Natural inflation from 5D SUGRA and low reheat temperature

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Abstract

Motivated by recent cosmological observations of a possibly unsuppressed primordial tensor component r of inflationary perturbations, we reanalyze in detail the 5D conformal SUGRA originated natural inflation model of Ref. [1]. The model is a supersymmetric variant of 5D extranatural inflation, also based on a shift symmetry, and leads to the potential of natural inflation. Coupling the bulk fields generating the inflaton potential via a gauge coupling to the inflaton with brane SM states we necessarily obtain a very slow gauge inflaton decay rate and a very low reheating temperature $T_r \lesssim \mathcal{O}(100)$ GeV. Analysis of the required number of e-foldings (from the CMB observations) leads to values of n_s in the lower range of present Planck 2015 results. Some related theoretical issues of the construction, along with phenomenological and cosmological implications, are also discussed.

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1. Introduction

Inflation solves the problems of early cosmology in a natural way [2] and besides that produces a primordial fluctuation spectrum [3] which allows to discuss structure formation successfully. In detailed models (i) a sufficient number of e-folds for the inflationary phase has to be produced, (ii) guided by bounds presented recently by the Planck Collaboration [4], the cosmic background radiation and a spectral index $n_s = 0.968 \pm 0.006$ should be generated.[2] And (iii), the normalization of fluctuations has to be reproduced. Rather flat potentials for the inflaton field lead to the "slow roll" needed for (i). Such potentials appear naturally in (tree level) global supersymmetric models; higher loop corrections can be controlled, but the inclusion of supergravity easily produces an inflaton mass of the order of the Hubble scale.

In models with an extra dimension the fifth component of a U(1) gauge field entering in a Wilson loop operator can act as an inflaton field of pseudo Nambu–Goldstone type which is protected against gravity corrections and avoids a transplanckian scale [6,7], present in the original model of "natural inflation" [8]. We have presented such a model [1] based on 5D conformal SUGRA on an orbifold $S^1/\mathbb{Z}_2$ with a predecessor based on global supersymmetry with a chiral "radion" multiplet on a circle in the fifth dimension [9]. We also made the interesting observation that a spectral index $n_s \sim 0.96$ as observed recently [different from a value very close to one usually obtained in straightforward SUSY hybrid inflation [11]] is obtained rather generically in gauge inflation. Actually, in the supersymmetric formulation we have a complex scalar field which besides the gauge inflaton A_5^1 contains a further "modulus" field M^1 which also might allow for successful inflation [1]. The main difference between the two inflation types is that gauge inflation leads to a large tensor to scalar ratio r (~ 0.12 in [1]) whereas modulus inflation leads to very small r ($\lesssim 10^{-4}$ in [1]).[3] Recently the BICEP2 data [12] gave strong indication of a large ratio $r = 0.2^{+0.07}_{-0.05}$ though recent joint analysis of BICEP2/Keck and Planck [13] gave a reduced upper bound $r \lesssim 0.12$,[4] with the likelihood curve for r having a maximum for $r \simeq 0.05$. Because of this, we here consider the gauge inflation of Ref. [1] again with particular emphasis on the required length of inflation. The well known 62 e-folds solving the horizon problem will turn out to require a substantial expansion during the reheating period within the natural inflation scenario emerged from 5D SUGRA.

Let us present the organization of the paper and summarize some of the results. In Section 2 we perform a detailed analysis of natural inflation with cos-type potential. For the calculation of the spectral index n_s and the tensor to scalar ratio r, we use a second order approximation with respect to slow roll (SR) parameters. Since these quantities (n_s, r) are determined at the point where the SR parameters are tiny, this approximation is sufficient for all practical purposes. However, near the end of the inflation, when SR breaks down, we perform an accurate numerical determination of the point via the condition $\epsilon_H = 1$ on the Hubble slow roll parameter (see [15–17] for definitions). This is needed to compute, with desired precision, the number of e-foldings (N_e^{inf}) before the end of the inflation. We carry out our analysis by using recent fresh

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[2] In the original version of this paper (see v1 of arXiv:1501.03520) we had the 2013 value [5] $n_s = 0.9603 \pm 0.0073$ which being about a standard deviation below this value makes quite a difference for our analysis.

[3] Genuine two field inflation was discussed in Ref. [10]. The two basic inflation types depending on initial conditions turn out to be still like in [1]. Since inclusion of the M^1 modulus into the inflation process is fully legitimate, one can reserve this scenario as an alternative with a tiny tensor perturbations, if it should be.

[4] Earlier, Planck's intermediate results [14] noted about a possible ordinary dust contribution instead of the light polarization effect really due to gravitational waves.

data [4] for n_s, N_e, the amplitude of curvature perturbations and bounds for r [13]. Our results are in agreement with a recent analysis of natural inflation by Freese and Kinney (see 2nd citation in [8]). For various cases we have also calculated the reheat temperature, which we use later on for contrasting with our 5D SUGRA emerged inflation scenario requiring a very low reheat temperature.

In Section 3 and Appendix A we shortly review our model of Ref. [1] in a more self-contained way and discuss how natural inflation emerges from 5D SUGRA. Using a superfield formulation, we do not need to go into the details of the component expressions in conformal 5D SUGRA of Fujita, Kugo and Ohashi (FKO) [18]. Indeed this emerged from our discussion [19,20] (see also Ref. [23]) bringing the 5D conformal SUGRA formulation closer to the 4D global SUSY language [20]. We concentrate here on gauge inflation, i.e. on the case $M^1 = 0$ (stabilized moduli in the origin or a choice of initial conditions[5]). In Section 3, discussing the realization of natural inflation within 5D SUGRA, we present a new mechanism for inflaton decay, which eventually leads to the reheat of the Universe. Note that, besides a specific string theory realization [25], the inflaton decay and reheating has never been discussed before in the context of natural inflation. We show that the inflaton's slow decay is a natural consequence of the 5D construction (with consistent UV completion), being realized by couplings of the heavy bulk supermultiplets generating the inflaton potential through their gauge coupling with brane SM states. Since the inflaton decay proceeds by 4-body decay and the decay width is strongly suppressed by the 2-nd power of the tiny $U(1)$ gauge coupling constant[6] (of the gauge inflaton-charged fields) and a relatively small inflaton mass coupled to the intermediate bulk fields, a strong suppression of the reheat temperature T_r comes out naturally. Our 5D SUGRA construction allows us to make an estimate $T_r \sim 0.34 \rho_{reh}^{1/4} \sim |\lambda|^2 \times 100\,\text{GeV}$ (where $\lambda \lesssim 1$ is a brane Yukawa coupling). At the end of Section 3 we show that, by the parameters we are dealing with, preheating is excluded within the considered scenario.

Appendix A discusses the Kaluza–Klein spectrum of the fields involved, as well as the SUSY breaking effects for brane fields. We also perform a derivation of higher-dimensional operators involving the inflaton ϕ_Θ and light (MSSM) states relevant for the inflaton decay. As it turns out, the dominant decay channel is $\phi_\Theta \to llhh$ (with l and h denoting SM lepton and Higgs doublets respectively). Section 4 includes a discussion and concluding remarks about some related issues.

2. Natural inflation

In this section we analyze inflation with the potential of natural inflation [8] given by:

$$\mathcal{V} = \mathcal{V}_0 \left(1 + \cos(\alpha \phi_\Theta)\right) , \tag{1}$$

where ϕ_Θ is a canonically normalized real scalar field of inflation. In the concrete scenario of Ref. [1], we focus later on, the inflaton originates from a 5D gauge superfield, while the parameters/variables of (1) are derived through the underlying 5D SUGRA. See Eqs. (24), (25), (A.17) and also the comment underneath Eq. (A.17).

The slow roll parameters ("VSR" – derived through the inflaton potential) are given by

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[5] For a discussion of moduli stabilization in the superfield formalism within 5D SUGRA see [24]. For a choice of initial conditions leading approximately to $M^1 = 0$ see Ref. [10].

[6] From a very recent paper [26] we learned that the 'weak gravity conjecture' (going back to Ref. [27]) based on magnetically charged black hole considerations and the dangerous neighborhood to a global symmetry, applies in disfavor of gauge (extranatural) inflation and might explain difficulties to embed the model in string theory.

$$\epsilon = \frac{M_{Pl}^2}{2}\left(\frac{\mathcal{V}'}{\mathcal{V}}\right)^2 = \frac{(M_{Pl}\alpha)^2}{2}\tan^2\frac{\alpha\phi_\Theta}{2}$$

$$\eta = M_{Pl}^2\frac{\mathcal{V}''}{\mathcal{V}} = \frac{(M_{Pl}\alpha)^2}{2}\left(\tan^2\frac{\alpha\phi_\Theta}{2} - 1\right) = \epsilon - \frac{1}{2}(M_{Pl}\alpha)^2 \ ,$$

$$\xi = M_{Pl}^4\frac{\mathcal{V}'\mathcal{V}'''}{\mathcal{V}^2} = -(M_{Pl}\alpha)^4\tan^2\frac{\alpha\phi_\Theta}{2} = -2(M_{Pl}\alpha)^2\epsilon \ , \qquad (2)$$

where $M_{Pl} = 2.4 \cdot 10^{18}$ GeV is the reduced Planck mass. In order to make notations compact, for the VSR parameters we do not use the subscript 'V' (denoting them by $\epsilon, \eta, \xi, \ldots$). However, for HSR parameters (derived through the Hubble parameter) we use subscript 'H' (e.g. $\epsilon_H, \eta_H, \xi_H, \ldots$), as adopted in literature [15,17,16]. The number of e-foldings during inflation, i.e. during exponential expansion, denoted further by N_e^{inf}, is calculated as

$$N_e^{\text{inf}} = \frac{1}{\sqrt{2}M_{Pl}}\int_{\phi_\Theta^e}^{\phi_\Theta^i}\frac{1}{\sqrt{\epsilon_H}}d\phi_\Theta \ . \qquad (3)$$

In this exact expression the HSR parameter ϵ_H (defined below), participates. The point ϕ_Θ^e, at which inflation ends, is determined by the condition $\epsilon_H = 1$. The point ϕ_Θ^i corresponds to the begin of the inflation. Also further, symbols with superscript or subscript 'i' will correspond to values at the beginning of the inflation, while superscript/subscript 'e' will indicate end of the inflation.

The observables n_s and r depend on the value of ϕ_Θ^i (the point at which scales cross the horizon). This allows to determine ϕ_Θ^i as follows. Via HSR parameters, the expressions for n_s and r are given by [15,17,16]:

$$n_s = 1 - 4\epsilon_{Hi} + 2\eta_{Hi} - 2(1 + C)\epsilon_{Hi}^2 - \frac{1}{2}(3 - 5C)\epsilon_{Hi}\eta_{Hi} + \frac{1}{2}(3 - C)\xi_{Hi},$$

$$r = 16\epsilon_{Hi}\left(1 + 2C(\epsilon_{Hi} - \eta_{Hi})\right) \ , \quad \text{with } C = 4(\ln 2 + \gamma) - 5 \simeq 0.0815 \ , \qquad (4)$$

where we have limited ourself with second order corrections. The HSR parameters $\epsilon_H, \eta_H, \xi_H$ are given by:

$$\epsilon_H = 2M_{Pl}^2\left(\frac{H'}{H}\right)^2 \ , \qquad \eta_H = 2M_{Pl}^2\frac{H''}{H}, \qquad \xi_H = 4M_{Pl}^4\frac{H'H'''}{H^2} \ , \qquad (5)$$

with the Hubble parameter H and it's derivative with respect to the inflaton field. The subscript 'i' in (4) indicates that the parameter is defined at the point at which scales cross the horizon. As it turns out, at this scale the slow roll parameters are small and second order corrections in n_s and r are small and the approximations made in (4) are pretty accurate. Exact relations between VSR ($\epsilon, \eta, \xi, \ldots$) and HSR parameters ($\epsilon_H, \eta_H, \xi_H, \ldots$) are given by [15,17,16]:

$$\epsilon = \epsilon_H\left(\frac{3 - \eta_H}{3 - \epsilon_H}\right)^2 \ , \qquad \eta = \frac{3(\epsilon_H + \eta_H) - \eta_H^2 - \xi_H}{3 - \epsilon_H},$$

$$\xi = 3\frac{3 - \eta_H}{(3 - \epsilon_H)^2}\left(3\epsilon_H\eta_H + \xi_H(1 - \eta_H) - \frac{1}{6}\sigma_H\right) \ , \quad \text{with } \sigma_H = 4M_{Pl}^4\epsilon_H\frac{H^{(iv)}}{H}. \qquad (6)$$

When the slow roll parameters are small, from (6), the HSR parameters to a good approximation can be expressed in terms of VSR parameters as

$$\epsilon_H \simeq \epsilon - \frac{4}{3}\epsilon^2 + \frac{2}{3}\epsilon\eta \,, \qquad \eta_H \simeq \eta - \epsilon + \frac{8}{3}\epsilon^2 - \frac{8}{3}\epsilon\eta + \frac{1}{3}\eta^2 + \frac{1}{3}\xi,$$
$$\xi_H \simeq 3\epsilon^2 - 3\epsilon\eta + \xi. \tag{7}$$

Using these approximations in (4), we can write n_s and r in terms of VSR parameters:

$$n_s = 1 - 6\epsilon_i + 2\eta_i + \frac{2}{3}(22 - 9C)\epsilon_i^2 - (14 - 4C)\epsilon_i\eta_i + \frac{2}{3}\eta_i^2 + \frac{1}{6}(13 - 3C)\xi_i$$
$$r = 16\epsilon_i\left(1 - (\frac{2}{3} - 2C)(2\epsilon_i - \eta_i)\right) \,, \tag{8}$$

where we have still restricted the approximations up to the second order. Applying these expressions, for the model (determining ϵ, η and ξ as given in Eq. (2)), we arrive at:

$$n_s = 1 - \left(1 + 2\tan^2\frac{\alpha\phi_\Theta^i}{2}\right)(M_{Pl}\alpha)^2$$
$$+ \left(\frac{1}{6} + (1 - \frac{1}{2}C)\tan^2\frac{\alpha\phi_\Theta^i}{2} + (\frac{1}{3} - \frac{1}{2}C)\tan^4\frac{\alpha\phi_\Theta^i}{2}\right)(M_{Pl}\alpha)^4 \,, \tag{9}$$

and

$$r = 8(M_{Pl}\alpha)^2\left(1 - (\frac{1}{3} - C)(1 + \tan^2\frac{\alpha\phi_\Theta^i}{2})(M_{Pl}\alpha)^2\right)\tan^2\frac{\alpha\phi_\Theta^i}{2} \,. \tag{10}$$

From Eq. (10) we can express $\tan\frac{\alpha\phi_\Theta^i}{2}$ in terms of r and $M_{Pl}\alpha$. As will turn out, the latter's value is small, so to a good approximation we find:

$$\tan^2\frac{\alpha\phi_\Theta^i}{2} \simeq \frac{r}{8(M_{Pl}\alpha)^2}\left(1 + (\frac{1}{3} - C)(\frac{r}{8} + (M_{Pl}\alpha)^2) + \frac{1}{8}(\frac{1}{3} - C)^2(M_{Pl}\alpha)^4\right). \tag{11}$$

Plugging this into Eq. (9) for the spectral index we get:

$$n_s - 1 = -\frac{r}{4} - (M_{Pl}\alpha)^2 + \frac{1}{6}(M_{Pl}\alpha)^4 - \frac{r^2}{64}(\frac{1}{3} - \frac{3}{2}C) + \frac{r}{8}(\frac{1}{3} + \frac{3}{2}C)(M_{Pl}\alpha)^2$$
$$+ \frac{r^2}{128}(\frac{10}{9} - C(\frac{13}{3} - 3C))(M_{Pl}\alpha)^2. \tag{12}$$

Using the recent value $n_s = 0.968 \pm 0.006$ from Planck [4][7,8] relation (12) provides an upper bound for the value of $M_{Pl}\alpha$:

$$M_{Pl}\alpha \lesssim 0.19 \quad \text{(obtained via } 2\sigma \text{ variations of } n_s) \,. \tag{13}$$

This will be used as orientation for further analysis and various predictions.

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[7] The central value of n_s, is larger, though the range (within 1σ) is consistent with Planck's old result $n_s = 0.9603 \pm 0.0073$ [5]. This required modification of our first version appeared before the new results.

[8] Recent joint analysis of BICEP2/Keck and Planck [13] gave an upper bound $r \lesssim 0.12$, while the likelihood curve for r has a maximum for $r \simeq 0.05$. Note that the value of r reported before by BICEP2 Collaboration [12] was $r = 0.2^{+0.07}_{-0.05}$ although, later on, Planck's intermediate results [14] warned about possible ordinary dust contribution instead of the light polarization effect really due to the gravitational waves.
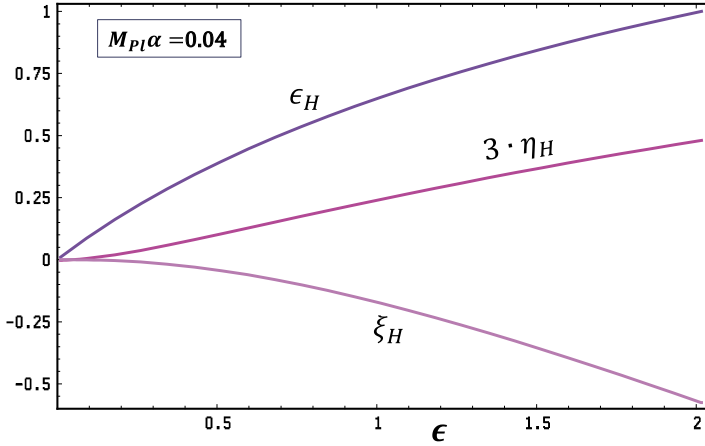
Fig. 1. Dependence of ϵ_H, $3\eta_H$ and ξ_H on the value of ϵ, for $M_{Pl}\alpha = 0.04$.

So far, we have performed calculations in a regime of small slow roll parameters, determining the value of ϕ_Θ^i via Eq. (11). As was mentioned, the value of ϕ_Θ^e is determined from the condition $\epsilon_H = 1$. Near this point both ϵ and η parameters turn out to be large and instead of an expansion we need to perform numerical calculations. This will be relevant upon the calculation of the number of e-foldings N_e^{inf}.

Since, within our model, via Eq. (2) VSR parameters are related to each other as

$$\eta = \epsilon - \frac{1}{2}(M_{Pl}\alpha)^2 , \qquad \xi = -2(M_{Pl}\alpha)^2\epsilon, \tag{14}$$

the three equation in (6) can be rewritten as

$$\epsilon_H\left(\frac{3 - \eta_H}{3 - \epsilon_H}\right)^2 = \epsilon$$

$$\frac{3(\epsilon_H + \eta_H) - \eta_H^2 - \xi_H}{3 - \epsilon_H} = \epsilon - \frac{1}{2}(M_{Pl}\alpha)^2$$

$$3\frac{3 - \eta_H}{(3 - \epsilon_H)^2}(3\epsilon_H\eta_H + \xi_H(1 - \eta_H)) = -2(M_{Pl}\alpha)^2\epsilon , \tag{15}$$

where σ_H has been dropped because of it's smallness. From the system of (15), for a fixed value of $M_{Pl}\alpha$, the parameters ϵ_H, η_H and ξ_H can be found in terms of the single parameter ϵ. The dependence of these parameters on the value of ϵ, for $M_{Pl}\alpha = 0.04$ are shown in Fig. 1 (for different values of $M_{Pl}\alpha$ shapes of the curves are similar). We see that $\epsilon_H = 1$ is achieved when $\epsilon = \epsilon_e \approx 2$ and thus, the expansion with respect to ϵ, η within this stage of inflation is invalid. On the other hand, the values of η_H and ξ_H remain relatively small. From the relation $2\epsilon = (M_{Pl}\alpha)^2 \tan^2 \frac{\alpha\phi_\Theta}{2}$ one derives:

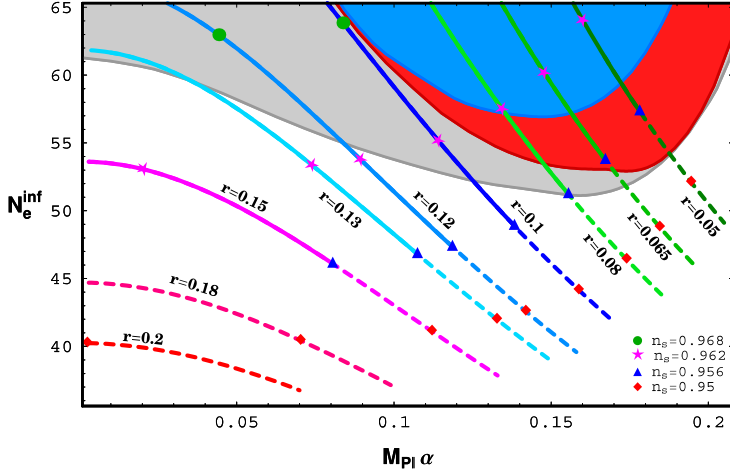$$d\phi_\Theta = \frac{M_{Pl}\sqrt{2}}{\sqrt{\epsilon}(2\epsilon + (M_{Pl}\alpha)^2)}d\epsilon. \tag{16}$$

Fig. 2. Number of e-foldings. Solid lines correspond to the values of n_s which fit with the current experimental data within 2σ error bars (with no restriction on r). Shaded areas correspond to the marginalized joint 68% CL regions, given recently in [4] for (n_s, r) pairs, mapped by us to the $(M_{Pl}\alpha, N_e^{\text{inf}})$ pairs for natural inflation. Gray background corresponds to the Planck TT + lowP, while red and blue colors represent Planck TT + LowP + BKP and Planck TT + LowP + BKP + BAO respectively (see Ref. [4] for an explanation of these combinations). (For interpretation of the references to color in this figure, the reader is referred to the web version of this article.)

Using this, the integral in (3) can be rewritten as

$$N_e^{\text{inf}} = \int_{\epsilon_e}^{\epsilon_i} \frac{1}{2\epsilon + (M_{Pl}\alpha)^2} \frac{d\epsilon}{\sqrt{\epsilon \epsilon_H}} \; . \tag{17}$$

Having the numerical dependence $\epsilon_H = \epsilon_H(\epsilon)$ (depicted in Fig. 1), we can evaluate the integral in (17) and find N_e^{inf} for various values of $M_{Pl}\alpha$. The results are given in Fig. 2. While BICEP2/Keck and Planck [13] reported the bound $r \lesssim 0.12$, upon generating the curves of Fig. 2 we also allowed larger values of r. Curves in Fig. 2 and also Table 1 demonstrate that, within natural inflation, with values $n_s \lesssim 0.962$ (the previous Planck 2013 value) and $r \gtrsim 0.1$ or $n_s \lesssim 0.953$ and $r \gtrsim 0.05$ there is an upper bound on N_e^{inf}:

$$N_e^{\text{inf}} \lesssim 55. \tag{18}$$

Turned around this also implies that violating the bound (18), say $N_e^{\text{inf}} \sim 60$, indicates larger n_s and/or smaller r. Present Planck 2015 data seems to favor this. Bound (18) (if realized) would lead to another striking prediction and constraint.

As discussed in Refs. [28,5], the N_e^{eff}, guaranteeing causality of fluctuations, should satisfy:

$$N_e^{\text{inf}} = 62 - \ln \frac{k}{a_0 H_0} - \ln \frac{10^{16}\text{GeV}}{\mathcal{V}_i^{1/4}} + \ln \frac{\mathcal{V}_i^{1/4}}{\mathcal{V}_e^{1/4}} - \frac{4 - 3\gamma}{3\gamma} \ln \frac{\mathcal{V}_e^{1/4}}{\rho_{reh}^{1/4}} \; , \tag{19}$$

where for the scale k we take $k = 0.002 \text{ Mpc}^{-1}$, while the present horizon scale is $a_0 H_0 \approx 0.00033 \text{ Mpc}^{-1}$. The factor γ accounts for the dynamics of the inflaton's oscillations [29,30] after inflation, and can be for our model approximated as $\gamma \simeq 1 - \frac{1}{16} \frac{\mathcal{V}_e}{\mathcal{V}_0}$ (will turn out to be a pretty good approximation).

Table 1

Numerical results for different values of r and $M_{Pl}\alpha$. For all cases $A_s^{1/2} = 4.686 \times 10^{-5}$.

r	$M_{Pl}\alpha$	n_s	$10^4 \times \frac{dn_s}{d\ln k}$	N_e^{inf}	$\frac{\mathcal{V}_0^{1/4}}{10^{16}\text{ GeV}}$	$\frac{\mathcal{V}_i^{1/4}}{10^{16}\text{ GeV}}$	$\frac{\mathcal{V}_e^{1/4}}{10^{16}\text{ GeV}}$	$\rho_{reh}^{1/4}$ (GeV)
0.15	0.001	0.962	−7.1	53.62	19.8	2.01	0.53	$3.14 \cdot 10^4$
	0.04	0.961	−7.71	51.47	3.2	2.01	0.54	53.7
	0.055	0.959	−8.25	49.73	2.77	2.01	0.54	0.31
	0.065	0.958	−8.71	48.39	2.58	2.01	0.55	0.006
0.125	0.04	0.967	−5.43	61.25	2.92	1.92	0.49	$2.77 \cdot 10^{14}$
	0.06	0.965	−6.06	57.92	2.45	1.92	0.5	$1.39 \cdot 10^{10}$
	0.08	0.962	−6.95	53.96	2.2	1.92	0.52	$1.14 \cdot 10^5$
	0.1	0.959	−8.09	49.79	2.04	1.92	0.54	0.5
	0.11	0.957	−8.76	47.72	2	1.92	0.55	0.001
0.1	0.097	0.966	−5.53	60	1.89	1.81	0.49	$9.2 \cdot 10^{12}$
	0.1	0.965	−5.68	59.14	1.87	1.81	0.5	$7.13 \cdot 10^{11}$
	0.12	0.961	−6.79	53.58	1.79	1.82	0.52	$5 \cdot 10^4$
	0.135	0.957	−7.77	49.7	1.74	1.82	0.54	0.52
	0.143	0.955	−8.33	47.76	1.72	1.82	0.55	0.0017
0.05	0.172	0.958	−4.53	59.5	1.35	1.53	0.47	$4.8 \cdot 10^{12}$
	0.19	0.952	−5.36	53.48	1.34	1.53	0.49	$8.7 \cdot 10^4$
	0.194	0.95	−5.55	50.93	1.34	1.53	0.49	44.8
	0.2	0.948	−5.86	50.51	1.33	1.53	0.5	13.6
	0.205	0.946	−6.12	49.12	1.33	1.53	0.5	0.225
	0.21	0.944	−6.39	47.78	1.33	1.53	0.51	0.0044

To reconcile the first two entries ($62 - \ln \frac{k}{a_0 H_0} \approx 60.2$) of Eq. (19) with the bound of Eq. (18) (see also Fig. 2), the remaining entries of Eq. (19) should be significant enough to bring N_e^{inf} down (at least) to ≈ 55. The 3rd and 4th entries on the r.h.s. of Eq. (19) can be calculated with help of another observable – the amplitude of curvature perturbation A_s, which according to the Planck measurements [4,5], should satisfy $A_s^{1/2} = 4.686 \times 10^{-5}$ (this value corresponds to the ΛCDM model). Generated by inflation, this parameter is given by:

$$A_s^{1/2} = \frac{1}{\sqrt{12}\pi} \left| \frac{\mathcal{V}^{3/2}}{M_{Pl}^3 \mathcal{V}'} \right|_{\phi_\Theta^i} \simeq \frac{4\sqrt{6}}{3\pi} \frac{\mathcal{V}_0^{1/2}}{M_{Pl}^2} \frac{M_{Pl}\alpha}{r(1 + 8(M_{Pl}\alpha)^2/r)^{1/2}} \ . \tag{20}$$

In order to obtain the observed value of $A_s^{1/2}$, for typical $r = 0.12$ and $M_{Pl}\alpha \sim 0.1$ we need to have $\mathcal{V}_0^{1/4} \sim 10^{-2} M_{Pl}$. This, on the other hand, gives $\mathcal{V}_i^{1/4} \sim 0.01 M_{Pl}$ and $\mathcal{V}_e^{1/4} \sim 2 \cdot 10^{-3} M_{Pl}$. Using these values in (19) we see that the sum of the 3rd and 4th terms is ≈ 3.4. Thus, the last term should be responsible for a proper reduction of N_e^{inf}. Namely, during the reheating process, the universe should expand by nearly 10 (or even more) e-foldings. This means that, for this case, the model should have a significant reheat history with $\rho_{reh}^{1/4} \sim 100$ GeV.[9] Within the scenario of natural inflation, this has not been appreciated before.[10] For lower r and appropriate

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[9] The reheating process can continue even till the epoch of nucleosynthesis. In this case one should have $\rho_{reh}^{1/4} \sim$ few $\times 10^{-3}$ GeV.

[10] See however some recent analysis in Ref. [21].

values of $M_{Pl}\alpha$ (and n_s) the reheating temperature can be big. The concise numerical results (compared to the rough evaluation below Eq. (20)) are given in Table 1, where we considered cases with $\rho_{reh}^{1/4}$ not smaller than 10^{-3} GeV, and $N_e^{\inf} \leq 62$. The values of the spectral index running $\frac{dn_s}{d\ln k} = 16\epsilon_i\eta_i - 24\epsilon_i^2 - 2\xi_i$ are also presented. The first three row-blocks correspond to the values of n_s within 2σ ranges of the current experimental data. The first three cases of the bottom block correspond to the n_s within 3σ range, while the last three lines of this block have lower values of n_s (beyond the 3σ deviation). Since the issue for the value of r is not fully settled yet, we have included moderately large values of $r(\leq 0.15)$. At the bottom block of the table we gave results for $r = 0.05$, which corresponds to the peak of the r's likelihood curve presented by the joint analysis of BICEP2/Keck and Planck [13]. Note that the results presented here are consistent with the analysis for natural inflation carried out before [8] (see 2nd citation of this reference).

Below we will show that within our scenario of natural inflation, a low reheat temperature is realized naturally.

Before closing this section, let us comment on the inflaton value during the course of inflation. Shifting the inflaton field ϕ_Θ around the vacuum $\phi_\Theta = \frac{\pi}{\alpha} + \hat{\phi}_\Theta$ [see Eq. (1)], in terms of $\mathcal{V}_0, \mathcal{V}$ one can calculate $\hat{\phi}_\Theta = \frac{2}{\alpha}\arcsin\sqrt{\frac{\mathcal{V}}{2\mathcal{V}_0}}$. With this and the values of $\mathcal{V}_0, \mathcal{V}_i, \mathcal{V}_e$ and α given in Table 1, we can see that during inflation $\hat{\phi}_\Theta \gg M_{Pl}$. For instance, for $\{r, M_{Pl}\alpha\} = \{0.15, 0.001\}$ fields are $\{\hat{\phi}_\Theta^i, \hat{\phi}_\Theta^e\} \approx \{455, 232\} \times M_{Pl}$, while for $\{r, M_{Pl}\alpha\} = \{0.1, 0.1\}$ we have $\{\hat{\phi}_\Theta^i, \hat{\phi}_\Theta^e\} \approx \{15, 7.5\} \times M_{Pl}$. Note, that here the role of the decay constant f (of PNGB inflation) plays $1/\alpha$ which, according to Table 1, is always greater than M_{Pl}, a well known problem of natural inflation as an effective field theory. However, as noted in [6], within a 5D construction the parameter f (i.e. $1/\alpha$ in our notations) can be seen as a derived quantity and expressed with 5D fundamental parameters. Also, the field $\hat{\phi}_\Theta$ can be expressed in terms of a 5D gauge field. As will be worked out in Section 3 and appendix Appendix A, within our 5D SUGRA construction $1/f = \alpha = \pi R g_4$. Large values of f corresponds to a weak coupling constant. Indeed, from Table 2 (given in the next section) we can see that successful inflation is realized with $g_4 \sim 10^{-3}$ and $g_4\hat{\phi}_\Theta = g_1\hat{A}_5^1$ [see Eq. (25)] is a subplanckian gauge field.

3. Natural inflation from 5D SUGRA

In order to address the details of inflaton decay, related to the reheat temperature, we need to specify the underlying theory natural inflation emerged from. A very good candidate is a higher-dimensional construction [6]. Here we present a 5D conformal SUGRA realization [1] of this idea, using the off-shell superfield formulation developed in Refs. [19,20].[11]

Lagrangian couplings, for the bulk $\mathbf{H} = (H, H^c)$ hypermultiplets', components are:

$$e_{(4)}^{-1}\mathcal{L}(H) = \int d^4\theta (T + T^\dagger)\left(H^\dagger H + H^{c\dagger}H^c\right)$$
$$+ \int d^2\theta \left(2H^c\partial_y H + g_1\Sigma_1(e^{i\hat{\theta}_1}H^2 - e^{-i\hat{\theta}_1}H^{c2})\right) + \text{h.c.} \quad (21)$$

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[11] For the component formalism of 5D conformal SUGRA see the pioneering work by Fujita, Kugo and Ohashi [18]. Note also, that the component off shell 5D SUGRA formulation, discussed by Zucker [22], was used in many phenomenologically oriented papers.

where the odd fields V_i are set to zero.[12] Σ_1 is the Z_2 even 5th component of the 5D $U(1)$ vector supermultiplet. With the parity assignments

$$Z_2: \quad H \to H, \qquad H^c \to -H^c, \tag{22}$$

the KK decomposition for H and H^c superfields is given by

$$H = \frac{1}{2\sqrt{\pi R}}H^{(0)} + \frac{1}{\sqrt{2\pi R}}\sum_{n=1}^{+\infty}H^{(n)}\cos\frac{ny}{R}, \qquad H^c = \frac{1}{\sqrt{2\pi R}}\sum_{n=1}^{+\infty}\overline{H}^{(n)}\sin\frac{ny}{R}. \tag{23}$$

With these decompositions, and steps given in Appendix A, we can calculate the mass spectrum of KK states, their couplings to the inflaton and with these, the one loop order inflation potential (dropping higher winding modes) having the form of (1) with

$$\alpha = \pi g_4 R, \qquad \mathcal{V}_0 = \frac{3}{16\pi^6 R^4}\mathcal{B} \quad \text{and} \quad \mathcal{B} = 1 - \cos(\pi R|F_T|). \tag{24}$$

The 4D inflaton field ϕ_Θ and the gauge coupling g_4 are related to the 5D $U(1)$ gauge field A_5^1 and the 5D gauge coupling g_1 as:

$$\phi_\Theta = \sqrt{2\pi R}A_5^1, \quad g_4 = \frac{g_1}{\sqrt{2\pi R}}. \tag{25}$$

Since the model is well defined, we also can write down the inflaton coupling with the components of H. The latter, having a coupling with the SM fields, would insure the inflaton decay and the reheating of the Universe. In our setup, we assume that all MSSM matter and scalar superfields are introduced at the $y = 0$ brane. Since H is even under orbifold parity and a singlet under all SM gauge symmetries, it can couple to the MSSM states through the following brane superpotential couplings

$$\mathcal{L}_{H\text{-}br} = \sqrt{2\pi R}\int d^2\theta\, dy\, \delta(y)\lambda l h_u H + \text{h.c.} \tag{26}$$

where l and h_u are 4D $N = 1$ SUSY superfields corresponding to lepton doublets and up type Higgs doublet superfields respectively. In Eq. (26), without loss of generality, only one lepton doublet (out of three lepton families) is taken to couple with the H,

$$\mathcal{L}_{H\text{-}br} \supset -\lambda\left((\frac{1}{\sqrt{2}}\psi_H^{(0)} + \sum_{n=1}^{+\infty}\psi_H^{(n)})(lh_u + \tilde{l}\tilde{h}_u) + (\frac{1}{\sqrt{2}}H^{(0)} + \sum_{n=1}^{+\infty}H^{(n)})l\tilde{h}_u \right.$$
$$\left. - (\frac{1}{\sqrt{2}}F_H^{(0)} + \sum_{n=1}^{+\infty}F_H^{(n)})\tilde{l}\tilde{h}_u\right) + \text{h.c.} \tag{27}$$

where l now denotes the fermionic lepton doublet and h_u an up-type Higgs doublet. States $\tilde{l}$ and $\tilde{h}_u$ stand for their superpartners respectively. $H^{(n)}$ and $\psi_H^{(n)}$ in Eq. (27) indicate scalar and fermionic components of the superfield H.[13]

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[12] The bulk hypermultiplet action of Eq. (21), derived from 5D off shell SUGRA construction [19], including coupling with a radion superfield T, in a rigid SUSY limit coincides with the one given in Ref. [32].

[13] In Eq. (27) we have omitted $HF_l h_u$ and $H\tilde{l}F_{h_u}$ type terms, which because of the smallness of the μ term ($\sim$ few $\times$ TeV) and suppressed lepton Yukawa couplings ($\lesssim 10^{-2}$) can be safely ignored in the inflaton decay process.
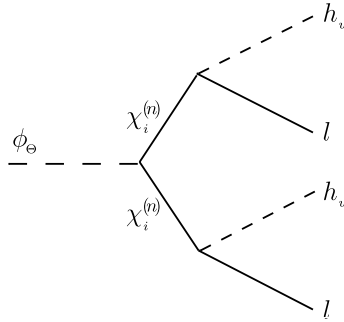
Fig. 3. Diagram responsible for the inflaton's dominant decay.

Upon eliminating all F-terms and heavy fermionic and scalar states (in the H and H^c superfields), we can derive effective operators containing the inflaton linearly. As it will turn out within the model considered (see discussion in A.1), the $\tilde{l}$ states are heavier than the inflaton and operators containing $\tilde{l}$ are irrelevant for the inflaton decay. Thus, the effective operators, needed to be considered, are

$$\phi_\Theta \left(C_0 (lh_u)^2 + C_1 (l\tilde{h}_u)^2 + \text{h.c.} \right) + C_2 \phi_\Theta (l\tilde{h}_u)(\bar{l}\bar{\tilde{h}}_u). \tag{28}$$

These terms should be responsible for the inflaton decay. Derivation and form of the C-coefficients are given in Appendix A.

3.1. Inflaton decay and reheating

As was mentioned above and shown in A.1, the slepton states $\tilde{l}$ have masses $\frac{1}{2}|F_T| \sim 1/(2R)$ and thus are heavier than the inflaton. Indeed, the latter's mass, obtained from the potential, is:

$$M_{\phi_\Theta} = \frac{g_4 \sqrt{3}(1 - \cos(\pi R|F_T|))^{1/2}}{4\pi^2 R} \ll \frac{1}{R} . \tag{29}$$

($g_4 \ll 1$ for successful inflation). Thus the inflaton decay in channels containing $\tilde{l}$ is kinematically forbidden. Anticipating, we note that the preheating process by inflaton decay in heavy states is excluded within our scenario with parameters we consider (this is shown at the end of this subsection). Thus, the reheating proceeds by perturbative 4-body decay of the inflaton.

Among operators generated via exchange of heavy fermionic $\chi_i^{(n)}$ and scalar $S_i^{(n)}$ states, only those given in Eq. (28) are relevant. For calculating the decay widths (in a pretty good approximation) it is enough to have the form of the C_i coefficients.

As shown in Appendix A, within our model $C_2 = 0$ and the corresponding operator does not play any role. Moreover, according to Eqs. (A.26) and (A.30) we have $C_0 \sim R^2$ (due to a $1/m_{\psi_H}^2$) and $C_1 \sim R^3$ (with $|F_T| \sim 1/R$, dictated from the inflation). Thus, we get an estimate for the following branching ratio

$$\frac{\Gamma(\phi_\Theta \to ll\tilde{h}_u\tilde{h}_u)}{\Gamma(\phi_\Theta \to llh_uh_u)} \sim \frac{|C_1|^2 M_{\phi_\Theta}^7}{|C_0|^2 M_{\phi_\Theta}^5} \sim (RM_{\phi_\Theta})^2 \sim \frac{3g_4^2}{16\pi^4} \ll 1 . \tag{30}$$

This means that the inflaton decay is mainly due to the C_0 operator [see Eqs. (28) and (A.26), with gauge coupling g_4 and Yukawa coupling λ], i.e. in the channel $\phi_\Theta \to llh_uh_u$ (the diagram

in Fig. 3. Remember: l denotes the SM lepton doublet and h_u the scalar up type Higgs doublet). For simplicity we assume that the state h_u includes the light SM Higgs doublet h with weight nearly equal to one, i.e. $h_u \supseteq h$.

For the decay width we get[14]:

$$\Gamma(\phi_\Theta) \simeq \Gamma(\phi_\Theta \to l l h_u h_u) = \frac{9}{9 \cdot 2^7 (2\pi)^5} |C_0|^2 M_{\phi_\Theta}^5 . \tag{31}$$

The factor 9 in the numerator accounts for the multiplicity of final states. (The final $l l h_u h_u$ channel includes three combinations $e^- e^- h^+ h^+$, $\nu \nu h^0 h^0$, $e^- \nu h^+ h^0$ and for each pair of identical final states a factor 2 should be included.) The denominator factors in (31) come from the phase space integration. Using the form of C_0, given by Eq. (A.26), in expression (31), we get:

$$\Gamma(\phi_\Theta) \simeq \frac{g_4^2 |\lambda|^4}{2^{18} \pi} (R M_{\phi_\Theta})^4 M_{\phi_\Theta} . \tag{32}$$

Expressing $\rho_{reh} = \frac{\pi^2}{30} g_* T_r^4$ through the reheat temperature [33]

$$T_r = \left(\frac{90}{\pi^2 g_*} \right)^{1/4} \sqrt{M_{Pl} \Gamma(\phi_\Theta)} \tag{33}$$

(g_* is the number of relativistic degrees at temperature T_r) and using expressions (32) and (29), we get

$$\rho_{reh}^{1/4} = 1.316 \, (M_{Pl} \Gamma(\phi_\Theta))^{1/2} = 5.85 \cdot 10^{-7} M_{Pl} \frac{g_4^{7/2} |\lambda|^2}{(R M_{Pl})^{1/2}} (1 - \cos(\pi R |F_T|))^{5/4} . \tag{34}$$

From this, with $R M_{Pl} \sim 10$, $R |F_T| \sim 1$ and $g_4 \sim 1.5 \cdot 10^{-3}$ we obtain $\rho_{reh}^{1/4} \sim |\lambda|^2 \times 100$ GeV.

Our 5D SUGRA construction allows more accurate estimates, because some of the parameters are related to each other. For instance, from (24) we have

$$R \simeq \frac{0.118}{\mathcal{V}_0^{1/4}} (1 - \cos(\pi R |F_T|))^{1/4} , \tag{35}$$

$$g_4 = \frac{\alpha}{\pi R} . \tag{36}$$

From (35) we see that in order to have $R M_{Pl} \gtrsim 10$ we need $\mathcal{V}_0^{1/4} \lesssim 3.4 \cdot 10^{16}$ GeV.[15] The latter value suites well with most of the values of $\mathcal{V}_0^{1/4}$ given in Table 1 (calculated from the inflation potential). At the same time, we see from (35) that $|F_T|$ cannot be suppressed and should be $|F_T| \sim 1/R$. Using Eqs. (35) and (36) in (34), we obtain

$$\rho_{reh}^{1/4} = 5.45 \cdot 10^{-5} M_{Pl} (\alpha M_{Pl})^{7/2} \left(\frac{\mathcal{V}_0^{1/4}}{M_{Pl}} \right)^4 |\lambda|^2 (1 - \cos(\pi R |F_T|))^{1/4}. \tag{37}$$

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[14] For 4-body phase space we have used an expression of [31] derived for the $K \to \pi \pi e e$ decay, setting $m_\pi, m_e \to 0$ and replacing $m_K \to M_{\phi_\Theta}$.

[15] For adequate suppression of undesirable non-local operators the large volume $R \gtrsim 10/M_{Pl}$ is needed [6]. Note that the 5D Planck mass is $M_5 = M_{Pl}/(2\pi R M_{Pl})^{1/3} \simeq (0.18 - 0.25) M_{Pl}$.

Table 2
Values of RM_{Pl}, g_4 and $\rho_{reh}^{1/4}$ for different cases of successful inflation.

(r, n_s)	$M_{Pl}\alpha$	$\dfrac{\mathcal{V}_0^{1/4}}{10^{16}\,\text{GeV}}$	$R\lvert F_T\rvert$	RM_{Pl}	g_4	$\dfrac{\rho_{reh}^{1/4}}{\lvert\lambda\rvert^2}$ (GeV)
$(0.15, 0.961)$	0.04	3.2	0.75	10.1	$1.26 \cdot 10^{-3}$	60.5
			0.9	10.5	$1.22 \cdot 10^{-3}$	62.6
$(0.15, 0.959)$	0.055	2.77	0.5	10.2	$1.71 \cdot 10^{-3}$	90.6
			0.9	12.1	$1.45 \cdot 10^{-3}$	107
$(0.15, 0.958)$	0.065	2.58	0.5	11	$1.88 \cdot 10^{-3}$	122
			0.9	13	$1.6 \cdot 10^{-3}$	145
$(0.125, 0.959)$	0.1	2.04	0.5	13.9	$2.29 \cdot 10^{-3}$	216
			0.9	16.4	$1.94 \cdot 10^{-3}$	255
$(0.1, 0.957)$	0.135	1.74	0.5	16.3	$2.64 \cdot 10^{-3}$	327
			0.9	19.3	$2.23 \cdot 10^{-3}$	386
$(0.05, 0.95)$	0.194	1.34	0.5	21	$2.91 \cdot 10^{-3}$	409
			0.9	25	$2.45 \cdot 10^{-3}$	483
$(0.05, 0.948)$	0.2	1.33	0.5	21.3	$3 \cdot 10^{-3}$	442
			0.9	25.2	$2.53 \cdot 10^{-3}$	522
$(0.05, 0.944)$	0.21	1.33	0.2	14.1	$4.74 \cdot 10^{-3}$	346
			0.9	25.2	$2.65 \cdot 10^{-3}$	619

This expression is useful to find the maximal value of $\rho_{reh}^{1/4}$. Using the pairs of $(\alpha M_{Pl}, \mathcal{V}_0)$ given in Table 1, from Eq. (37) it turns out that $\rho_{reh}^{1/4} \lesssim \lvert\lambda\rvert^2 \times 619$ GeV. This is an upper bound on the reheating energy density obtained within our 5D SUGRA scenario. In Table 2 we give the values of RM_{Pl}, g_4 and $\rho_{reh}^{1/4}$ for various cases. Input values of $\mathcal{V}_0^{1/4}$ and $M_{Pl}\alpha$ were taken from Table 1, which correspond to successful inflation. Also, we have selected the values of $R\lvert F_T\rvert$ in such a way as to get $RM_{Pl} \gtrsim 10$. We see that within 2σ deviations of n_s we have $\rho_{reh}^{1/4} \lesssim \lvert\lambda\rvert^2 \times 386$ GeV, corresponding to reheat temperatures $T_r \lesssim \lvert\lambda\rvert^2 \times 130$ GeV. These values can be easily reconciled with those low values of $\rho_{reh}^{1/4}$, given in Table 1, by natural selection of the brane Yukawa coupling λ in a range $1/300 \lesssim \lambda \lesssim 1$.

3.2. Excluding preheating

Since in the presented 5D SUGRA scenario the inflaton has direct couplings with heavy KK states of H and H^c superfields, we need to make sure that after inflation, during the inflaton oscillation there is no production of these heavy states and no reheat is anticipated by the preheating process. Below we show that indeed, within our model preheating does not take place.

Starting from the fermionic KK states (which turned out to dominate in reheating), their masses are given by Eq. (A.16). with $\langle M^1\rangle = 0$ and shift of the inflaton around the vacuum

$$g_1 A_5^1 = g_1 \langle A_5^1\rangle + g_1 \hat{A}_5^1 = \frac{1}{R} + g_4 \hat{\phi}_\Theta \tag{38}$$

for fermion masses we get

$$m_{\chi_1}^{(n)} = \frac{1}{2}\left\lvert \frac{2n+1}{R} + g_4\hat{\phi}_\Theta\right\rvert, \qquad m_{\chi_2}^{(n)} = \frac{1}{2}\left\lvert \frac{2n-1}{R} - g_4\hat{\phi}_\Theta\right\rvert. \tag{39}$$

The $\hat{\phi}_\Theta$ is the quantum part oscillating around the potential's minimum (after the end of inflation) and finally relaxing to $\hat{\phi}_\Theta = 0$. Our aim is to see if either of the masses in (39) become zero during inflaton damped oscillation. As was shown in Ref. [34], this is the criterion for the fermionic preheating.

The amplitude of $\hat{\phi}_\Theta$ has a well defined value at the end of the inflation when slow roll breaks down, i.e. at the point $\epsilon = \epsilon_f \simeq 2$. With $\epsilon = \frac{1}{2}(M_{Pl}\alpha)^2 \cot^2(\pi R g_4 \hat{\phi}_\Theta/2)$, for times $t > t_f$ we have

$$|g_4 \hat{\phi}_\Theta^f| = \frac{1}{R}\frac{2}{\pi}\text{ArcTan}\left(\frac{M_{Pl}\alpha}{\sqrt{2\epsilon_f}}\right) \simeq \frac{1}{R}\frac{M_{Pl}\alpha}{\pi} \ . \tag{40}$$

On the other hand, from our Table 2 we have $M_{Pl}\alpha \lesssim 0.2$. Using this in (40), we get

$$|g_4 \hat{\phi}_\Theta^f| \lesssim \frac{0.064}{R}. \tag{41}$$

The kinetic energy of the oscillation is still at most comparable at the end of inflation and there is also damping. Thus, Eq. (41) is a good estimate for the maximal amplitude of $\hat{\phi}_\Theta$. With this bound, we can see that the term $g_4 \hat{\phi}_\Theta$ in Eq. (39) will not be able to nullify fermion masses during inflaton oscillations. This fact, as was shown in Ref. [34], prevents KK fermion production and no fermionic preheating takes place.

Now we turn to the scalar KK states. With Eqs. (A.10), (38) and $\langle M^1 \rangle = 0$ for the scalar masses we get

$$(m_{1,2}^{(n)})^2 = \frac{1}{4}\left(\frac{2n+1}{R} \pm |F_T| + g_4 \hat{\phi}_\Theta\right)^2,$$

$$(m_{3,4}^{(n)})^2 = \frac{1}{4}\left(\frac{2n-1}{R} \mp |F_T| - g_4 \hat{\phi}_\Theta\right)^2 \ . \tag{42}$$

Therefore, with Eq. (41) and the values of $|F_T|$ given in Table 2, we see that masses in (42) never cross zero and for the positively defined mass^2 of the S_i states we have

$$(m_i^{(n)})^2 \gtrsim \left(\frac{0.036}{R}\right)^2 \gg 10^4 \times (M_{\phi_\Theta})^2 \ , \tag{43}$$

where the inflaton mass $M_{\phi_\Theta} \sim 10^{13}$ GeV. Therefore, all modes from the scalar KK tower are much heavier (by a factor > 100) than the inflaton mass for any time during the inflaton's oscillation. As was shown in Refs. [33,35] for these conditions the amplification and/or production of the scalar modes never happens. This excludes preheating also via the scalar production. Within our scenario this result is insured by the gauge symmetry, because the inflaton in the heavy KK states' masses contributes in the combination $g_4 \hat{\phi}_\Theta$ [see e.g. Eqs. (39) and (42)].

Thus, we finally conclude that within our scenario reheating occurs by the perturbative inflaton four-body decays discussed at the beginning of Section 3.1

4. Discussion and concluding remarks

In the effective action of our 5D conformal SUGRA model the 5-th component (Σ_1) of a $U(1)$ vector supermultiplet couples to a charged hypermultiplet (H, H^c). This, due to a fixed

compactification radius R leads to the potential of natural inflation for the CP odd part of Σ_1, neglecting the suppressed higher winding modes. We analyzed this potential like in [8] putting emphasis on the potential of inflation and the number of e-folds of perturbations leaving the horizon. This we compared with the number of e-folds required by a causal connection between the observed universe background fluctuations and by the size of observed curvature perturbations. For a large tensor component $r \gtrsim 0.1$ and $n_s \lesssim 0.962$ or $r \gtrsim 0.05$ and $n_s \lesssim 0.953$ close to the lower bound of present Planck 2015 data a small $N_e^{\text{inf}} \lesssim 55$ results. This requires a small reheating temperature. We inspected the decay of the gauge inflaton to the light MSSM fields living on a brane. These decays are mediated by the bulk hypermultiplet H. The very same H hypermultiplet, together with it's $SU(2)_R$ partner H^c, generates the inflation potential. The H is assumed to have superpotential Yukawa couplings to brane fields with a Yukawa strength $\lambda \lesssim 1$. Due to a very small gauge coupling and to a relatively light inflaton this led to a suppressed decay width and reheat temperature $T_r \sim |\lambda|^2 \times 100$ GeV. Within the considered scenario the dominant 4-body decays of the inflaton are mediated by fermionic components ψ_H (of H) with $llhh$ final states (two lepton and two Higgs doublets' components). Other channels are either kinematically forbidden due to heavy sleptons $\tilde{l}$ gaining large masses through the large F_T term, a case of split SUSY, or are suppressed (due to the small inflaton mass $M_{\phi_\Theta} \ll 1/R$) by an additional small factor $(RM_{\phi_\Theta})^2$. Therefore, a similar mechanism can be realized also for extranatural inflation [6] without supersymmetry with a bulk fermionic ψ_H generating the inflation potential and brane Yukawa coupling $\lambda l h \psi_H$. Within our model (as shown in Appendix A), due to specific bulk couplings and degeneracy, the lepton number is conserved and neutrinos stay massless (and this remains true for extranatural inflation). The situation can be changed by introducing a brane Majorana mass term $\frac{1}{2} M_{br} H H$ and it is inviting to exploit such a possibility. Since this is not directly related to inflation, on one side, and trying to keep the calculus simple on the other side, we have not pursued this possibility in this paper and reserved it for future studies.

The model of [1], reanalyzed here in more detail, is by no means complete. A concrete mechanism for radion stabilization like in Ref. [24] has to be presented beyond our remarks in appendix Appendix A and the breaking of 4D SUSY has to be worked out in more detail. Here and in [1] we concentrated on the aspects that our model originates in a very straightforward way from 5D conformal SUGRA – which can be also interpreted as a result of M-theory [20] – and that the inflaton is related to a gauge field. If the new BICEP2 data, advertising large tensorial fluctuations, will turn out not to be mainly dust effects and if in the future, the value of r turns out to be unsuppressed (i.e. $r \gtrsim 0.05$ or so) and if n_s will not remain very close to the presently favored higher values, then some form of natural inflation derived from 5D gauge inflation would be indeed a suitable and attractive candidate for inflationary model building.[16] Also baryogenesis at low reheating temperature would be an interesting subject again.[17] If further findings will indicate a really small value of r, then as an alternative, the 'modulus' inflation of [1] should be pursued. This would mean that the inflaton is the real part of the Σ_1 chiral supermultiplet scalar component. Also, a more general two field inflation [10] from complex Σ_1 could get into focus again.

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[16] See however footnote 6 mentioning the 'weak gravity conjecture' and Ref. [26] proposing a 5D gauge field model leading to natural inflation without need of a tiny gauge coupling.

[17] Electroweak and Affleck–Dine baryogenesis at low temperatures were e.g. discussed some time ago in [36]. Leptogenesis at $\sim$ TeV scales were considered e.g. in Refs. [37].

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Appendix A. KK spectrum and the inflaton effective couplings

First let us discuss the emergence of the non-zero F_T term of the T radion superfield. This can be easily understood by the effective 4D SUGRA description developed in [20]. The 4D supergravity action is given by [40]

$$\mathcal{L}_D^{(4D)} + \mathcal{L}_F^{(4D)} \quad \text{with}$$

$$\mathcal{L}_D^{(4D)} = -3 \int d^4\theta \, e^{-\mathcal{K}/3} \phi^+ \phi,$$

$$\mathcal{L}_F^{(4D)} = \int d^2\theta \phi^3 \, W + \frac{1}{4} \int d^2\theta f_{IJ} \mathcal{W}^{\alpha I} \mathcal{W}_\alpha^J + \text{h.c.} \tag{A.1}$$

where $\mathcal{K}$ and $W(\Phi)$ are the Kähler potential and the superpotential respectively, while $f_{IJ}(\Phi)$ is the gauge kinetic function. ϕ is the 4D compensator chiral superfield. Being a 4D effective theory, (A.1) would include zero modes of the 5D supermultiplets and the brane fields as well. Therefore, for the bulk states the form of (A.1) will be dictated by the 5D construction [20]. For instance, the 4D compensator ϕ is related with the 5D compensator as $\phi = \sqrt{2\pi R} \kappa_5^{-\frac{1}{3}} h^{\frac{2}{3}}$. The Kähler potential, corresponding to the zero modes of the hypermultiplets of Eq. (21), is given by [20]:

$$\mathcal{K}_H = -2\ln\left(1 - \frac{H^\dagger H}{h^\dagger h}\right).$$

For our computing purposes, it is more convenient to work with hypermultiplet couplings in the form of Eq. (21) (which includes all modes – even and odd under Z_2 parity). Because of this and the fact that Eq. (21) includes mixed $H^c \partial_y H$ term, we will not attempt to write here 4D superpotential for the bulk hypermultiplets.

From (A.1) we find the expressions for the F-terms:

$$F^I = -M_P e^{\mathcal{K}/3} \mathcal{K}^{I\bar{J}} D_{\bar{J}} \bar{W}, \qquad F_\phi = M_P^2 e^{\mathcal{K}/3} \left(\bar{W} - \frac{1}{3} \mathcal{K}^{I\bar{J}} \mathcal{K}_I D_{\bar{J}} \bar{W}\right), \tag{A.2}$$

where I runs over all scalars. By plugging Eq. (A.2) back in to (A.1), one derives the F-term scalar potential (by setting $\phi = M_P$ and going to the 4D Einstein-frame, rescaling the metric $g_{\mu\nu} \to e^{\mathcal{K}/3} g_{\mu\nu}$):

$$V_F = M_P^4 \, e^{\mathcal{K}} \left(\mathcal{K}^{I\bar{J}} D_I W D_{\bar{J}} \bar{W} - 3|W|^2\right), \quad \text{with } D_I \equiv \partial_I + \mathcal{K}_I. \tag{A.3}$$

[Here, we consider the dimensionless Kähler and superpotentials. With rescalings $\mathcal{K} \to \mathcal{K}/M_{Pl}^2$, $W \to W/M_{Pl}^3$, the potential will get the form $V_F \to e^{\mathcal{K}/M_{Pl}^2}\left(\mathcal{K}^{I\bar{J}} D_I W D_{\bar{J}} \bar{W} - 3\frac{|W|^2}{M_{Pl}^2}\right)$.] For the T modulus (the radion) the Kähler potential is $\mathcal{K} = -3\ln(T + T^\dagger)$. For the

time being we take $W = $ const. for the superpotential.[18] With these, it is easy to check that we get a flat potential $V_F = 0$ with $F_\phi = 0$ and $F_T = M_P W^*$. Thus, we have fixed a non-zero F_T which plays a crucial role for the generation of the inflaton potential. This is enough for performing a calculation of the KK spectrum and the 1-loop inflaton potential. We will come back to the SUSY breaking at the end, upon discussion of the superpartners' spectrum from the MSSM brane fields. As worked out in our paper [24] [Eq. (25)] an additional non-perturbative Kähler and superpotential [Eq. (29) of Ref. [24]] due to gaugino condensation $w_{np} = \tilde{w} e^{-\frac{3}{2c}T}$ leads to fixation of the scalar radion component at some R. Uplifting the potential (see Chapter 4 of Ref. [24]) is than a further step.

Any bulk state transforming non-trivially under $SU(2)_R$ feels F_T SUSY breaking. This happens of course with the bulk hypers described by the terms in (21). With the parametrization

$$F_T = -|F_T|e^{i\alpha}, \quad \text{with } \alpha = \text{Arg}(F_T) + \pi, \tag{A.4}$$

setting the scalar component of T to one, and making a phase redefinition of the scalar components H, H^c:

$$H \to e^{-i(\hat{\theta}_1 + \alpha)/2} H, \qquad H^c \to e^{i(\hat{\theta}_1 - \alpha)/2} H^c, \tag{A.5}$$

the couplings in (21) give the potential:

$$V(H) = 2 \left| \partial_5 H - \frac{g_1}{2}(M^1 - \frac{i\Theta}{2\pi R})H^c - \frac{1}{2}|F_T|H^{c*} \right|^2$$
$$+ 2 \left| \partial_5 H^c - \frac{g_1}{2}(M^1 - \frac{i\Theta}{2\pi R})H + \frac{1}{2}|F_T|H^* \right|^2$$
$$+ \frac{1}{2}g_1 M^1 |F_T|(H^2 - H^{c2}) + \frac{1}{2}g_1 M^1 |F_T|(H^{*2} - H^{c*2}). \tag{A.6}$$

With the decomposition of Eq. (23) and integrating along the fifth dimension $\int_0^{2\pi R} dy \mathcal{L}^{(5)}$ we obtain

$$V(H^{(n)}) = \left| g_1 \Sigma_1 H^{(0)} - \frac{1}{2}|F_T|H^{(0)*} \right|^2 + \sum_{n=1}^{+\infty} \left| \frac{n}{R} H^{(n)} + g_1 \Sigma_1 \overline{H}^{(n)} + \frac{1}{2}|F_T|\overline{H}^{(n)*} \right|^2$$
$$+ \sum_{n=1}^{+\infty} \left| \frac{n}{R} \overline{H}^{(n)} - g_1 \Sigma_1 H^{(n)} + \frac{1}{2}|F_T|H^{(n)*} \right|^2$$
$$+ \frac{1}{4}g_1 M^1 |F_T| \left(\sum_{n=0}^{+\infty}(H^{(n)})^2 - \sum_{n=1}^{+\infty}(\overline{H}^{(n)})^2 + \text{h.c.} \right) \tag{A.7}$$

$$H^{(0)} = \frac{1}{\sqrt{2}}(S_1^{(0)} + i S_2^{(0)}) \quad \text{for } n = 1, \ldots, +\infty,$$

---

[18] As shown in [1], this system (at zero mode level and for the purpose of discussing SUSY breaking) is equivalent to 5D SUGRA with gauged $U(1)_R$ and with suitable couplings of a linear supermultiplet.

$$\begin{pmatrix} \mathrm{Re}H^{(n)} \\ \mathrm{Im}H^{(n)} \\ \mathrm{Re}\overline{H}^{(n)} \\ \mathrm{Im}\overline{H}^{(n)} \end{pmatrix} = \frac{1}{2\sqrt{2}} \begin{pmatrix} 1 & 1 & -1 & -1 \\ -1 & 1 & 1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} S_1^{(n)} \\ S_2^{(n)} \\ S_3^{(n)} \\ S_4^{(n)} \end{pmatrix}, \tag{A.8}$$

(where $S_i^{(n)}$ are real scalars) and the potential mass terms will get diagonal and canonical forms:

$$V(S^{(n)}) = \frac{1}{2}(m_1^{(0)})^2(S_1^{(0)})^2 + \frac{1}{2}(m_2^{(0)})^2(S_2^{(0)})^2 + \frac{1}{2}\sum_{i=1}^{4}\sum_{n=1}^{+\infty}(m_i^{(n)})^2(S_i^{(n)})^2, \tag{A.9}$$

with

$$(m_1^{(0)})^2 = \frac{1}{4}(g_1 A_5^1 + |F_T|)^2 + \frac{1}{4}(g_1 M^1)^2, \qquad (m_2^{(0)})^2 = \frac{1}{4}(g_1 A_5^1 - |F_T|)^2 + \frac{1}{4}(g_1 M^1)^2,$$

$$\text{for } n = 1, \dots, +\infty: \quad (m_{1,2}^{(n)})^2 = \frac{1}{4}\left(\frac{2n}{R} + g_1 A_5^1 \pm |F_T|\right)^2 + \frac{1}{4}(g_1 M^1)^2,$$

$$(m_{3,4}^{(n)})^2 = \frac{1}{4}\left(\frac{2n}{R} - g_1 A_5^1 \mp |F_T|\right)^2 + \frac{1}{4}(g_1 M^1)^2. \tag{A.10}$$

As for the spectrum of the fermionic components of the H, H^c superfields, with the phase redefinition

$$\psi_H \to e^{-i\hat{\theta}_1/2}\psi_H, \qquad \psi_{H^c} \to e^{i\hat{\theta}_1/2}\psi_{H^c}, \tag{A.11}$$

from Eq. (21) we get the couplings

$$\mathcal{L}_\psi^{(5)} \supset -2\psi_{H^c}\partial_y\psi_H - \frac{1}{2}g_1(M^1 - iA_5^1)(\psi_H\psi_H - \psi_{H^c}\psi_{H^c}) + \text{h.c.} \tag{A.12}$$

With the mode expansion of Eq. (23) and integration over the fifth dimension $\int_0^{2\pi R} dy \mathcal{L}_\psi^{(5)} = \mathcal{L}_\psi^{(4)}$, from (A.12) we get terms

$$\mathcal{L}_\psi^{(4)} \supset -\sum_{n=1}^{+\infty}\frac{n}{R}\psi_{\overline{H}}^{(n)}\psi_H^{(n)} - \frac{1}{4}g_1(M^1 - iA_5^1)\left(\sum_{n=0}^{+\infty}\psi_H^{(n)}\psi_H^{(n)} - \sum_{n=1}^{+\infty}\psi_{\overline{H}}^{(n)}\psi_{\overline{H}}^{(n)}\right) + \text{h.c.} \tag{A.13}$$

Now, with the substitution

$$\psi_H^{(0)} = e^{i\omega_0}\chi_1^{(0)}, \quad \text{with } \omega_0 = -\frac{1}{2}\mathrm{Arg}(M^1 - iA_5^1),$$

$$\text{for } n = 1, \dots, +\infty: \quad \psi_H^{(n)} = \frac{1}{\sqrt{2}}\left(e^{i\omega_n}\chi_1^{(n)} + e^{i\bar{\omega}_n}\chi_2^{(n)}\right),$$

$$\psi_{\overline{H}}^{(n)} = \frac{i}{\sqrt{2}}\left(-e^{i\omega_n}\chi_1^{(n)} + e^{i\bar{\omega}_n}\chi_2^{(n)}\right),$$

$$\text{with } \omega_n = -\frac{1}{2}\mathrm{Arg}\left(g_1 M^1 - i(g_1 A_5^1 + \frac{2n}{R})\right),$$

$$\bar{\omega}_n = -\frac{1}{2}\mathrm{Arg}\left(g_1 M^1 - i(g_1 A_5^1 - \frac{2n}{R})\right), \tag{A.14}$$

from Eq. (A.13) we will get diagonal and canonically normalized mass terms:

$$\mathcal{L}_\psi^{(4)} \supset -\frac{1}{2} \sum_{n=0}^{+\infty} m_{\chi_1}^{(n)} \chi_1^{(n)} \chi_1^{(n)} - \frac{1}{2} \sum_{n=1}^{+\infty} m_{\chi_2}^{(n)} \chi_2^{(n)} \chi_2^{(n)} + \text{h.c.} \tag{A.15}$$

with

$$m_{\chi_1}^{(n)} = \frac{1}{2} \left| g_1 M^1 - i(g_1 A_5^1 + \frac{2n}{R}) \right|, \qquad m_{\chi_2}^{(n)} = \frac{1}{2} \left| g_1 M^1 - i(g_1 A_5^1 - \frac{2n}{R}) \right|. \tag{A.16}$$

With this spectrum, integrating out the corresponding KK states (including zero modes) leads to the 1-loop effective potential [1,9]:

$$\mathcal{V}^{\text{eff}}(\phi_\Theta) = \frac{3}{16\pi^6 R^4} \sum_{k=1}^{\infty} \frac{1}{k^5} (1 - \cos(\pi k R |F_T|)) \cdot \cos(\pi k g_4 R \phi_\Theta)$$

$$\times e^{-\pi k g_4 R |\phi_M|} \left(1 + \pi k g_4 R |\phi_M| + \frac{1}{3}(\pi k g_4 R |\phi_M|)^2 \right), \tag{A.17}$$

written in terms of canonically normalized 4D scalar fields $\phi_\Theta = \sqrt{2\pi R} A_5^1$, $\phi_M = \sqrt{2\pi R} M^1$ and dimensionless 4D gauge coupling $g_4 = g_1/\sqrt{2\pi R}$. In (A.17) summation is performed with k winding modes. The dominant contribution comes from $k = 1$ [38]. With this leading term, the minimum of the potential is achieved for $g_4 R \langle \phi_\Theta \rangle = g_1 R \langle A_5^1 \rangle = 1$ and $\langle \phi_M \rangle = 0$. Further, we assume that the modulus ϕ_M (i.e. M^1) is sitting in its minimum and study only the motion of ϕ_Θ's quantum part as the inflaton. We add to the potential (A.17) a constant term in such a way as to set the ground state vacuum energy to be zero (usual fine tuning of 4D cosmological constant). With these, the inflaton potential (part with $k = 1$) gets the form of Eq. (1) with the parametrization given in Eq. (24).

Further, we work out the effective couplings of the inflaton with the MSSM states. For this purpose, in couplings (A.9), (A.16) (and in any relevant term) we make the substitution

$$g_1 A_5^1 \to g_1 \langle A_5^1 \rangle + g_1 A_5^1 = \frac{1}{R} + g_4 \phi_\Theta \tag{A.18}$$

and put $\langle M^1 \rangle = 0$. With this, we obtain the linear couplings of the inflaton with the heavy S_i states:

$$\mathcal{L}_{\phi_\Theta SS} = -\frac{1}{2} g_4 \phi_\Theta \left(\sum_{n=0}^{+\infty} [m_1^{(n)}(S_1^{(n)})^2 + m_2^{(n)}(S_2^{(n)})^2] - \sum_{n=1}^{+\infty} [m_3^{(n)}(S_3^{(n)})^2 + m_4^{(n)}(S_4^{(n)})^2] \right),$$

$$\text{with} \quad m_{1,2}^{(n)} = \frac{1}{2} \left(\frac{2n+1}{R} \pm |F_T| \right), \quad m_{3,4}^{(n)} = \frac{1}{2} \left(\frac{2n-1}{R} \mp |F_T| \right). \tag{A.19}$$

At the same time, with (A.18) from (A.14) we have $\omega_n = -\overline{\omega}_n = \pi/4$, and Eq. (A.15) gives inflaton couplings with heavy χ_i states:

$$\mathcal{L}_{\phi_\Theta \chi\chi} = \frac{1}{4} g_4 \phi_\Theta \left(\sum_{n=0}^{+\infty} \chi_1^{(n)} \chi_1^{(n)} - \sum_{n=1}^{+\infty} \chi_2^{(n)} \chi_2^{(n)} \right) + \text{h.c.} \tag{A.20}$$

Furthermore, we derive couplings of S_i and χ_i states with the corresponding components of the brane superfields l, h_u. As shown in A.1, the $\tilde{l}$ states are heavy. Because of this, they will not

be relevant for the inflaton decay and we will omit any term containing the $\tilde{l}$. From the part of Eq. (27) involving ψ_H states we obtain

$$\mathcal{L}_{\chi l h_u} \supset -\frac{\lambda e^{i(\pi - 2\hat{\theta}_1)/4}}{\sqrt{2}} \left(\sum_{n=0}^{+\infty} \chi_1^{(n)} - i \sum_{n=1}^{+\infty} \chi_2^{(n)} \right) l h_u + \text{h.c.} \tag{A.21}$$

On the other hand, making (A.5) phase redefinitions, the part of Eq. (27) involving H gives:

$$\mathcal{L}_{H l h_u} \supset -\frac{\lambda e^{-i(\hat{\theta}_1 + \alpha)/2}}{\sqrt{2}} \left(H^{(0)} + \sqrt{2} \sum_{n=1}^{+\infty} H^{(n)} \right) l \tilde{h}_u + \text{h.c.} \tag{A.22}$$

From Eq. (A.22) we get $S_i l \tilde{h}_u$ type couplings:

$$\mathcal{L}_{S l h_u} \supset -\frac{\lambda}{2} e^{-i(\hat{\theta}_1 + \alpha)/2} \left(S_1^{(0)} + i S_2^{(0)} + e^{-i\pi/4} \sum_{n=1}^{+\infty} (S_1^{(n)} + i S_2^{(n)} - S_3^{(n)} - i S_4^{(n)}) \right) l \tilde{h}_u$$
$$+ \text{h.c.} \tag{A.23}$$

Now, we integrate out the heavy χ_i and S_i states, in order to obtain effective operators. Starting with the integration of the fermionic modes, at relatively low energies, we can ignore kinetic terms for the χ_i states. With this, via equations of motions $\frac{\delta\mathcal{L}}{\delta\chi_1^{(n)}} = \frac{\delta\mathcal{L}}{\delta\chi_2^{(n)}} = 0$, we can solve $\chi_{1,2}^{(n)}$ and plug them back into the Lagrangian. Doing so [using couplings of Eqs. (A.15), (A.20) and (A.21)] and keeping terms up to the first power of ϕ_Θ, we obtain:

$$\frac{i\lambda^2}{4} e^{-i\hat{\theta}_1} \left(\sum_{n=0}^{+\infty} \left(\frac{1}{m_{\chi_1}^{(n)}} - \frac{1}{m_{\chi_2}^{(n+1)}} \right) + \frac{1}{2} g_4 \phi_\Theta \sum_{n=0}^{+\infty} \left(\frac{1}{(m_{\chi_1}^{(n)})^2} + \frac{1}{(m_{\chi_2}^{(n+1)})^2} \right) \right) (l h_u)^2 + \text{h.c.} \tag{A.24}$$

With $\langle M^1 \rangle = 0$, $g_1 \langle A_5^1 \rangle = 1/R$, from (A.16) we have $m_{\chi_1}^{(n)} = m_{\chi_2}^{(n+1)} = (2n+1)/(2R)$. Using this in (A.24), we see that the first sum-term (coefficient in front of $d = 5$ operator) cancels out, i.e. no $d = 5$ lepton number violating operator emerges. This is understandable, because the whole theory has $U(1)$ gauge symmetry and the lepton number is a residual global symmetry (with $\langle M^1 \rangle = 0$) at $d = 5$ level. [19] Thus, from Eq. (A.25) we obtain

$$\mathcal{L}_{(\chi)}^{d=6} = \frac{i\lambda^2}{4} e^{-i\hat{\theta}_1} \left(\sum_{n=0}^{+\infty} \frac{1}{(m_{\chi_1}^{(n)})^2} \right) g_4 \phi_\Theta (l h_u)^2 + \text{h.c.} \tag{A.25}$$

where subscript (χ) indicates that this $d = 6$ operator is obtained through the integration of the heavy χ_i states. The sum in (A.25) is well convergent because $\sum_{n=0}^{+\infty} \frac{1}{(2n+1)^2} = \frac{\pi^2}{8}$. It turns out that a $\phi_\Theta (l h_u)^2$ type operator emerges only via integration of the χ_i states. Taking into account these, comparing Eqs. (A.25) and (28) we have

$$C_0 = \frac{1}{8} g_4 i \lambda^2 e^{-i\hat{\theta}_1} (\pi R)^2 . \tag{A.26}$$

---

[19] Different result would emerge if we have had included brane H^2 coupling which explicitly violates the lepton number. We do not consider such terms for the sake of simplicity.

Next, by integrating out heavy S_i states, the $\phi_\Theta(l\tilde{h}_u)^2$ and $\phi_\Theta(l\tilde{h}_u)(\bar{l}\bar{\tilde{h}}_u)$ type dimension 7 operators will be

$$
\begin{aligned}
C_1\phi_\Theta(l\tilde{h}_u)^2 &+ \text{h.c.} + C_2\phi_\Theta(l\tilde{h}_u)(\bar{l}\bar{\tilde{h}}_u), \quad \text{with} \\
C_1 &= \frac{1}{8}g_4\lambda^2 e^{-i(\hat\theta_1+\alpha)} \\
&\times \left(-\sum_{n=0}^\infty \frac{e^{-i\frac{\pi}{2}(1-\delta_{0n})}}{(m_1^{(n)})^3} + \sum_{n=0}^\infty \frac{e^{-i\frac{\pi}{2}(1-\delta_{0n})}}{(m_2^{(n)})^3} - i\sum_{n=1}^\infty \frac{1}{(m_3^{(n)})^3} + i\sum_{n=1}^\infty \frac{1}{(m_4^{(n)})^3}\right), \\
C_2 &= \frac{1}{4}g_4|\lambda|^2\left(-\sum_{n=0}^\infty \frac{1}{(m_1^{(n)})^3} - \sum_{n=0}^\infty \frac{1}{(m_2^{(n)})^3} + \sum_{n=1}^\infty \frac{1}{(m_3^{(n)})^3} + \sum_{n=1}^\infty \frac{1}{(m_4^{(n)})^3}\right).
\end{aligned} \tag{A.27}
$$

Taking into account $m_3^{(n+1)} = m_2^{(n)}$ and $m_4^{(n+1)} = m_1^{(n)}$, we see that the sums in C_2 precisely cancel out, while C_1 ($\sim R^3$) remains non-zero. From the identity [39]

$$
\sec^2\frac{\pi x}{2} = \frac{4}{\pi^2}\sum_{n=0}^{+\infty}\left(\frac{1}{(2n+1+x)^2} + \frac{1}{(2n+1-x)^2}\right), \tag{A.28}
$$

taking first derivatives on both sides, we get

$$
\sum_{n=0}^{+\infty}\left(\frac{1}{(2n+1-x)^3} - \frac{1}{(2n+1+x)^3}\right) = \frac{\pi^3}{8}(1+\tan^2\frac{\pi x}{2})\tan\frac{\pi x}{2} . \tag{A.29}
$$

Using Eq. (A.29), from (A.27) we finally obtain:

$$
\begin{aligned}
C_1 &= \frac{1}{4}g_4\lambda^2 e^{-i(\hat\theta_1+\alpha)} \\
&\times \left(8\sqrt{2}e^{i\frac{\pi}{4}}R^4|F_T|\frac{3+(R|F_T|)^2}{(1-(R|F_T|)^2)^3} - i(\pi R)^3\tan\frac{\pi R|F_T|}{2}(1+\tan^2\frac{\pi R|F_T|}{2})\right), \\
C_2 &= 0 .
\end{aligned} \tag{A.30}
$$

While the C_2 is precisely zero, the C_1 vanishes in the $F_T \to 0$ limit. However, with $|F_T| \sim 1/R$, we have $C_1 \sim R^3$.

Remaining operators, as discussed in Section 3.1, will not have any relevance for the inflaton decay and we will not present them here.

A.1. SUSY breaking on a brane

We assume that all MSSM states, that are matter $\{f\}$, gauge $\{V\}$ and Higgs h_u, h_d superfields, live on a 4D brane. Matter superfields can be included in the Kähler potential as follows

$$
\mathcal{K} = -\ln(T+T^\dagger)^3 - \ln\left(1 - \frac{2}{M_P^2}f^\dagger e^{-V}f\right) + \mathcal{K}(h_u) + \mathcal{K}(h_d) , \tag{A.31}
$$

where $\mathcal{K}(h_{u,d})$ account for part of the Higgs superfields and will be specified below. With (A.31), from Eq. (A.3), for squark and slepton masses we get

$$
M_{\tilde f}^2|\tilde f|^2, \quad \text{with} \quad M_{\tilde f}^2 = \frac{1}{4}M_P^2|W|^2 = \frac{1}{4}|F_T|^2 . \tag{A.32}
$$

Due to the brane superpotential coupling of l, h_u with H state, there will be also a loop induced contribution to the soft mass[2], which we do not display here. Thus, with the large F_T-term all squark and sleptons are heavier than the inflaton field and they play no role for the inflaton decay. On the other hand we need to keep at least one Higgs doublet to be light. Since the SUSY breaking scale is very high, this can be achieved only by price of fine tuning: assuming for instance that the light Higgs mainly resides in h_u, and selecting its Kähler potential as

$$\mathcal{K}(h_u) = \left(1 + \alpha(T + T^\dagger)^3\right) \frac{2 h_u^\dagger e^{-V} h_u}{(1 + 8\alpha) M_P^2} . \tag{A.33}$$

Note, that with this selection, the kinetic term for h_u is canonically normalized for arbitrary values of α. For the soft mass[2] of h_u we obtain

$$M_{h_u}^2 = \frac{27 - (16\alpha + 5)^2}{8(1 + 8\alpha)^2} |F_T|^2 . \tag{A.34}$$

With the selection $16\alpha + 5 = \sqrt{27} - \mathcal{O}(\frac{M_W^2}{F_T^2})$, we obtain $M_{h_u} \sim \mathcal{O}(100 \text{ GeV})$ – the needed value.

As far as the gaugino masses are concerned, since the MSSM gauge supermultiplets are introduced on a brane they will not have direct couplings neither with the T modulus nor with the compensator. By selecting, in Eq. (A.1), the gauge kinetic function $f_{IJ} = \delta_{IJ}$, the corresponding gauginos will remain light. By the same token, the higgsino mass – the μ parameter, can be around the TeV scale. Therefore, the lightest neutralino can be a dark matter candidate. This is the split SUSY scenario, which, as was shown [41], can have various remarkable phenomenological features and interesting implications.

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