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Forward Compton scattering with weak neutral current: Constraints from sum rules

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ABSTRACT

We generalize forward real Compton amplitude to the case of the interference of the electromagnetic and weak neutral current, formulate a low-energy theorem, relate the new amplitudes to the interference structure functions and obtain a new set of sum rules. We address a possible new sum rule that relates the product of the axial charge and magnetic moment of the nucleon to the 0th moment of the structure function $g_5(\nu, 0)$. For the dispersive γZ-box correction to the proton's weak charge, the application of the GDH sum rule allows us to reduce the uncertainty due to resonance contributions by a factor of two. The finite energy sum rule helps addressing the uncertainty in that calculation due to possible duality violations.

The study of Compton scattering within dispersion relation formalism has led to a derivation of the celebrated sum rules that model-independently relate low-energy properties of the nucleon to its excitation spectrum. Forward Compton amplitude that contains parity-conserving (PC) and parity-violating (PV) interactions is expressed in terms of scalar PC amplitudes f, g and PV amplitudes $\tilde{f}, \tilde{g}$, see *e.g.* Refs. [1,2],

$$T(\nu) = f(\nu)(\vec{\varepsilon}\,'^* \cdot \vec{\varepsilon}) + g(\nu) i\vec{\sigma} \cdot [\vec{\varepsilon}\,'^* \times \vec{\varepsilon}] + \tilde{f}(\nu) i\hat{q} \cdot [\vec{\varepsilon}\,'^* \times \vec{\varepsilon}] + \tilde{g}(\nu)(\vec{\sigma}\hat{q})(\vec{\varepsilon}\,'^* \cdot \vec{\varepsilon}), \quad (1)$$

with M the nucleon mass, $\vec{\sigma}$ the nucleon spin, $\hat{q}$ the unit vector pointing in the direction of the photon three momentum, and $\vec{\varepsilon}\,', \vec{\varepsilon}$ the final (initial) photon polarization vectors. We define the electromagnetic forward Compton amplitude as

$$T_{\gamma\gamma} = \frac{i}{8\pi M e^2} \int d^4x e^{iqx} \langle p | T\, j^\mu_{EM}(x) j^\nu_{EM}(0) | p \rangle \varepsilon_\mu \varepsilon\,'^*_\nu, \quad (2)$$

with M the nucleon mass and e related to the fine structure constant $\alpha_{em} = e^2/(4\pi) \approx 1/137$. Only PC amplitudes f, g are present in the electromagnetic case. The γZ-interference forward Compton amplitude is normalized as

$$T^{\gamma Z} = \frac{i \sin 2\theta_W}{4\pi M e^2} \int d^4x e^{iqx} \langle p | T\, j^\mu_{NC}(x) j^\nu_{EM}(0) | p \rangle \varepsilon_\mu \varepsilon\,'^*_\nu, \quad (3)$$

with θ_W the weak mixing angle. We focus on transverse Z^0 here, whereas the longitudinal component may be related to pion photo production through PCAC. The vector coupling of the Z^0 contributes to the amplitudes f, g that already appeared in the electromagnetic case. To disambiguate we will use the superscript γZ for the interference case. The PV amplitudes $\tilde{f}, \tilde{g}$ arise from an interference of the electromagnetic current with the axial vector current, and correspond to nucleon spin-independent and nucleon spin-dependent contributions, respectively. Under crossing $\nu \to -\nu$ the amplitudes $f, \tilde{g}$ are even, while the amplitudes $\tilde{f}, g$ are odd.

We wish to emphasize that although we consider a Z^0 boson in the final or initial state, the kinematics of the Compton process that we study here is such that the on-shell Z^0 cannot be produced since $q^2 = 0$. PV sum rules have been considered either in the Compton process $\gamma + N \to \gamma + N$ with hadronic PV effects [3,4] or Compton-like process $\gamma + \nu \to W^+ + e^-$, $\gamma + e^- \to Z^0 + e^-$ and such, with an on-shell weak boson produced in the final state [5,6]. In the process that we consider, the Z^0 may originate, *e.g.*, from neutrino or charged lepton scattering off the nucleon accompanied with a radiation of a real photon in the final state, as, *e.g.*, virtual Compton scattering is accessed in a process $e^- + N \to e^- + N + \gamma$.

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In the context of, *e.g.*, MiniBooNE [7] and other neutrino oscillation experiments, $\nu + N \to \nu + N + \gamma$ is an important background that has been addressed in phenomenological calculations [8–12]. Forward γZ interference Compton amplitude enters the calculation of some electroweak corrections, in particular the dispersion γZ-box correction to the weak charge of the proton [13–17] for the kinematics of the Q-Weak experiment currently under analysis [18]. At present, the theory of uncertainty is dominated by that due to the γZ-box [16]. PV and PC γZ-interference structure functions are not constrained by experimental data, especially at low Q^2, and to perform calculations they have to be modeled applying symmetry and isospin structure assumptions to the electromagnetic data. The estimates for the theory of uncertainty due to the γZ-correction for the Q-Weak vary between 0.5% [17] and 2.8% [16] of the Standard Model value of the proton's weak charge, and we investigate here, to what extent the sum rules for the interference Compton process constrain this calculation.

The low energy limit of the Compton amplitudes is obtained by considering the ground state contribution, and it depends only on the nucleon mass M, charge (electric e_N, weak Q_W^N and axial g_A^N) and magnetic moment (electromagnetic κ_N or weak κ_N^Z). Parametrizing the next-to-leading order in the photon energy ν in terms of polarizabilities, one obtains the low-energy expansion (LEX) up to order ν^2: the well-known result for the electromagnetic case [19,20],

$$f(\nu) = -\frac{e_N^2}{4\pi M} + \frac{1}{e^2}(\alpha+\beta)^{\gamma\gamma}\nu^2 + \ldots,$$
$$g(\nu) = -\nu\frac{(\kappa_N^\gamma)^2}{8\pi M^2} + \ldots, \tag{4}$$

and the new results for the γZ-interference,

$$f^{\gamma Z}(\nu) = -\frac{e_N Q_W^N}{4\pi M} + \frac{1}{e^2}(\alpha+\beta)^{\gamma Z}\nu^2 + \ldots,$$
$$g^{\gamma Z}(\nu) = -\nu\frac{\kappa_N^\gamma \kappa_N^Z}{8\pi M^2} + \ldots,$$
$$\tilde f(\nu) = 0 + \frac{1}{e^2}\delta_1^{\gamma Z}\nu + \ldots,$$
$$\tilde g(\nu) = -\frac{g_A^N \mu_N}{4\pi M} + \frac{1}{e^2}\delta_2^{\gamma Z}\nu^2 + \ldots \tag{5}$$

Above, according to the definition of the interference Compton amplitude, we use $Q_W^p = 1 - 4\sin^2\theta_W$, $Q_W^n = -1$, $g_A^p = -g_A^n = 1.2701(25)$, $\kappa_p^Z = (1-4\sin^2\theta_W)\kappa_p^\gamma - \kappa_n^\gamma - \mu_s$, and $\kappa_n^Z = (1-4\sin^2\theta_W)\kappa_n^\gamma - \kappa_p^\gamma - \mu_s$. The strangeness contribution to the magnetic moment μ^s, according to a recent global analysis of Ref. [21], is $\mu_s = 0.37 \pm 0.27$. A recent lattice calculation of Ref. [22] obtains a significantly lower value, $\mu_s = 0.20 \pm 0.08$. In what follows, the strange quark contribution is only used to assess the uncertainties, and the nucleon axial charge is taken without radiative corrections. Above, the nucleon magnetic moment was defined as $\mu_N = e_N + \kappa_N^\gamma$, and two new polarizabilities $\delta_{1,2}^{\gamma Z}$ were introduced. The optical theorem relates the imaginary parts of the forward amplitudes to the inelastic structure functions $F_{1,3}(\nu, Q^2), g_{1,5}(\nu, Q^2)$ taken in the limit $Q^2 = 0$:

$$\mathrm{Im}\, f = \frac{1}{4M}F_1, \qquad \mathrm{Im}\, g = \frac{1}{4M}g_1,$$
$$\mathrm{Im}\, \tilde f = \frac{1}{8M}F_3^{\gamma Z}, \qquad \mathrm{Im}\, \tilde g = -\frac{1}{2M}g_5^{\gamma Z}. \tag{6}$$

The amplitudes $f, g, \tilde f, \tilde g$ are analytic functions of complex energy and obey dispersion relations

$$\mathrm{Re}\, f(\nu) = f(0) + \frac{\nu^2}{4\pi M}\int_{\nu_\pi}^\infty \frac{d\nu'^2}{\nu'^2(\nu'^2-\nu^2)}F_1(\nu',0)$$
$$\mathrm{Re}\, g(\nu) = \frac{\nu}{2\pi M}\int_{\nu_\pi}^\infty \frac{d\nu'}{\nu'^2-\nu^2}g_1(\nu',0),$$
$$\mathrm{Re}\, \tilde f(\nu) = \frac{\nu}{4\pi M}\int_{\nu_\pi}^\infty \frac{d\nu'}{\nu'^2-\nu^2}F_3^{\gamma Z}(\nu',0)$$
$$\mathrm{Re}\, \tilde g(\nu) = \tilde g(0) - \frac{\nu^2}{2\pi M}\int_{\nu_\pi}^\infty \frac{d\nu'^2}{\nu'^2(\nu'^2-\nu^2)}g_5^{\gamma Z}(\nu',0), \tag{7}$$

where $\nu_\pi = m_\pi + m_\pi^2/2M$ is the first inelastic threshold due to pion production. The high-energy behavior of F_1, g_5 requires subtractions for $f, \tilde g$.

These dispersion relations can now be evaluated for low energies $\nu \ll \nu_\pi$. Taylor-expanding the dispersion integrals in powers of ν^2 and equating the coefficients in this expansion to the LEX of Eq. (5) the sum rules follow,

$$(\alpha+\beta)^{\gamma\gamma,\gamma Z} = \frac{2\alpha_{em}}{M}\int_{\nu_\pi}^\infty \frac{d\nu}{\nu^3}F_1^{\gamma\gamma,\gamma Z}(\nu,0) \tag{8}$$

$$\kappa_N^\gamma \kappa_N^{\gamma,Z} = -4M\int_{\nu_\pi}^\infty \frac{d\nu}{\nu^2}g_1^{\gamma\gamma,\gamma Z}(\nu,0), \tag{9}$$

$$\delta_1^{\gamma Z} = \frac{\alpha_{em}}{M}\int_{\nu_\pi}^\infty \frac{d\nu}{\nu^2}F_3^{\gamma Z}(\nu,0), \tag{10}$$

$$\delta_2^{\gamma Z} = -\frac{4\alpha_{em}}{M}\int_{\nu_\pi}^\infty \frac{d\nu}{\nu^3}g_5^{\gamma Z}(\nu,0). \tag{11}$$

Eqs. (8), (9) are Baldin [23] and Gerasimov–Drell–Hearn [24] sum rules, respectively, and their straightforward generalization to the case of the γZ interference. Both sum rules were checked experimentally for the electromagnetic case [25,26] and the agreement was found to be better than 4% for Baldin sum rule, and to be within 10% for the GDH sum rule. The GDH sum rule was checked perturbatively in electroweak theory [5,6,28]. Note that a GDH-like sum rule for PV Compton scattering was considered, *e.g.*, in Refs. [3,4] but in the context of hadronic parity violation, and not due to $\gamma - Z^0$ interference. The other two sum rules equate the PV polarizabilities δ_1 and δ_2 to the 1st moment of the structure function F_3 and 2nd moment of g_5, respectively, and both integrals are certainly convergent.

Finally, the finite energy sum rule (FESR) for the amplitudes f and $\tilde g$ results from extracting Regge-behaved part $f^R(\tilde g^R)$ of the amplitude $f(\tilde g)$ explicitly, and writing a dispersion relation for the difference $f - f^R$ and $\tilde g - \tilde g^R$. At the asymptotically high energy such an amplitude can be at most a constant that is denoted by $C_\infty(\tilde C_\infty)$, and one obtains a dispersion representation for this constant (the $J = 0$ pole) [27],

$$C_\infty = -\frac{e_N^2}{4\pi M} - \frac{1}{2\pi M}\int_{\nu_{thr}}^N \frac{d\nu}{\nu}[F_1(\nu,0) - F_1^R(\nu,0)]. \tag{12}$$

Above, $F_1^R = \sum_i c_i \nu^{\alpha_i}$ with α_i strictly positive. The leading high-energy behavior is described by the Pomeron with $\alpha_P \approx 1.09$ and the f_2-trajectory exchange with $\alpha_{f_2} \approx 0.5$, and was obtained from a Regge fit at $\nu \geq N \approx 2$ GeV [29]. Note that due to different normalization of the Compton amplitude, C_∞ in Eq. (12) differs from that in [27,29] by a factor $(4\pi\alpha_{em})^{-1}$. Quite analogously we obtain for the interference PC amplitude,

$$C_\infty^{\gamma Z} = -\frac{e_N Q_W^N}{4\pi M} - \frac{1}{2\pi M}\int_{\nu_\pi}^{N}\frac{d\nu}{\nu}[F_1^{\gamma Z}(\nu,0) - F_1^{\gamma Z,R}(\nu,0)], \quad (13)$$

and for the interference PV amplitude,

$$\tilde{C}_\infty^{\gamma Z} = -\frac{g_A^N \mu_N}{4\pi M} + \frac{1}{\pi M}\int_{\nu_{thr}}^{N}\frac{d\nu}{\nu}[g_5^{\gamma Z}(\nu,0) - g_5^{\gamma Z,R}(\nu,0)]. \quad (14)$$

It is necessary to stress that the FESR of Eq. (14) is based on the assumption that at asymptotically high energy $\nu \to \infty$, $g_5^{\gamma Z,R}$ grows as ν^α with $\alpha > 0$. Should this assumption not hold, and the structure function g_5 decrease at high energies, then an unsubtracted sum rule would have to be postulated,

$$g_A^N \mu_N = 4\int_{\nu_{thr}}^{\infty}\frac{d\nu}{\nu} g_5^{\gamma Z}(\nu,0). \quad (15)$$

To assess these options, we examine the high-energy asymptotics of g_5 more closely. At high energy and in the Regge framework, the structure accompanying g_5 may come about due to an exchange of an axial vector meson. Possible lowest mass candidates are $h_1(1170)$, $b_1(1235)$, and $a_1(1260)$. Due to lack of sufficient higher spin states for these channels the Chew–Frautschi plot for these trajectories is not fully constrained, and we will give a range for the intercept of these trajectories. The upper limit stems from relating an axial vector to the pion trajectory, thus $\alpha_0 \approx -\alpha' m_\pi^2 \approx -0.02$. The lower limit results from a linear extrapolation $\alpha_0^M = 1 - \alpha' m_M^2$ that range from -0.1 to -0.4 for the three candidates. We refer to two recent studies of the properties of Regge trajectories in [30] and [31]. The latter reference includes an analysis of polarized NN data up to high energy. Our simple estimate is in line with these two studies. Other works, *e.g.* [32], use $\alpha_{b_1}(0) \approx 0.5$ which would require a much smaller Regge slope or a substantial nonlinearity of the respective trajectory. Apart from nucleon and meson scattering, the high-energy behavior of g_5 enters parametrizations of the polarized quark PDFs. Ref. [33] obtains for the two lightest flavors, $g_5^u \sim (\Delta u - \Delta\bar{u})(x \to 0) \sim x^{-0.308}$ and $g_5^d \sim (\Delta d - \Delta\bar{d})(x \to 0) \sim x^{-0.836}$. In terms of possible Regge exchanges, this may suggest, upon assuming a universal Regge slope, an existence of two degenerate axial vector meson trajectories (isoscalar and isovector) realized as particles with masses below $\rho(770)$. No such states have been observed or predicted. On the other hand, the most recent analysis of polarized DIS data performed in Ref. [34] obtains the low-x behavior of the polarized valence PDF's for which an unsubtracted dispersion integral converges. Finally, PDF fits are driven by large Q^2-data that may exhibit quite different asymptotic energy behavior than real photon data, as is the case for the "soft" and "hard" Pomerons [35]. The situation remains inconclusive, as existing polarized DIS data do not constrain the high-energy asymptotics of $g_5(\nu,0)$.

As an informative check, we consider the contribution of the $\Delta(1232)$ resonance to the isoscalar (in the isovector combination it drops out) sum rule of Eq. (15). Accounting for the dominant magnetic $\gamma N\Delta$ coupling $c_{1\Delta}$ and the axial $Z^0 N\Delta$ coupling h_A (we refer the reader to Refs. [36,37] and references therein for details), we obtain,

$$g_A(\mu_p - \mu_n) = -\frac{32}{9}h_A c_{1\Delta}\left(M + M_\Delta + \frac{M_\Delta^2 - M^2}{2M_\Delta}\right)\frac{1}{\pi} \times \int_{\nu_\pi}^{\infty} d\nu\, \mathrm{Im}\left[\frac{1}{W^2 - M_\Delta^2 + iM_\Delta\Gamma_\Delta(W)}\right], \quad (16)$$

with $W^2 = M^2 + 2M\nu$ the invariant mass of the intermediate hadronic state. Using the values for the Δ parameters from Refs. [36,37], $h_A = 1.40$, $c_{1\Delta} = 1.21$ and treating the $\Delta(1232)$ as a narrow resonance leads to r.h.s. $\approx 16/9 h_A c_{1\Delta}(1 + M_\Delta/M + [M_\Delta^2 - M^2]/2MM_\Delta) \approx 7.79$; using the experimental width and accounting for its energy dependence reduces the result to 5.93, to be compared to the l.h.s. $g_A(\mu_p - \mu_n) = 5.96$. The agreement is remarkable. The only viable Regge exchange, $h_1(1170)$ seems to be consistent with negative intercept in all analyses known to us, while a_1, b_1 do not contribute being isovectors. Further contributions to the sum rule still have to be incorporated, such as non-resonant πN contributions, and higher resonance states. One may notice a similarity to the Fubini–Furlan–Rossetti sum rule [38] expressing $g_A\kappa_N$ as an integral over a forward pion photoproduction amplitude, related by PCAC to the γZ-interference Compton amplitude with the longitudinal Z. We consider the transverse part.

Sum rule for $\delta_2^{\gamma Z}$

Our numerical estimate in the model with the narrow $\Delta(1232)$ leads to $\delta_2^{\gamma Z,p} = \delta_2^{\gamma Z,n} \approx -2.0 \times 10^{-3}$ fm^3. For comparison, the proton's electric polarizability is about half that size, $\alpha_E^p = (1.12 \pm 0.04) \times 10^{-3}$ fm^3 [40]. A more realistic estimate of the polarizability should include, *e.g.* the threshold pion production mechanism that is expected to be important numerically due to $1/\nu^3$ weighting under the integral.

Sum rule for $\delta_1^{\gamma Z}$

The low-energy limit of the amplitude $\tilde{f}$ has been estimated in Ref. [8] upon introducing an anomalous $\gamma Z^0 \omega$ vertex, and in Ref. [9] with the $\Delta(1232)$ isobar. The latter mechanism turns out to be numerically more important. Our numerical estimate in the model with the narrow $\Delta(1232)$, $\delta_1^{\gamma Z,p} = \delta_1^{\gamma Z,n} \approx 4.5 \times 10^{-3}$ fm^2, is consistent with that of Ref. [9]. It has been argued [8] that this polarizability can induce an effective $\gamma\nu\bar{\nu}$ interaction that may provide an additional channel for energy loss from neutron stars.

GDH sum rule and the parametrization of resonance data

A parametrization of the inelastic structure functions $F_{1,2}^{\gamma\gamma}$ on the proton target in the resonance region has been proposed by Christy and Bosted in Ref. [39]. This parametrization was used to predict the interference structure functions $F_{1,2}^{\gamma Z}$ entering the calculation of the dispersive γZ-box correction to the weak charge of the proton in the kinematics of the QWEAK experiment [18]. The procedure involves a rotation of the transition helicity amplitudes for individual resonances in the weak isospin space [16]. It is based on the conservation of the vector current (CVC) and on the identification of quantum numbers of the resonances. The latter was taken from the original parametrization of Ref. [39]. We will assess this identification with the use of the GDH sum rule. The parametrization of Ref. [39] features two close resonances in the second resonance region, $S_{11}(1535)$ and $D_{13}(1520)$, of which the former dominates carrying $\approx 90\%$ of the strength in the sum of the two in the total cross section. The commonly accepted picture [40] is nearly opposite, and this difference can be disentangled with the GDH sum rule: $S_{11}(1535)$ being a $J = 1/2$ resonance cannot be excited in the $A_{3/2}$ channel, thus its contribution to the

Table 1
Values of the parameter A_T for the 5 of 7 resonances used in the fit of Ref. [39]. $A_T^I(0)$ stands for the original values and is referred to as Model I in the text. $A_T^{II}(0)$ shows the values modified in accord with the PDG as described in the text and referred to as Model II.

	$S_{11}(1535)$	$D_{13}(1520)$	$F_{15}(1680)$	$S_{11}(1650)$	$P_{11}(1440)$
$A_T^I(0)$	6.335	0.603	2.330	1.979	0.0225
$A_T^{II}(0)$	3.3	3.5	3.1	2.0	2.422

GDH sum rule is strictly negative (the spin structure function g_1 is related to the helicity-dependent photo absorption cross section as $g_1(\nu,0)=\frac{M\nu}{2\pi e^2}[\sigma_{1/2}-\sigma_{3/2}]$). Similarly, we consider $P_{11}(1440)$, $S_{11}(1650)$ and $F_{15}(1680)$. We display in Table 1 how resonance parameters should be changed to be in agreement with the helicity difference cross section $\sigma_{3/2}-\sigma_{1/2}$ without affecting the description of the data for the total cross section, see Fig. 1. The curves are compared to the data from Ref. [25] that with certainty exclude the blue dashed curve (Model I). The red solid curve (Model II) compares favorably to the data. Each curve can be used to evaluate the r.h.s. of the GDH sum rule. Model I leads to $\kappa_p^2\approx 0.9$, whereas Model II leads to $\kappa_p^2\approx 3.28$, a result close to the sum rule value, $\kappa_p^2=1.793^2\approx 3.215$. Note that for this evaluation we supplemented the threshold region with the non-resonant background contribution from MAID [41] that gives a sizable negative contribution. This contribution, being the helicity-difference cannot be directly obtained from the parametrization of Ref. [39] which only deals with the total cross section.

We evaluate the PV analogue of the GDH sum rule with the isospin-rotated cross sections, and compare it to $\kappa_p^\gamma\kappa_p^Z$. The evaluation with Model I parametrization leads to $\kappa_p^\gamma\kappa_p^Z\approx 2.247$, and that with Model II gives $\kappa_p^\gamma\kappa_p^Z\approx 3.615$, to be compared to the l.h.s. $\kappa_p^\gamma\kappa_p^Z=(1-4s_W^2)(\kappa_p^\gamma)^2-\kappa_n^\gamma\kappa_p^\gamma\approx 3.666$. The Model II is consistent with both sum rules. In this evaluation we neglected the strangeness contribution, and we did consistently so both in the l.h.s. low-energy coefficient $\kappa_p^\gamma\kappa_p^Z$ and in the r.h.s. integral. Strange magnetic moment $\mu_s=0.20\pm 0.08$ [22] can be used to assess the uncertainty of the l.h.s. of the GDH sum rule as $\mu_s\kappa_p\sim 0.36$, significantly smaller than the deviation of the evaluation with Model I from the sum rule value.

Now, we are in a position to update the value and the uncertainty of the dispersion evaluation of the resonance contribution to the $\mathrm{Re}\,\Box_{\gamma Z}^V$ correction to the QWEAK measurement [16]. The sum of the resonance contributions to $\mathrm{Re}\,\Box_{\gamma Z_V}$ with the resonance parametrization of Model I amounted to $\mathrm{Re}\,\Box_{\gamma Z_V,\,Res.}^{Mod.\,I}=2.24^{+0.53}_{-0.43}\cdot 10^{-3}$. After the modification described above we arrive at $\mathrm{Re}\,\Box_{\gamma Z_V,\,Res.}^{Mod.\,II}=2.23^{+0.28}_{-0.23}\cdot 10^{-3}$, with the uncertainty halved.

Assessing duality violation with FESR

Historically, FESR of Eq. (12) has been used to address duality: vanishing of the (duality) integral implies the equality of the $J=0$ pole and the Thomson term [27]. Thomson term describes an effective two-photon coupling to an "elementary" proton, while $J=0$ pole – that to elementary charged constituents, quarks. Then, this equality may be naïvely understood as the equality between charges, $(\sum_{q\in p}e_q)^2=\sum_{q\in p}e_q^2=1$. Note the similarity to the nuclear Thomas–Reiche–Kuhn sum rule in this interpretation [29,42]. Using this reasoning, we may compare the interference Thomson term $(\sum_{q\in p}e_q)\cdot(\sum_{q\in p}g_V^q)\approx 0.07$ and the respective quantity on quarks, $\sum_{q\in p}e_q g_V^q=5/3-4\sin^2\theta_W\approx 0.72$. This indicates that the duality violation for the interference case may be more pronounced.

The numerical evaluation of Eq. (12) with resonance contributions from Model II and the background parametrization from [16,29] leads to $C_\infty=-12.2$ μb GeV, which agrees reasonably well with the extraction of the $J=0$ pole in [16], $C_\infty=-8.2\pm 3.8$ μb GeV. To summarize the two evaluations,

$$C_\infty=-10.2\pm 3.8(stat.)\pm 2.0(syst.)\ \text{μb GeV},\qquad (17)$$

where we estimate the systematical uncertainty by averaging over the two evaluations. The parametrization of Model I gives a larger result, $C_\infty=-18.3$ μb GeV. Next we evaluate the γZ-interference analog of the $J=0$ pole, Eq. (13) using the isospin rotation as described in [16]. Model II leads to

$$C_\infty^{\gamma Z}=28.5\pm 22.0(back.)^{+10.1}_{-8.5}(res.)\ \text{μb GeV},\qquad (18)$$

where the first uncertainty is due to the isospin structure of the background, and the second one due to that of the resonances. It is seen that the Model II evaluation may in principle be used to constrain the background contribution since the uncertainty of the latter dominates over that due to resonances. To do that, the information about the l.h.s. of Eq. (18) is necessary. It has been argued in the literature that the $J=0$ pole, if exists, should be due to an effective two-photon-quark coupling. Then, knowing the $J=0$ pole for Compton scattering, Eq. (17) we can try to model the γZ-interference $J=0$ pole. Varying the dominant physical picture between the pure valence quarks and a symmetric SU(6) quark collection one would expect $C_\infty^{\gamma Z}\sim(0.85-0.72)C_\infty$, whereas the exact duality limit would correspond to the $J=0$ pole being equal to the interference Thomson term. These estimates indicate that the γZ-interference $J=0$ pole is likely to be somewhat smaller

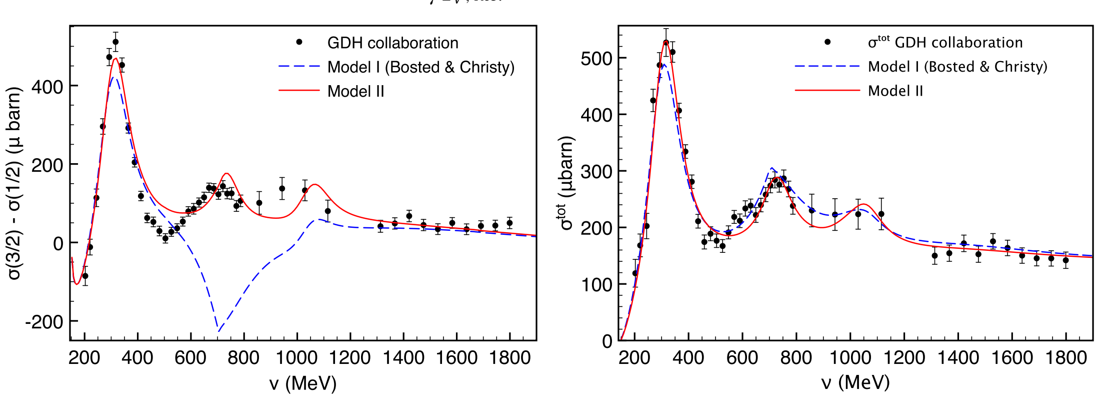

Fig. 1. Left panel: helicity difference $\sigma_{3/2}-\sigma_{1/2}$ photoabsorption cross section from the original parametrization of Ref. [39] (dashed blue line) and the modified one adjusted in accord with the PDG [40] (solid red line) in comparison with data by GDH collaboration [25]. Right panel: the same for the total photo absorption cross section. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

than the electromagnetic one, and to have the same sign. We can then assume that, conservatively,

$$C_\infty^{\gamma Z} = -5.1 \pm 5.1\ \mu\text{b GeV}. \tag{19}$$

A comparison with the evaluation of Eq. (18) suggests that for the two to agree, the background contribution should be taken at its lower range, suggesting that the model of Ref. [16] is likely to overestimate the interference structure functions $F_{1,2}^{\gamma Z}$ at $Q^2 = 0$. The recent measurement of the PV asymmetry in the resonance region on the deuteron [43] observed that models tend to overshoot the data in the $\Delta(1232)$ region by up to 25–30%, although the disagreement is not striking because of large uncertainties. Our analysis implies that this discrepancy may need to be taken seriously, since another physical constraint from FESR suggests the same behavior.

In summary, we derived a set of sum rules for forward Compton scattering generalized to the case of electromagnetic-weak neutral current interference. Along with a straightforward generalization of the GDH, Baldin and finite energy sum rules, we proposed a new sum rule that relates the product of the nucleon's axial charge and magnetic moment to an integral over the parity-violating structure function g_5. We analyzed the Regge asymptotics of that amplitude and found that currently, no solid statement about the convergence of this sum rule can be made. A model calculation for the isoscalar sum rule (its convergence is more reliable from the Regge stand point) with the $\Delta(1232)$ resonance leads to a very good agreement. This sum rule deserves further study: if confirmed it may give a constraint on the low-x behavior of the polarized PDF's parametrizations. We showed that accounting for GDH and finite energy sum rules for electromagnetic and electroweak Compton amplitudes can help constraining parametrizations of inclusive electromagnetic and interference structure functions. The latter are important for calculating nucleon structure-dependent electroweak corrections to precision low-energy tests, *e.g.*, the proton's weak charge measurement. Analysis of the electroweak GDH sum rule allowed for reducing the uncertainty in that calculation due to nucleon resonance contributions by a factor of 2. The analysis of FESR indicated that duality violation in the $\gamma - Z$ interference Compton process may be significantly larger than in the pure electromagnetic case.

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