0\left(1 + \frac{\sigma_\rho}{\rho_0}\xi_\rho\right) \tag{36}$$

where ξ is zero mean value Gaussian random. The following material parameters are used in this study: $E_0 = 21 \times 10^{10}\,\mathrm{N/m^2}$ and $\rho_0 = 7800\,\mathrm{kg/m^3}$ are the mean values of the structural parameters, $\sigma_E = 5\%$ and $\sigma_\rho = 5\%$ are the standard deviations. The first two moments (mean and variance) of eigenvalues and eigenvectors are calculated by second order perturbation method. The results obtained by Craig Bampton method (CB), Free interface method (FA), Craig Bampton method with reduction of junction dof (CBR) and Free interface

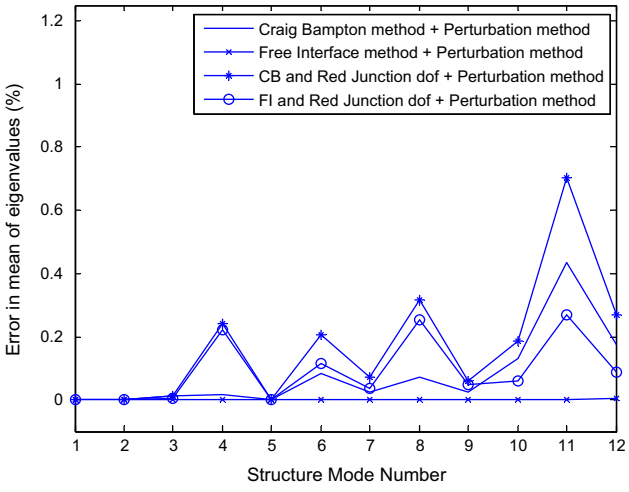

Figure 8 Percent error in mean of eigenvalues, perturbation method with the whole structure WS and with CB, FI, CB-Red and FI-Red component mode synthesis. $f_i \leqslant 3f_{\max}, \sigma_E = 5\%$ and $\sigma_\rho = 5\%$.

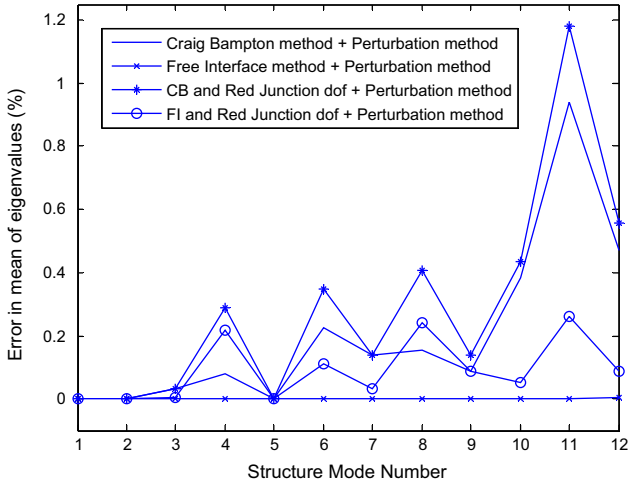

Figure 7 Percent error in mean of eigenvalues, perturbation method with the whole structure WS and with CB, FI, CB-Red and FI-Red component mode synthesis. $f_i \leqslant 2f_{\max}, \sigma_E = 5\%$ and $\sigma_\rho = 5\%$.

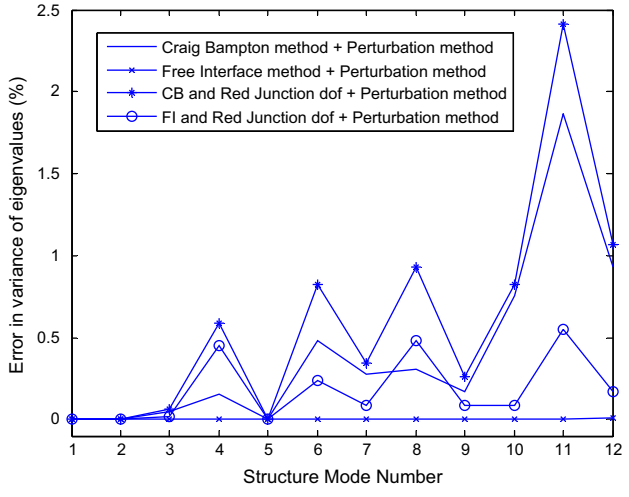

Figure 9 Percent error in variance of eigenvalues, perturbation method with the whole structure WS and with CB, FI, CB-Red and FI-Red component mode synthesis. $f_i \leqslant 2f_{\max}, \sigma_E = 5\%$ and $\sigma_\rho = 5\%$.

method with reduction of junction dof (FAR) are compared to the results obtained by the whole structure.

The useful frequency band of the whole structure is a priori fixed between 0 and $f_u = 2000$ Hz containing 12 global eigenmodes.

For Craig Bampton method and free interface method, to select the substructure normal modes, we use the criterion that consists in selecting all the substructure modes whose frequencies are smaller than a cutout frequency defined by $f_{cs} = c_i f_u$. Two cases are selected corresponding respectively to $c_i = 2$ ($f_{cs} = 4000$ Hz) and $c_i = 3$ ($f_{cs} = 6000$ Hz). For $c_i = 2$, we retain respectively 10 and 13 normal modes for each substructure. The size of reduced system is 98 for Craig Bampton method and 104 for free interface method (total number of substructure modes and junction dof). For $c_i = 3$, 15 and 20 normal modes are retained for each substructure. The size of reduced system is 108 for Craig Bampton method and 118 for free interface method (total number of substructure modes and junction dof).

For methods with reduction of junction dof the interface normal modes are selected by using a similar criterion with a cutout frequency defined by $f_{cs} = 3f_u$ ($f_{cs} = 6000$ Hz), and 10 normal modes are retained. For case 1 ($c_i = 2$), the size of reduced system is 30 for Craig Bampton method and 36 for free interface method (total number of substructure modes and interface modes). For case 2 ($c_i = 3$), the size of reduced system is 40 for Craig Bampton method with interface modes, and 36 for free interface method with interface modes (total number of substructure modes and interface modes). The relative errors in the mean of eigenvalues are plotted in Figs. 7

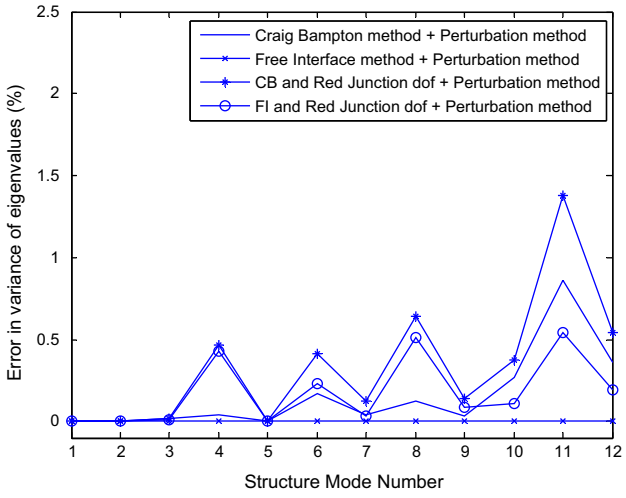

Figure 10 Percent error in variance of eigenvalues, perturbation method with the whole structure WS and with CB, FI, CB-Red and FI-Red component mode synthesis. $f_i \leqslant 3f_{\max}, \sigma_E = 5\%$ and $\sigma_\rho = 5\%$.

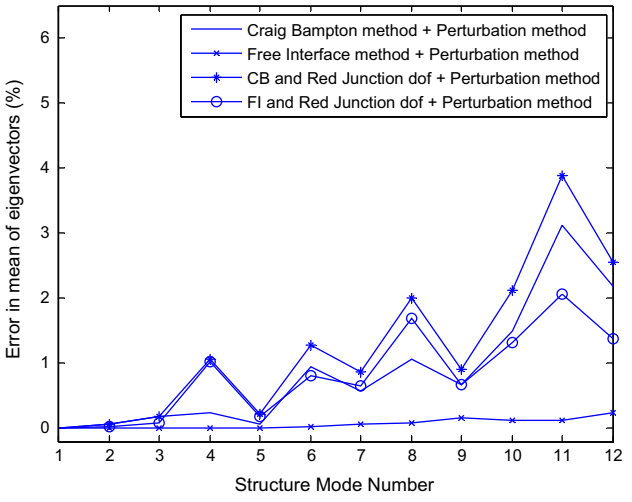

Figure 12 Percent error in mean of eigenvectors, perturbation method with the whole structure WS and with CB, FI, CB-Red and FI-Red component mode synthesis. $f_i \leqslant 3f_{\max}, \sigma_E = 5\%$ and $\sigma_\rho = 5\%$.

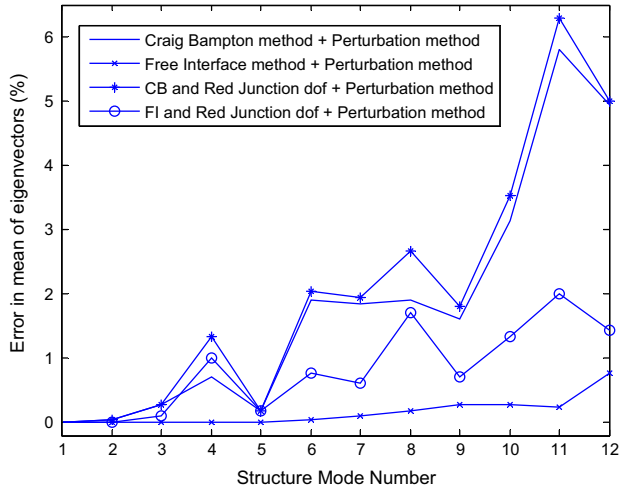

Figure 11 Percent error in mean of eigenvectors, perturbation method with the whole structure WS and with CB, FI, CB-Red and FI-Red component mode synthesis. $f_i \leqslant 2f_{\max}, \sigma_E = 5\%$ and $\sigma_\rho = 5\%$.

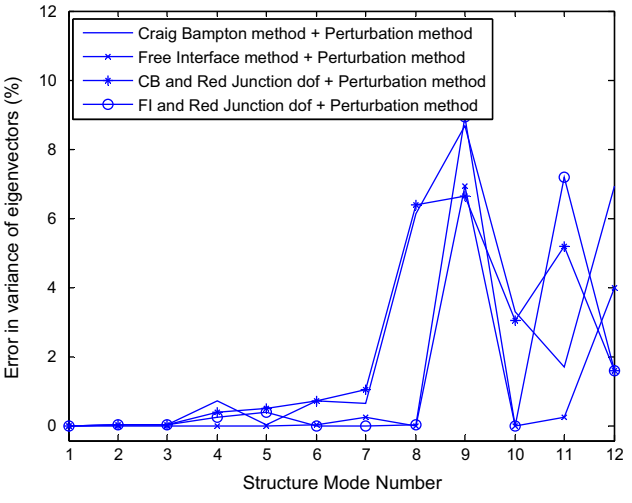

Figure 13 Percent error in variance of eigenvectors, perturbation method with the whole structure WS and with CB, FI, CB-Red and FI-Red component mode synthesis. $f_i \leqslant 2f_{\max}, \sigma_E = 5\%$ and $\sigma_\rho = 5\%$.

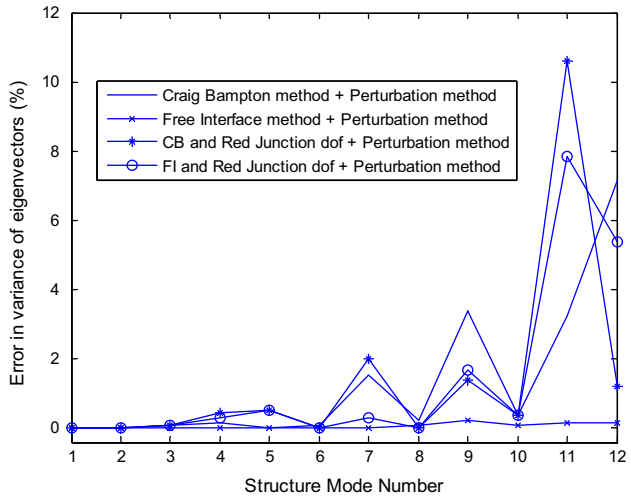

Figure 14 Percent error in variance of eigenvectors, perturbation method with the whole structure WS and with CB, FI, CB-Red and FI-Red component mode synthesis. $f_i \leqslant 3f_{\max}, \sigma_E = 5\%$ and $\sigma_\rho = 5\%$.

and 8, of the variance of eigenvalues are plotted in Figs. 9 and 10, the mean of eigenvectors are plotted in Figs. 11 and 12 and of the variance of the eigenvectors are plotted in Figs. 13 and 14. These figures show that when we increase the cutout frequency the relative errors in the mean and variance of eigenmodes decrease.

5. Conclusion

A methodological approach based on a coupling of component mode synthesis methods and perturbation method is developed and used to investigate the stochastic eigenmodes of structures with uncertain parameters for large linear FE models of beams and assembled plates with linear and nonlinear stochastic parameters. For the first two moments of eigenvalues and eigenvectors for stochastic structures, it is shown that by increasing the cutout frequency of the choice of the substructure normal mode, the accuracy is increasing. Agreement between results obtained by these methods and by the direct Monte Carlo simulation is demonstrated. The presented approaches are efficient and fast computational ones for the stochastic eigenmodes of structures with uncertain parameters for large structural systems with linear and nonlinear random parameters.

Acknowledgement

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Appendix A. Component mode synthesis

A.1. Fixed interface method

In the fixed interface method, the displacements of each substructure are expressed as

$$\mathbf{y} = \mathbf{Y}\boldsymbol{\eta} + \boldsymbol{\psi}_c \mathbf{y}_j \tag{A.1}$$

The matrix $\mathbf{Q}$ is given by

$$\mathbf{Q} = [\boldsymbol{\psi}_c \quad \mathbf{Y}] \tag{A.2}$$

in which $\mathbf{Y}$ is the matrix of truncated undamped normal modes of the substructure SS with a fixed interface as boundary condition. $\boldsymbol{\psi}_c$ is the matrix of the constrained mode associated with the interface, which is the static deformation shapes of SS obtained by imposing successively a unit displacement on one interface, while holding the remaining interface coordinates fixed.

A.2. Free interface method

In the free interface method, the displacements of each substructure are expressed as

$$\mathbf{y} = \mathbf{Y}\boldsymbol{\eta} + \boldsymbol{\psi}_r \xi_r + \boldsymbol{\psi}_a \xi_a \tag{A.3}$$

$\mathbf{Y}$ is the matrix of truncated undamped normal modes of the substructure SS with a free interface as boundary condition. $\boldsymbol{\psi}_r$ is the matrix of rigid body modes for an unconstrained substructure with a free interface. $\boldsymbol{\psi}_a$ is the matrix of attachment modes associated with the interface, which are the static deformation shapes of SS obtained by applying successively a unit force to one coordinate of the interface.

$$\boldsymbol{\psi}_a = \mathbf{G}\mathbf{F}_j$$

where

$$\mathbf{F}_j = \begin{bmatrix} \mathbf{I_j} \\ \mathbf{0} \end{bmatrix} \tag{A.4}$$

in which $\mathbf{G}$ is the residual flexibility matrix. The expression of $\mathbf{G}$ depends on the nature of the problem.

If the substructure is statically determined (i.e. no rigid body modes) then

$$\mathbf{G} = \mathbf{K}^{-1} \tag{A.5}$$

Else $\mathbf{G} = {}^T\mathbf{A}\mathbf{K}_{(c)}^{-1}\mathbf{A}$

where $\mathbf{A} = \mathbf{I} - \boldsymbol{\varphi}^{(r)\,T}\boldsymbol{\varphi}^{(r)}\mathbf{M}$ and ${}^T\boldsymbol{\varphi}^{(r)}\mathbf{M}\boldsymbol{\varphi}^{(r)} = \mathbf{I}$, $\mathbf{I}$: unit matrix and $\boldsymbol{\varphi}^{(r)}$: matrix of rigid modes

$\mathbf{K}_{(c)}$: stiffness matrix obtained by fixing arbitrary dof to make the structure isostatic and replacing the corresponding part of the initial stiffness matrix by zero.

To preserve the interface dof, the following partition is used:

$$\mathbf{Y} = \begin{bmatrix} \mathbf{Y}_j \\ \mathbf{Y}_i \end{bmatrix}$$

$$\boldsymbol{\psi}_r = \begin{bmatrix} \boldsymbol{\psi}_{rj} \\ \boldsymbol{\psi}_{ri} \end{bmatrix}$$

$$\boldsymbol{\psi}_a = \begin{bmatrix} \boldsymbol{\psi}_{aj} \\ \boldsymbol{\psi}_{ai} \end{bmatrix} \tag{A.6}$$

Using this partition one obtains the following:

$$\xi_a = \boldsymbol{\psi}_{aj}^{-1}\mathbf{y}_j - \boldsymbol{\psi}_{aj}^{-1}\mathbf{Y}_j\boldsymbol{\eta} + \boldsymbol{\psi}_{aj}^{-1}\boldsymbol{\psi}_{rj}\xi_r \tag{A.7}$$

The matrix $\mathbf{Q}$ is then given by

$$\mathbf{Q} = \begin{bmatrix} \boldsymbol{\psi}_a\boldsymbol{\psi}_{aj}^{-1} & \boldsymbol{\psi}_r - \boldsymbol{\psi}_a\boldsymbol{\psi}_{aj}^{-1}\boldsymbol{\psi}_{rj} & \mathbf{Y} - \boldsymbol{\psi}_a\boldsymbol{\psi}_{aj}^{-1}\mathbf{Y}_j \end{bmatrix} \tag{A.8}$$

The residual attachment modes $\boldsymbol{\psi}_{ar}$, obtained by removing in the attachment modes the components of the normal mode already retained in $\mathbf{Y}$, can be used to get

$$\mathbf{y} = \mathbf{Y}\boldsymbol{\eta} + \boldsymbol{\psi}_r\boldsymbol{\xi}_r + \boldsymbol{\psi}_{ar}\boldsymbol{\xi}_{ar} \tag{A.9}$$

$\boldsymbol{\psi}_{ar}$ is the residual attachment modes obtained by

$$\boldsymbol{\psi}_{ar} = \mathbf{R}\mathbf{F}_j \tag{A.10}$$

And

$$\mathbf{R} = \mathbf{G} - \mathbf{Y}\boldsymbol{\Lambda}^{-1}{}^T\mathbf{Y} \tag{A.11}$$

where $\boldsymbol{\Lambda}$ is the matrix of the retained eigenvalues. The matrix $\mathbf{Q}$ can be written as

$$\mathbf{Q} = \left[\boldsymbol{\psi}_{ar}\boldsymbol{\psi}_{arj}^{-1} \quad \boldsymbol{\psi}_r - \boldsymbol{\psi}_{ar}\boldsymbol{\psi}_{arj}^{-1}\boldsymbol{\psi}_{rj} \quad \mathbf{Y} - \boldsymbol{\psi}_{ar}\boldsymbol{\psi}_{arj}^{-1}\mathbf{Y}_j\right] \tag{A.12}$$

Appendix B

Based on the second order perturbation method the three algebraic systems given in Eqs. (28)–(30) have to be solved. The zero order corresponds to the deterministic reduced eigenproblem. For orders 1 and 2 one simplifies the problems by integrating the orders 1 and 2 equations after having multiplied them by the density of joint probability of α. The eigenvectors are assumed to be $\mathbf{M}_0$ normalized.

The first order derivative of eigenvalues is given by

$$\lambda_i^n = {}^T\phi_i^0(\mathbf{K}^n - \lambda_i^0\mathbf{M}^n)\phi_i^0 \tag{B.1}$$

There are then I systems to solve for first order in order to calculate the first derivative of eigenvalues.

The eigenvalue of second order is defined as the double sum of the second order partial derivative multiplied by covariance of the random variables; the second order derivative of eigenvalues is given by [7]

$$\begin{aligned}\lambda_i^{(2)} = {}&{}^T\phi_i^0[(\mathbf{K}^{np} - 2\lambda_i^n\mathbf{M}^p - \lambda_i^0\mathbf{M}^{np})\phi_i^0 \\ &+ 2{}^T\phi_i^0(\mathbf{K}^n - \lambda_i^n\mathbf{M}^0 - \lambda_i^0\mathbf{M}^n)\phi_i^p]\mathrm{cov}(\alpha_n, \alpha_p)\end{aligned} \tag{B.2}$$

Note that there is only one system for second order. The derivative of the random eigenvectors is expressed as a linear combination of eigenvectors of the deterministic eigenvectors. One forms the equations then giving the coefficients of this linear combination by using the orthogonality conditions to the stiffness matrix $\mathbf{K}$ and mass matrix $\mathbf{M}$.

The first order derivative of eigenvectors is given by

$$\phi_i^n = \sum_{l=1}^{L} C_{il}^n\phi_l^0 \tag{B.3}$$

$$\text{with :} \quad C_{il}^n = \frac{{}^T\phi_l^0\{R_i\}_n^I}{\lambda_l^0 - \lambda_i^0} \quad l \neq i$$

$$\text{and :} \quad C_{ii}^n = -\frac{1}{2}{}^T\phi_i^0\mathbf{M}^n\phi_i^0$$

where

$$\{R_i\}_n^I = -(\mathbf{K}^n - \lambda_i^n\mathbf{M}^0 - \lambda_i^0\mathbf{M}^n)\phi_i^0$$

The second order derivative of eigenvectors is obtained in the same way and given by

$$\phi_i^{(2)} = \sum_{l=1}^{L} D_{il}^{np}\phi_l^0 \tag{B.4}$$

$$\text{with :} \quad D_{il}^{n,p} = \frac{{}^T\phi_l^0\{R_i\}_{n,p}^{II}}{\lambda_l^0 - \lambda_i^0} \quad l \neq i$$

$$\text{and :} \quad D_{ii}^{n,p} = -\left[\frac{1}{2}{}^T\phi_i^0\mathbf{M}^{np}\phi_i^0 - 2{}^T\phi_i^0\mathbf{M}^n\phi_i^p + C_{ii}^nC_{ii}^p\right]\mathrm{cov}(\alpha_n, \alpha_p)$$

where

$$\begin{aligned}\{R_i\}_{k,m}^{II} = {}&-[(\mathbf{K}^{np} - \lambda_i^{np}\mathbf{M}^0 - 2\lambda_i^n\mathbf{M}^p - \lambda_i^0\mathbf{M}^{np})\phi_i^0 + 2(\mathbf{K}^n \\ &- \lambda_i^n\mathbf{M}^0 - \lambda_i^0\mathbf{M}^n)\phi_i^p]\mathrm{cov}(\alpha_n, \alpha_n)\end{aligned}$$

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Dr. Driss Sarsri is Associate Professor of Mechanic, Department of Electrical and Industrial Engineering at National School for Applied Sciences, Abdelmalek Essaadi University, Tangier, Morocco. He received his PhD from the Faculty of Sciences and Techniques, Tangier, Morocco. His research interests are in the field of vibration analysis of structures with uncertain parameters and FEM and component mode synthesis methods.

Dr. Lahcen Azrar is a **Full Professor** of Applied Mathematics at the Faculty of Sciences and Techniques (FST) of Tangier, Abdelmalek Essaadi University, Morocco and a Research Professor to King Abdulaziz University (KAU), Jeddah, Saudi Arabia, under a research professorship project. Fulbright visiting Scholar 2001 and 2008. He received his PhD in Applied Mathematics from the University of Metz in France in February 1993 and a Habilitation (Thèse d'Etat) from EMI, Mohammad V university, Morocco in June 1999. **Leader** of a research group on Mathematical Modeling and Control at the FST of Tangier and **Expert** at the National Research Center (CNRST) in Morocco. **Research interests:** - Mathematical modeling, Numerical simulation, integral equations, stochastic process, nonlinear dynamics - Micromechanical modeling of multifunctional materials, - Stability of structures, active and passive vibration control, crack and damage identification, etc. More than sixthly referred journal papers and more than eighty proceeding papers. Organizer of more than six national and international conferences.