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Uniform structures in the beginning of the third millenium

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Abstract

It is the aim of the present survey article to discuss briefly results published by various researchers in the area of uniform structures over the past five years.

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Keywords: Quasi-uniformity; Uniformity; Quasi-pseudometric; Pseudometric; Bitopological space; Asymmetric norm; Ball structure; Apartness space; Lax algebra; Uniform frame; Balanced topological group; Metrically generated theory; Uniform selection principle; Uniform approximate resolution; Partial (quasi-)metric; Lattice of quasi-uniformities; Uniform hyperspace; Uniform function space; H-equivalence; u-equivalent; Uniformly approachable function

Introduction

It is the goal of the present article to discuss recent work in the area of uniform spaces and some related topological structures. We shall mainly concentrate on publications that became available after our survey article entitled "Quasi-uniform spaces in the year 2001" had been completed. That article [85] appeared in the year 2002 as a chapter of the book 'Recent Progress in General Topology II'.

In order to fulfill our task we have classified the work done in the area of uniform mathematical structures over the past years into several sections. Although the composition of these sections may often look somewhat arbitrary, we have found the classification quite useful when compiling the material for our article. Of course, the selection of the results and their presentation was mainly influenced by the taste and the knowledge of the author. Indeed the area is huge and clearly not everything related to our topic could be covered in this short survey. Often the selected papers that we mention in the following have to serve as typical examples representing further ongoing work in the corresponding field. Some areas where uniformities were studied, like for instance nonstandard analysis [128], were completely ignored.

Let us start by recalling that unfortunately in the year 2005 three well-known mathematicians whose names are closely related to the theory of uniform spaces and their generalizations passed away:

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J. Isbell (1931–2005) had published the monograph "Uniform Spaces" [73] in 1964, which is still after forty years a most valuable general reference to the area.

J. Pelant (1950–2005) made numerous important contributions to the theory of uniform spaces; for instance, he had shown in 1987 [103] that the class of subfine uniform spaces and the class of locally fine uniform spaces coincide.

With the work done for his PhD thesis [119], S. Salbany (1941–2005) had popularized many ideas from bitopology and asymmetric topology.

In the year 2005 also Dvalishvili's book [49] dealing with bitopological spaces appeared. It studies bitopological spaces in the sense of Kelly [79], that is, sets equipped with two arbitrary topologies τ_1 and τ_2. Dvalishvili's monograph contains a wealth of ideas stemming from many different fields of mathematics and expresses the firm belief of its author that the theory of bitopological spaces will still find further fruitful applications throughout mathematics in the near future. Since the book treats several important basic facts about bitopological spaces (for instance, related to quasi-pseudometrics and quasi-uniformities) only in passing, it may however be more useful to the specialist than the beginner.

1. Preliminaries

Throughout this article we assume that the reader is familiar with the standard theory of uniformities and quasi-uniformities (see, for instance, [51,56,84]). Nevertheless we start by recalling some basic concepts in order to avoid misunderstandings. That preparation seems necessary, since terms like 'quasi-pseudometric' and 'uniform' have several conflicting meanings throughout the mathematical literature.

Let X be a (nonempty) set. A filter $\mathcal{U}$ on $X \times X$ is called a *quasi-uniformity* on X if each member $U \in \mathcal{U}$ contains the diagonal $\Delta = \{(x,x)\colon x \in X\}$ of X and for each $U \in \mathcal{U}$ there is $V \in \mathcal{U}$ such that $V^2 := V \circ V \subseteq U$. Here $\circ$ denotes the usual composition of relations so that for any $V, W \subseteq X \times X$, $V \circ W = \{(x,z) \in X \times X\colon$ there is $y \in X$ such that $(x,y) \in W$ and $(y,z) \in V\}$.

If $\mathcal{U}$ is a quasi-uniformity on X, then $\mathcal{U}^{-1} = \{U^{-1}\colon U \in \mathcal{U}\}$ is the so-called *conjugate quasi-uniformity* of $\mathcal{U}$, where $U^{-1} = \{(x,y) \in X \times X\colon (y,x) \in U\}$ whenever $U \in \mathcal{U}$.

A quasi-uniformity $\mathcal{U}$ will be called *symmetric* provided that $\mathcal{U} = \mathcal{U}^{-1}$, that is, if it is a *uniformity*.

The topology $\tau(\mathcal{U})$ induced by a quasi-uniformity $\mathcal{U}$ on X is determined by the neighborhood filters $\mathcal{U}(x) = \{U(x)\colon U \in \mathcal{U}\}$ of the points $x \in X$, where $U(x) = \{y \in X\colon (x,y) \in U\}$ whenever $x \in X$ and $U \in \mathcal{U}$.

A non-negative real-valued function $d : X \times X \to [0,\infty)$ is called a *quasi-pseudometric* on X if it satisfies

(i) $d(x,x) = 0$ whenever $x \in X$, and
(ii) $d(x,z) \leqslant d(x,y) + d(y,z)$ whenever $x, y, z \in X$.

The quasi-pseudometric quasi-uniformity $\mathcal{U}_d$ on X is generated by the filter base $\{U_\varepsilon\colon \varepsilon > 0\}$ on $X \times X$ where $U_\varepsilon = \{(x,y) \in X \times X\colon d(x,y) < \varepsilon\}$.

A quasi-pseudometric d satisfying the symmetry condition $d(x,y) = d(y,x)$ (whenever $x, y \in X$) is called a *pseudometric*.

Each quasi-uniformity can be represented as the supremum filter of a family of quasi-pseudometric quasi-uniformities.

Various topics that we shall discuss in this survey were already treated in our former survey articles [84,85]. In such cases we shall mainly concentrate on recent developments in those fields and refer the reader to the cited two articles for further information. On the other hand, for topics that were not dealt with in our previous surveys, we shall also include references to older work.

2. Uniformities related to algebraic structures

The number of investigations conducted in the area of asymmetric functional analysis increased considerably in recent years. Many of these studies were motivated by the new need to apply results from classical (functional) analysis in a non-Hausdorff setting, as it is often encountered during investigations in theoretical computer science and general order theory (see, e.g., [8,83]). The area is vast and unfortunately at present no textbook is available which develops these ideas systematically. Therefore, it has become increasingly difficult to compare the different

results obtained by the various authors. Since the topic is somewhat peripheral to our present intentions, we here only mention a few typical new publications in that field [5,6,32–34,58–61]. In these articles fundamental concepts and ideas from classical functional analysis (like separation results for convex sets or extension theorems for linear bounded functionals) were explored in an asymmetric setting.

As an example for such a study, let us mention that in this field the theory of semi-Lipschitz maps replaces the theory of Lipschitz maps [115,116]. Here a real-valued function f on a quasi-pseudometric space (X, d) is called *semi-Lipschitz* if there exists $k \geqslant 0$ such that $f(x) - f(y) \leqslant kd(x, y)$ whenever $x, y \in X$. For instance, in [115] Romaguera and Sanchis investigated the problem of the existence of semi-Lipschitz utility functions on a quasi-pseudometric space.

A tool from theoretical computer science, namely Matthews's partial metrics [94] gained further popularity in the last years [118,122]. The notion of a partial metric is a modification of the usual concept of a metric which allows one to assume that the distance from a point to itself may be non-zero. A positive self-distance can be interpreted as the weight or measure of the point. The notion of a partial metric has a direct connection to the concept of a metric space with a base point (see [82]).

In the last years Matthews's partial metrics were generalized to partial semimetrics and partial quasi-metrics [134, 87] and to partial metrics attaining values in quantales [81]. New applications of the concept were found in functional analysis [99,100].

We finish this section with remarks on (para)topological groups. We recall that a group equipped with a topology is called a *paratopological group* if the group operation (multiplication) is jointly continuous. A paratopological group in which the inversion is also continuous is called a *topological group*. Every paratopological group comes with two standard compatible quasi-uniformities, the so-called left and right quasi-uniformities, which turn out to be uniformities in the case of a topological group.

A topological group G is said to be *balanced* if its left and right uniformities coincide and it is said to be *functionally balanced*, if for each bounded, continuous function $f : G \to \mathbb{R}$, f is left uniformly continuous if and only if f is right uniformly continuous. Whether functional balancedness of a topological group implies its balancedness is known as the Itzkowitz problem [75,76] in the pertinent literature about topological groups. Observe that the problem can be reformulated as follows: If the left and the right uniformity of a topological group induce the same proximity, are they necessarily equal (compare, e.g., [75, Theorem 2.6])?

Work of many researchers has led to several positive partial solutions to this problem. The implication holds for topological groups G which are locally compact, or almost metrizable, or quasi-k, or locally connected. It is also known that each functionally balanced group is balanced if and only if each product of two functionally balanced groups is functionally balanced.

For instance, Bouziad and Troallic recently published several articles related to this problem [20,21,126], and in another related investigation [67] Hernández studied the question whether given a (complete metrizable) uniform space X the Banach space of all real-valued bounded uniformly continuous functions on X characterizes the uniform structure of X. Several articles concerning Itzkowitz's problem can be found in the latest issues of the journal "Topology Proceedings" (see, e.g., [68], where it is shown that the generalization of the problem to homogeneous factor-spaces of topological groups has a negative answer). But the problem remains unresolved.

We neither know the answer if Itzkowitz's problem is generalized to paratopological groups.

Problem 2.1. If the left and the right quasi-uniformity of a paratopological group induce the same quasi-proximity, are they necessarily equal?

Nevertheless in recent years much work and progress can be reported from the theory of paratopological groups (see, for instance, [12]), an area that was already covered in some detail in our former surveys [84,85]. We note however that while it is fairly common to apply uniformities in a study about topological groups, methods from the theory of quasi-uniformities have hardly been used so far in similar investigations on paratopological groups (see [117] for an exception).

3. Categorical methods and completions

It is well known that each separated uniform space has a unique completion. This construction can readily be generalized to quasi-uniformities where it is called the *bicompletion* (see, e.g., [56]). The latter construction has been

further generalized to many other categories: For instance in the last years to some class of fuzzy quasi-uniform spaces [50], to approach spaces [26] and to some kind of locally quasi-uniform spaces [127]. Since the idea underlying the bicompletion is basically a symmetric one, various authors have tried to construct other, possibly larger completions for quasi-uniform spaces that are based on asymmetric concepts. In this context many different notions of quasi-uniform completeness have been proposed (compare, e.g., [11,39]) and several competing completion theories with reasonable (categorical) properties have been developed (see, e.g., [84,85]). These theories all agree for the class of uniform spaces, but deviate from each other for general quasi-uniform spaces.

The search for such completions continued over the past years. For instance, Andrikopoulos [10] suggested another completion theory for quasi-uniform spaces which is based on some type of Cauchy-pairs of nets. For every generalized metric space Vickers [130] constructed a completion with the help of Cauchy filters of formal balls. The studied completion generalizes the usual one for metric spaces, and for quasi-pseudometric (T_0-)spaces it is equivalent to the Yoneda completion in its netwise form [88], but his construction additionally provides explicit characterizations of the points of that completion. In [65] Gregori, Mascarell and Sapena introduced a construction of a completion in the area of fuzzy quasi-metrics which imitates the so-called Doitchinov completion for balanced quasi-metric spaces. (Of course, Doitchinov's concept of 'balancedness' has no evident connections to the notion of a balanced topological group.)

At the present stage it is not clear which of the numerous completion theories for quasi-pseudometric and quasi-uniform spaces that have been developed by various researchers over the last decades are worth further studies. Unfortunately many of them seem to be difficult to use in practical applications. So a careful analysis and classification of the various completions with their different properties are needed.

Investigations that tried to connect or unify different topological and uniform theories have flourished in recent years [29,71,109]. Naturally such investigations were often inspired by ideas from category theory. Let us discuss a few specific examples.

Having as a starting point Barr's description of topological spaces as lax algebras for the ultrafilter monad, the article of Clementino, Hofmann and Tholen [31] showed how a generalization of Barr's presentation can similarly describe many other topological structures like uniform spaces and approach spaces.

Also the categorical approach to completeness was further developed, for instance, by Giuli. In [63] he used the categorical theory of closure operators to study separated, complete and compact objects with respect to the Zariski closure operator defined in any category $\mathcal{X}(A, \Omega)$ obtained by a given complete category $\mathcal{X}$ (endowed with a proper factorization structure for morphisms) and by a given $\mathcal{X}$-algebra (A, Ω) by forming the affine $\mathcal{X}$-objects modeled by (A, Ω).

Colebunders and Lowen, with their students, started investigations about so-called metrically generated theories [36,35]. The basic idea is that many known natural functors K describe the transition from categories $\mathcal{C}$ of generalized metric spaces to the "metrizable" objects in some given topological construct $\mathcal{X}$. If K preserves initial morphisms and if $K(\mathcal{C})$ is initially dense in $\mathcal{X}$, then $\mathcal{X}$ is said to be $\mathcal{C}$-metrically generated. While, for instance, (pre)topological, (pre)approach and (quasi-)uniform spaces are all metrically generated, bornological, pseudotopological and nearness spaces are not.

In another development Miyata and Watanabe [96] defined a category of approximate resolutions appropriate for uniform spaces and proved that it is equivalent to the category of uniform spaces. They showed how approximate resolutions can be used in uniform space theory, for instance in uniform shape theory.

In numerous articles the usefulness of Lowen's approach spaces [91] in quantified analysis was further explored. For instance, in [47] DiMaio, Lowen, Naimpally and Sioen considered proximity notions and Samuel compactifications in the context of approach spaces, in [92] Lowen discussed an extension of Ascoli's theorem to uniform gauge spaces, and in [93] Lowen and Sioen studied monoidal closed structures on the category of uniform approach spaces.

The classical idea of the Samuel compactification of a uniform space attracted some new interest [1,57]. Finally in [4] Alcaraz and Sanchis studied a special kind of extensions of dynamical systems which were introduced by considering completions of totally bounded uniform spaces.

4. Uniform structures in analysis and geometry

Naturally several of the articles discussed in this section are—in some sense or another—related to the modern theory called *analysis in metric spaces* [9].

In recent years the phenomenon of concentration of measure on high-dimensional structures was normally formulated in terms of a metric space with a Borel measure, also called an mm-space (see, e.g., [90]). In [124] Stojmirović extended some mm-space concepts to the setting of a quasi-metric space with a probability measure (called pq-space). His motivation came from biology: He showed that many common similarity measures on biological sequences can be converted to quasi-metrics. He also observed that a high-dimensional pq-space is very close to being an mm-space.

Similarly as in classical analysis the metric setting however turns out to be too restricted for many investigations and the idea of uniformities arises very naturally in this context. In that sense [107] Plaut developed the theory of quotients of uniform spaces via sufficiently nice group actions. He generalized two important constructions: quotients of topological groups via closed normal subgroups and quotients of metric spaces via actions by isometries. Results about inverse limits of topological groups were extended to inverse limits of group actions on uniform spaces, and notions of prodiscrete action and generalized covering map were defined.

In recent years *balleans* independently appeared in combinatorics [13] and in asymptotic topology (see, for instance, [48]). Let us recall [95] that a set X is called a *coarse space* if there is a distinguished collection $\mathcal{E}$ of subsets of the product $X \times X$ called entourages such that any finite union of entourages is contained in an entourage, the union of all entourages is the entire space $X \times X$, the inverse M^{-1} of an entourage M is contained in an entourage and the composition $M_1 \circ M_2$ of entourages M_1 and M_2 is contained in an entourage.

Certainly the reader will have recognized the obvious analogy between this concept and the notion of a uniform space introduced earlier. Protasov's definition of a ball structure helps to unify the two corresponding theories (see, e.g., [111,112]).

A *ball structure* is a triple $\mathbb{B} = (X, P, B)$, where X and P are nonempty sets and, for any $x \in X$ and $\alpha \in P$, $B(x, \alpha)$ is a subset of X which is called the *ball of radius α around x*.

It is supposed that $x \in B(x, \alpha)$ for all $x \in X$ and $\alpha \in P$. The set X is called the *support of* $\mathbb{B}$, and P is called the *set of radii*.

Given any $x \in X$, $A \subseteq X$ and $\alpha \in P$ we put $B^*(x, \alpha) = \{y \in X\colon\ x \in B(y, \alpha)\}$, where B^* is said to be the dual of B, and set $B(A, \alpha) = \bigcup_{a \in A} B(a, \alpha)$ and $B^*(A, \alpha) = \bigcup_{a \in A} B^*(a, \alpha)$.

A ball structure $\mathbb{B} = (X, P, B)$ is called

lower symmetric if, for any $\alpha, \beta \in P$, there exist $\alpha', \beta' \in P$ such that, for every $x \in X$, $B^*(x, \alpha') \subseteq B(x, \alpha)$, $B(x, \beta') \subseteq B^*(x, \beta)$;

upper symmetric if, for any $\alpha, \beta \in P$, there exist $\alpha', \beta' \in P$ such that, for every $x \in X$, $B(x, \alpha) \subseteq B^*(x, \alpha')$, $B^*(x, \beta) \subseteq B(x, \beta')$;

lower multiplicative if, for any $\alpha, \beta \in P$, there exists $\gamma \in P$ such that, for every $x \in X$, $B(B(x, \gamma), \gamma) \subseteq B(x, \alpha) \cap B(x, \beta)$;

upper multiplicative if, for any $\alpha, \beta \in P$, there exists $\gamma \in P$ such that, for every $x \in X$, $B(B(x, \alpha), \beta) \subseteq B(x, \gamma)$.

The connection between uniformities and ball structures is now readily described. If $\mathbb{B} = (X, P, B)$ is a lower symmetric and lower multiplicative ball structure, then the family

$$\left\{ \bigcup_{x \in X} B(x, \alpha) \times B(x, \alpha)\colon\ \alpha \in P \right\}$$

is a base of entourages for some (uniquely determined) uniformity on X.

On the other hand, if $\mathcal{U}$ is a uniformity on X, then the ball structure $(X, \mathcal{U}, B)$ is lower symmetric and lower multiplicative, where for each $U \in \mathcal{U}$ and $x \in X$, $B(x, U) = \{y \in X\colon\ (x, y) \in U\}$. Indeed the lower symmetric and lower multiplicative ball structures can be identified with the uniform topological spaces.

Similarly, a (quasi-)pseudometric space (X, d) can be regarded as a ball structure $(X, [0, \infty), B_d)$, where $[0, \infty)$ is the set of all nonnegative real numbers and, for any $x \in X$, $r \in [0, \infty)$, $B_d(x, r)$ is the usual "closed" ball of radius r and center x. The ball structure associated with a quasi-pseudometric is lower multiplicative, but in general not lower symmetric, whereas the ball structure associated with a pseudometric space is both lower symmetric and lower multiplicative.

A ball structure is said to be a *ballean* if it is upper symmetric and upper multiplicative. (Connected) balleans correspond to coarse spaces and form the asymptotical counterparts of uniform topological spaces.

They have been used to prove asymptotical analogues of classical topological results (see, for instance, Protasov's investigations related to normality [110], where continuous functions in the classical results are replaced by slowly oscillating functions and disjoint sets by so-called asymptotically disjoint sets).

5. Uniform structures in point-free and constructive topology

The attempts continued to create more computational or constructive variants of well known theorems in classical uniform mathematics. In particular, further classical results about uniform structures were generalized to a point-free setting in the past years (see, for instance, [98,106]). Again we shall concentrate on a few relevant articles.

In [106] Picado characterized gauge structures for frames by describing frame uniformities with the help of families of metric diameters. Moreover extended studies about (possibly non-symmetric) uniform structures in the point-free context were conducted in several joint papers by Ferreira and Picado (see, e.g., [52,53]). It is well known that in general, unlike a uniformity, a quasi-uniformity on a set is not determined by its quasi-uniform covers. However, a simple, but basic construction, due to Fletcher [55] assigns a transitive quasi-uniformity to each subbasic family of interior-preserving open covers (on a topological space). It allows one to describe all compatible transitive quasi-uniformities on topological spaces in terms of interior-preserving families of open covers. In [52] Ferreira and Picado developed a point-free generalization of this fundamental construction. By using their method, many kinds of interior-preserving open covers (e.g., locally finite, point-finite, spectrum, well-monotone) were shown to induce compatible quasi-uniformities on an arbitrary frame. The extension of the Fletcher construction from spaces to frames is by no means immediate and an important role is played by the sublocale lattice, or equivalently the frame of congruences. Based on their method they subsequently [53] developed a point-free formulation of the theory of functorial transitive quasi-uniformities for frames, as it had been created by Brümmer [25] more than twenty years ago for topological spaces.

In [14] Banaschewski surveyed work done in the theory of uniform and nearness frames and their completions. His paper is part of a book that contains several other articles that are of interest to specialists working in the theory of uniform (fuzzy) structures. Indeed a large number of new articles about fuzzy uniform structures appeared recently. They can roughly be classified by the criterion of whether they use entourages, covers or operators in their definition. For more details we refer the interested reader to two very recent articles in this area [66,136] and the references listed there.

In [15] it was shown that many results about epimorphisms from frame theory carry over to the setting of uniform frames.

Textures were originally introduced by Brown (see, e.g., [24]) as a point-based setting for the study of fuzzy sets and have proved to be an appropriate framework for the development of complement-free mathematical concepts. In [101] Özçağ and Brown laid the foundation for a theory of uniformities on textures, giving descriptions in terms of direlations, dicovers and dimetrics. The interaction between di-uniformities and a complementation on a texture was studied in [102]. The distinction between quasi-uniformities and uniformities, which is one of symmetry in the classical representation, becomes a matter of complementation in the description using direlations.

The theory of *apartness spaces*, a counterpart of the classical theory of proximity spaces, was developed by Bridges and Vîţă in a series of articles (see, e.g., [22,23]). The aim of these investigations was a systematic development of computable topology using apartness as the fundamental notion.

In [135] Yasugi and Tsujii introduced two extended computability notions for real number functions. One is based on an effective uniformity and the other one is defined with the help of a limiting recursive modulus of convergence (relative to the usual Euclidean topology). They showed that the two derived notions of sequential computability are indeed equivalent.

In [45] DiConcilio and Gerla proposed an approach to point-free geometry in which the primitives are the regions and a non-symmetric distance between regions. The intended models are the bounded regular closed subsets of a metric space together with the Hausdorff excess function.

6. Uniform structures and foundations

In the second half of the twentieth century in Prague several interesting results dealing with the lattice of uniformities were obtained by Pelant, Reiterman, Rödl and Simon (see, e.g., [104,105]). For instance, Pelant and Reiterman

established that every separable metrizable uniformity on an uncountable set X has a complement in the lattice of uniformities on X; however a non-discrete uniformity on a set X that induces the discrete proximity does not have such a complement.

Recently de Jager and Künzi conducted related investigations about the larger lattice of quasi-uniformities on a given set X. Note that the (dual of the) lattice of preorders on X embeds into this lattice as a sublattice, via the map $T \mapsto \mathrm{fil}\{T\}$ where T is a preorder on X and $\mathrm{fil}\{T\}$ denotes the quasi-uniformity on X generated by the filterbase $\{T\}$.

Their studies [40–42], mainly about atoms, anti-atoms and complements in such lattices, were already summarized in a short survey paper [43], to which we refer the interested reader. In that article also some of the many interesting open questions in this area were mentioned.

For instance, it remains unknown whether, given two distinct uniformities $\mathcal{U}_1$ and $\mathcal{U}_2$ on a set X such that $\mathcal{U}_1 \subseteq \mathcal{U}_2$, there always exists a non-symmetric quasi-uniformity $\mathcal{Q}$ on X satisfying $\mathcal{U}_1 \subseteq \mathcal{Q} \subseteq \mathcal{U}_2$.

Seemingly, this problem has some intriguing similarities with the Itzkowitz problem stated above and the question about H-equivalence of uniformities discussed below. Indeed all these problems essentially try to determine conditions under which two similar uniform structures coincide. Let us finish the discussion of this topic with the following open problem, which was not listed in [43].

Problem 6.1. (Compare [123].) What are the possible cardinalities of maximal families of mutually complementary families of quasi-uniformities on a given set X?

In [27] Burdick investigated the use of nets indexed by preorders in uniform spaces. He showed that different Cauchy conditions and different convergence conditions for these nets characterize various well-known completeness properties, which can also be used to describe functors associated with these completeness properties.

By considering uniform covers instead of open covers, in [80] Kočinac started first investigations about uniform variants of the following three classical topological *selection properties* which are named after Rothberger, Menger and Hurewicz, respectively:

If X is a topological space, and $\{\mathcal{U}_n\colon n \in \mathbb{N}\}$ is an arbitrary sequence of open covers of X, then

(R) for each $n \in \mathbb{N}$, there exists $U_n \in \mathcal{U}_n$ such that $\bigcup_{n\in\mathbb{N}} U_n = X$;
(M) for each $n \in \mathbb{N}$, there exists a finite subfamily $\mathcal{V}_n \subseteq \mathcal{U}_n$ such that $\bigcup_{n\in\mathbb{N}} \bigcup \mathcal{V}_n = X$;
(H) for each $n \in \mathbb{N}$, there exists a finite subfamily $\mathcal{V}_n \subseteq \mathcal{U}_n$ such that each point $x \in X$ belongs to all but finitely many members of $\{\bigcup \mathcal{V}_n\colon n \in \mathbb{N}\}$.

Sichler and Trnková [120] constructed uniform spaces X and Y such that every uniformly continuous map $Y \times Y \to Y$ depends only on one of its variables, the space X fails to have this property, and the respective monoids of all uniformly continuous self-maps of X and Y are the same. Their investigations were motivated by problems originating in the theory of clones in universal algebra.

7. Uniform functions and related topics

In recent years many articles treated problems related to the standard concept of a uniform(ly continuous) function (see, for instance, the articles [37,38,97,113]). Some aspects of this subject are discussed next.

Let X be a set and $\mathcal{F}$ a family of real-valued functions on X. In [62] Garrido and Montalvo denoted by $\mu_{\mathcal{F}}X$ the set X endowed with the weak uniformity given by $\mathcal{F}$. They provided a method of generating the set $U(\mu_{\mathcal{F}}X)$, of all real-valued uniformly continuous functions on $\mu_{\mathcal{F}}X$, by means of the family $\mathcal{F}$. To this end they studied uniform approximation of real uniformly continuous functions on subsets of $\mathbb{R}^n$. As a consequence, they obtained an internal condition for $\mathcal{F}$ to be uniformly dense in $U(\mu_{\mathcal{F}}X)$. Their article is related to former work of Hager.

Thanks to several articles due to Berarducci, Ciesielski, Dikranjan and Pelant (see, e.g., [16,17,30]) our knowledge about a property called *uniform approachability* increased considerably over the past years. A map $f : X \to Y$ between metric or uniform spaces is called *uniformly approachable* if for every compact subset K of X and every subset $M \subseteq X$, there exists a uniformly continuous function $g : X \to Y$ such that $g(x) = f(x)$ whenever $x \in K$ and $g(M) \subseteq f(M)$. The weaker property that only the singletons of X instead of all compact subsets of X satisfy this condition, defines the class of *weakly uniformly approachable functions*.

These properties are stronger than continuity, but weaker than uniform continuity, and are preserved under composition. Functions from $\mathbb{R}$ to $\mathbb{R}$ are uniformly approachable if and only if they are continuous. However Burke had noted that the continuous map $f:\mathbb{R}^2 \to \mathbb{R}$ defined by $f(x,y) = x \cdot y$ is not weakly uniformly approachable.

In [16] it was established that a polynomial function $f:\mathbb{R}^n \to \mathbb{R}$ is uniformly approachable if and only if any two distinct fibers $f^{-1}\{x\}$ and $f^{-1}\{y\}$ are at positive distance; furthermore every real-valued continuous function with distant fibers on a uniformly locally connected metric space is uniformly approachable.

According to [17] every metric space on which every real-valued continuous function is uniformly approachable is thin (that is, every closed subspace has connected uniform quasi-components).

A metric space X is called *straight* [18,19] if for each finite cover of X by closed sets, and for each real-valued continuous function on X, if f is uniformly continuous on each set of the cover, then f is uniformly continuous on the whole space X. In [18] Berarducci, Dikranjan and Pelant characterized the spaces with this property within the class of all locally connected metric spaces as the uniformly locally connected spaces. On the other hand, in the class of all totally disconnected spaces, these spaces coincide with the Atsuji spaces (that is, those metric spaces on which every real-valued continuous function is uniformly continuous). In the literature the latter metric spaces are also called UC-spaces. In [18] it was observed that a metric space X is a UC-space if and only if each closed subspace of X is straight.

The same authors also noted in [19] that the completion of any straight metric space is straight and that the class of straight spaces is not closed under finite products.

Alas and DiConcilio [2] studied local forms of the concept of uc-ness in the metric context, where uc-ness means that some form of continuity is uniform. In particular, they were concerned with applications of these ideas in the theory of hyperspaces and function spaces. Numerous characterizations of those metric spaces having an Atsuji-completion were listed by Jain and Kundu [78].

Let X be a Tychonoff space and let $\mathcal{C}_p(X)$ be the space of all real-valued continuous functions on X with the topology of pointwise convergence. Uniformities are an important ingredient of $\mathcal{C}_p$-theory. Recall that Tychonoff spaces X and Y are called *u-equivalent* provided that the spaces $\mathcal{C}_p(X)$ and $\mathcal{C}_p(Y)$ are uniformly homeomorphic.

Recently Górak [64] proved that for every positive n, a Tychonoff space X is u-equivalent to the n-dimensional cube $\mathbf{I}^n = [0,1]^n$ if and only if the following conditions are satisfied:

(a) X is n-dimensional, compact and metrizable.
(b) Every nonempty closed subset of X contains a nonempty relatively open subset which can be embedded into $\mathbf{I}^n$.

Van Mill, Pelant and Pol [129] gave an example of a function space $C_p(X)$, where X is a Cook continuum, such that $C_p(X)^m$ is uniformly homeomorphic to $C_p(X)^n$ if and only if $n = m$.

Tkachuk [125] called a set $A \subseteq C_p(X)$ *uniformly dense* in $C_p(X)$ if it is dense in the topology of uniform convergence on $C(X)$, that is, if for any $f \in C_p(X)$ and any $\varepsilon > 0$, there is $g \in A$ such that $|g(x) - f(x)| < \varepsilon$ whenever $x \in X$. He proved that, for many properties $\mathcal{P}$, if a uniformly dense subspace of $C_p(X)$ has property $\mathcal{P}$, then $C_p(X)$ has property $\mathcal{P}$.

8. Hyperspace structures

Uniform and related structures continued to play a major role in the study of hyperspaces and (multi-)function spaces (see, e.g., [7,44,46,54,89]). For instance, in [7] it was shown that two compact metric spaces are homeomorphic if and only if their canonical complements (in the hyperspace of nonempty closed subsets equipped with the Hausdorff metric) are uniformly homeomorphic.

Recently in particular the problem of H-equivalence of uniformities and its generalizations led to several new publications. Let us recall that in 1966 Smith [121] had first raised the question whether two distinct uniformities on a set X really always determine Hausdorff uniformities that induce distinct topologies (on the set $\mathcal{P}_0(X)$ of nonempty subsets of X), as it had been suggested in an exercise in Isbell's aforementioned book on Uniform Spaces [73]. As Hitchcock later pointed out in [69], first results dealing with this problem had in fact been obtained by Albrecht [3] in 1952. The latter article however remained unnoticed for a long time and therefore had no influence on the subsequent developments of the subject.

In his paper Smith [121] showed that the answer to his question is positive in several important cases, for instance, for two distinct uniformities with a countable base, or for two distinct uniformities one of which is totally bounded, as well as for distinct left and right uniformities of a topological group.

However Isbell, Ivanov and Ward [74,77,131] soon presented examples showing that the answer to Smith's question is negative in general. Hursch also observed that his height relation between uniformities (see [72]) could be used to settle Smith's question.

Ward [132] then called uniformities on a set X *H-equivalent* if the topologies induced by their Hausdorff uniformities agree and presented a characterization of H-equivalent uniformities. It turns out (compare, e.g., [86, Corollary 2]) that two uniformities $\mathcal{U}$ and $\mathcal{V}$ on a set X are H-equivalent if and only if $\tau(\mathcal{U}_+) = \tau(\mathcal{V}_+)$ and $\tau(\mathcal{U}_-) = \tau(\mathcal{V}_-)$ on $\mathcal{P}_0(X)$, where as usual $\mathcal{U}_+$ resp. $\mathcal{U}_-$ denotes the upper quasi-uniformity resp. lower quasi-uniformity of $\mathcal{U}$. The first equality has a simple interpretation. It is known that two uniformities $\mathcal{U}$ and $\mathcal{V}$ on a set X satisfy $\tau(\mathcal{U}_+) = \tau(\mathcal{V}_+)$ on $\mathcal{P}_0(X)$ if and only if they induce the same proximity on X (compare, e.g., [28, Corollary 2.2]). In particular, it follows that H-equivalent uniformities on a set X induce the same proximity on X, which had already been observed by Smith [121].

It should be mentioned that on the other hand Poljakov [108] established that if for two uniformities $\mathcal{U}$ and $\mathcal{V}$ on a set X the topologies induced by the Hausdorff uniformities $(\mathcal{U}_H)_H$ of $\mathcal{U}_H$ and $(\mathcal{V}_H)_H$ of $\mathcal{V}_H$ on $\mathcal{P}_0(\mathcal{P}_0(X))$ agree, then $\mathcal{U}$ and $\mathcal{V}$ must be equal.

Recently now Hitchcock [70] continued his earlier work [69] about the comparability of Hausdorff uniform topologies on hyperspaces of uniform spaces. He gave simple conditions on a uniform space that are sufficient for H-singularity (that is, no other uniformity induces the same hyperspace topology). The latter topic had mainly been studied before in two papers of Ward [132,133], where among other things it had been shown that the left uniformity of a locally compact topological group is H-singular. In particular, Hitchcock related properties of a map $f : X \to Y$ between uniform spaces X and Y to properties of the induced hyperspace map $Hf : HX \to HY$, thereby generalizing and unifying results of Albrecht, Smith and Poljakov. For instance, he observed that for Hf to be continuous, f must be at least a proximity map.

The concept of H-equivalence was also generalized to the setting of quasi-uniformities. More precisely, in [28] Cao, Künzi and Reilly called two quasi-uniformities $\mathcal{U}$ and $\mathcal{V}$ on a set X *QH-equivalent* if the associated Hausdorff(–Bourbaki) quasi-uniformities $\mathcal{U}_H$ and $\mathcal{V}_H$ induce the same topology $\tau(\mathcal{U}_H) = \tau(\mathcal{V}_H)$ on $\mathcal{P}_0(X)$. Furthermore a quasi-uniformity is called *QH-singular* if there is no other quasi-uniformity QH-equivalent to it.

These authors obtained several results which are similar to those known in the uniform case. For instance, they noted that QH-equivalent quasi-uniformities on a set X induce the same quasi-proximity on X. It is also known that the Pervin quasi-uniformity of a topological space X is QH-singular if and only if X is hereditarily compact, and that for any topological space X the Pervin quasi-uniformity and the well-monotone quasi-uniformity each induce the Vietoris topology on $\mathcal{P}_0(X)$ (compare [114, Theorem 1]). Examples show that a H-singular uniformity need not be QH-singular.

At present no useful characterization of the concept of QH-equivalence of quasi-uniformities is known and the theory of QH-equivalence is clearly less satisfactory than the one of H-equivalent uniformities. For instance (see [86, Problems 1 and 2]), it is not clear whether the equality $\tau(\mathcal{U}_-) = \tau(\mathcal{V}_-)$ holds provided that $\mathcal{U}$ and $\mathcal{V}$ are QH-equivalent quasi-uniformities on a set X; it also seems unknown whether a quasi-uniformity $\mathcal{U}$ can be distinct from its conjugate $\mathcal{U}^{-1}$, but QH-equivalent to $\mathcal{U}^{-1}$.

In the light of Smith's result cited above the following problem posed in [28] is interesting, too.

Problem 8.1. If the left and right quasi-uniformities of a paratopological group are distinct, can they be QH-equivalent?

In [86] Künzi observed that the result of Poljakov about iterated Hausdorff uniformities does not generalize to quasi-uniformities. He exhibited two distinct quasi-uniformities $\mathcal{U}$ and $\mathcal{V}$ on the set ω of non-negative integers such that the Hausdorff quasi-uniformities $(\mathcal{U}_H)_H$ and $(\mathcal{V}_H)_H$ induce the same topology on $\mathcal{P}_0(\mathcal{P}_0(\omega))$. His result led to several natural questions about higher iterations of the Hausdorff–Bourbaki hyperspace construction in the area of quasi-uniformities which can be found in [86]. Let us restate here two of these questions in self-explanatory notation.

Problem 8.2. Do there exist two distinct quasi-uniformities $\mathcal{U}$ and $\mathcal{V}$ on a set X such that $\tau(\mathcal{U}_{H^n}) = \tau(\mathcal{V}_{H^n})$ on $\mathcal{P}_0^n(X)$ whenever $n \in \omega$?

Problem 8.3. For each $n \in \omega$, do there exist two quasi-uniformities $\mathcal{U}$ and $\mathcal{V}$ on a set X such that $\tau(\mathcal{U}_{H^n}) = \tau(\mathcal{V}_{H^n})$ on $\mathcal{P}_0^n(X)$, but $\tau(\mathcal{U}_{H^{n+1}}) \neq \tau(\mathcal{V}_{H^{n+1}})$ on $\mathcal{P}_0^{n+1}(X)$?

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