Analysis of optimal initial glide conditions for hypersonic glide vehicles

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Abstract  Hypersonic glide vehicles (HGVs) are launched by a solid booster and glide through the atmosphere at high speeds. HGVs will be important means for rapid long-range delivery in the future. Given that the glide is unpowered, the initial glide conditions (IGCs) are crucial for flight. This paper aims to find the optimal IGCs to improve the maneuverability and decrease the constraints of HGVs. By considering the IGCs as experiment factors, we design an orthogonal table with three factors that have five levels each by using the orthogonal experimental design method. Thereafter, we apply the Gauss pseudospectral method to perform glide trajectory optimization by using each test of the orthogonal table as the initial condition. Based on the analytic hierarchy process, an integrated indicator is established to evaluate the IGCs, which synthesizes the indexes of the maneuverability and constraints. The integrated indicator is calculated from the trajectory optimization results. Finally, optimal IGCs and valuable conclusions are obtained by using range analysis, variance analysis, and regression analysis on the integrated indicator.

1. Introduction

Due to flexible maneuverability and time sensitivity, hypersonic glide vehicles (HGVs) for long-range delivery missions have obtained considerable attention in recent years. The US Department of Defense and Air Force launched the “Falcon” program in 2003. The proposed common aero vehicle (CAV) is a type of HGV that has a lifting body configuration and is launched by a solid booster rocket. The CAV is able to glide without power through the atmosphere by relying on aerodynamic control.1,2 To demonstrate hypersonic technologies that will help achieve a prompt global-reach capability, the United States has conducted two experimental flight tests of the hypersonic technology vehicle 2 (HTV-2). The Minotaur Lite launch system successfully delivered the Falcon HTV-2 to the desired location. However, the HTV-2 failed to complete the whole flight. Although neither of the tests was fully successful, the future of HGV technology is promising.3,4 Considering that HGVs glide through the atmosphere unpowered, initial gliding conditions (IGCs) significantly influence the maneuverability and gliding trajectory constraints. Thus, analyzing the optimal glide conditions is necessary to enlarge delivery range while decreasing aerodynamic load and...
aerodynamic heating. The IGCs include initial height, speed, path angle, and azimuth. To determine the optimal IGC, we use the orthogonal experimental design method to arrange the experiment and generate an orthogonal table, which includes a number of experiment sets and the IGCs as experiment factors. By applying the Gauss pseudospectral method (GPM), optimal trajectories are accomplished by employing experiment sets as initial conditions. The evaluation indexes of optimal IGCs are established and acquired from the GPM optimization result. We achieve a synthetic indicator by adopting the analytic hierarchy process (AHP). The optimal IGCs and valuable conclusions are obtained by using range analysis, variance analysis, and regression analysis. Although the analysis conclusion is drawn for the CAV, the conclusion is universally applicable for HGVs.

2. HGV model

The CAV is a type of HGV that can achieve high terminal accuracy, extended cross range, and high maneuverability. The CAV carries approximately 1000 lbs of munitions with a cross range of approximately 3000 NM. The CAV is widely used in research concerning glide trajectory optimization and re-entry guidance. The CAV-H is a high lift-to-drag scheme in the two CAV design. Fig. 1 shows the lift coefficient and drag coefficient of the CAV-H, respectively (AoA means angle of attack, $C_L$ means lift coefficient, $C_D$ means drag coefficient). The maximum lift-to-drag ratio of the CAV-H is 3.5 with a reference area of 0.48 m$^2$ and mass of 907 kg.

3. Orthogonal experimental design (OED)

The OED is an analysis and optimization method for researching multiple factors and levels. This method utilizes an orthogonal table to arrange the experiment scientifically and evaluate the effect of multiple factors. Based on orthogonality, some representative tests can be chosen from the overall tests. Results from the representative tests can be used to find optimal schemes, discover unanticipated important information, and achieve valuable conclusions through range analysis, variance analysis, and regression analysis method.

The IGCs include height, speed, path angle, and azimuth angle. We arrange four factors in the orthogonal design and use five levels for each factor to cover the factor value domain. The factors and their levels are shown in Table 1. In the table, $V$ is the velocity, $H$ is the altitude, $\gamma$ is the flight path angle, $\psi$ is the velocity azimuth angle, subscripts "0" and "f" indicate the initial and final values, respectively. The orthogonal table $L_{25}(5^6)$ is very suitable for the desired design and contains 25 tests. Without considering the interaction among the factors, four arbitrary columns from $L_{25}(5^6)$ are chosen to arrange four factors. The remaining columns can be used to represent the degree of experiment error. The orthogonal array is shown in Table 2.

4. Evaluation indexes of optimal IGCs

Table 2 shows 25 sets of IGCs. To analyze the optimal IGC, the evaluation indexes of optimality are required. Considering that the HGV is used to complete long-range missions, the maximum downrange and maximum cross range should first be considered. Second, the HGV glides through the atmosphere at high speeds for a long time. The HGV endures very serious aerodynamic load and aerodynamic heating, thus leading to hazardous conditions. Therefore, aerodynamic load and aerodynamic heating should be minimized and not exceed the maximum constraints. Considering the above requirements into account, the maximum downrange, maximum cross range, peak normal load, peak dynamic pressure, peak heat flux are chosen as evaluation indexes (see Fig. 2). By applying the AHP method, the evaluation indexes can be synthesized into a total indicator.

According to the characteristics of the evaluation indexes, the indexes can be divided into two types:

1. Benefit index. A bigger index is preferable for maximum downrange and maximum cross range.
2. Cost index. The index is as small as possible for peak normal load, peak dynamic pressure, peak heat flux. These four cost indexes can be synthesized into a total cost index by the AHP method.

![Fig. 1 Aero coefficient of CAV-H.](image)

<table>
<thead>
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<th>Table 1 Factors and their OED levels.</th>
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Table 2 Orthogonal array.

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<th>( \psi_0 )</th>
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Fig. 2 Criterion structure of IGCs.

5. Glide trajectory optimization by GPM

5.1. Dynamic equations

The dynamics of atmospheric entry are expressed as a set of translational equations of motion defined in an Earth-fixed coordinate frame with time as the independent variable. Given that the vehicle during entry is unpowered, the energy monotonically decreases along with trajectory. Energy is an appropriate independent variable for the dynamics because no concern is placed on the entry time at the beginning or end, and the target final conditions are specified at either final velocity or final energy. \(^8\text{-}^{10}\) For each initial glide conditions, the initial energy is also specified. We define energy \( E \) as follows:

\[
E = \frac{1}{2} V^2 - \left( \frac{\mu}{r} - \frac{\mu}{R_d} \right) \tag{1}
\]

where \( V \) is the earth-relative velocity magnitude; \( r \) and \( R_d \) are the radial distances from the planet center to the center of mass of the vehicle and the surface of the planet, respectively; \( \mu \) is the gravitational constant. This is an appropriate formulation for the entry problem because the terminal conditions are given at an energy value and time plays no role. \( E \) is normalized as follows:

\[
\bar{E} = \frac{E - E_0}{E_t - E_0} \tag{2}
\]

where \( E_0 \) and \( E_t \) are the beginning and ending entry energy, respectively; \( \bar{E} \) is the normalized energy, \( \bar{E} \in [0, 1] \).

Denoting \( \frac{d(\cdot)}{dE} \) by \( (\cdot)' \) through \( (\cdot)' = \frac{1}{E} \frac{d(\cdot)}{dt} \) and \( \bar{E} = -DV \) with normalized energy as the independent variable, the translational motion of the vehicle can be modeled by using five state equations:

\[
\tilde{r}' = E_t - E_0 \quad \text{Ref}
\]

\[
\tilde{V}' = \frac{E_t - E_0}{R_d} \quad \left( -\cos \gamma \sin \psi \right) \quad \text{Ref}
\]

\[
\phi' = \frac{E_t - E_0}{R_d} \quad \left( -\cos \psi \cos \psi \right) \quad \text{Ref}
\]

\[
\gamma' = -\frac{E_t - E_0}{V^2 D} \left[ L \cos \sigma - \left( g - \frac{V^2}{R_d^2} \right) \cos \gamma + 2a_v V \cos \psi \sin \psi \right] \quad \text{Ref}
\]

\[
\psi' = -\frac{E_t - E_0}{V^2 D} \quad \left[ L \sin \gamma \cos \gamma + \frac{V^2}{R_d^2} \cos \gamma \sin \psi \tan \phi - 2a_v V \tan \gamma \cos \psi \cos \phi - \sin \phi \right] \quad \text{Ref}
\]

where \( r \) is normalized by \( R_d \), that is, \( \tilde{r} = r/R_d \). The longitude and latitude are \( \tilde{\lambda} \) and \( \tilde{\phi} \), respectively. The flight path angle is \( \gamma \); and the bank angle is \( \sigma \). The velocity azimuth angle \( \psi \) is measured from the north in a clockwise direction. \( a_v \) is the angular rate of planet rotation. \( L \) and \( D \) represent the lift and drag accelerations (specific forces) and are expressed as

\[
D = \frac{1}{2m} \rho V^2 C_D S_{ref} \quad \text{Ref}
\]

\[
L = \frac{1}{2m} \rho V^2 C_l S_{ref} \quad \text{Ref}
\]

where \( \rho \) is the air density as an exponential function of height, \( S_{ref} \) the reference area and \( m \) the vehicle mass.

5.2. GPM

The basic discretization method employed in this study is based on the GPM, which has been presented in great detail in Ref.\(^{11}\text{-}^{13}\) We provide a highlight of the basic ideas of this method in subsequent sections of the paper.

An approximation to the state \( X(t) \) is formed based on \( N + 1 \) Lagrange interpolating polynomials \( L_i(t)(i = 0, 1, \cdots, N) \):

\[
x(t) \approx X(t) = \sum_{i=0}^{N} X(t_i)L_i(t) \quad \text{Ref}
\]

where \( t \in [-1, 1] \). For the Bolza problem, the independent variable is defined in the interval \( t \in [\tau_0, \tau_t] \). The independent
variable can be mapped to the \( \tau \) interval via the affine transformation:

\[
\tau = \frac{2t}{t_f - t_0} - \frac{t_f + t_0}{t_f - t_0} \tag{11}
\]

where \( t_f = t_f + t_0 \).

\( L_i(\tau) (i = 0, 1, \ldots, N) \) can be expressed as follows:

\[
L_i(\tau) = \sum_{j=0}^{N} \frac{\tau - \tau_j}{t_j - \tau_j} \tag{12}
\]

The controls are approximated at the collocation points by using a basis of \( K \) Lagrange interpolating polynomials \( U(x)(i = 1, 2, \ldots, N) \):

\[
u(\tau) \approx U(\tau) = \sum_{i=0}^{N} U(\tau_i) L_i(\tau) \tag{13}
\]

where

\[
L_i(\tau_j) = \begin{cases} 1, & i = j \\ 0, & i \neq j \end{cases} \quad L_i^j(\tau) = \begin{cases} 1, & i = j \\ 0, & i \neq j \end{cases} \tag{14}
\]

By differentiating Eq. (10), the derivation of states at a set of collocation points need to be satisfied exactly as the dynamic equations.

\[
x(\tau) \approx \dot{X}(\tau) = \sum_{i=0}^{N} X(\tau_j) L_i(\tau) \tag{15}
\]

The differential of each Lagrange polynomials at a set of collocation points is a matrix.

\[
L_i(\tau_k) = P_{ik} = \sum_{j=0}^{N} \prod_{\substack{l \neq i \atop l \neq k}} \frac{\tau - \tau_l}{\tau_k - \tau_l} \tag{16}
\]

\[
(\{k, i = 0, 1, \ldots, N\})
\]

By using the matrix, the differential equations of dynamics can be transformed into algebra constraints:

\[
R_i = \sum_{j=0}^{N} P_{ik} X_j - \frac{t_j - t_0}{2} f(X_j, U_k, t_j; t_0, t_f) = 0 \tag{17}
\]

\[
(\{k = 1, 2, \ldots, N\}; X_k \equiv X(\tau_k) ; U_k \equiv U(\tau_k))
\]

Notice that the state constraints above are acted at collocation points and do not include two boundary points. The beginning boundary constraints of the states are \( X_0 = \dot{X}(-1) \). The final boundary constraints of the states can be discretized and approximated via Gauss quadrature.

\[
X_f \equiv X_0 + \frac{t_f - t_0}{2} \sum_{k=1}^{N} \omega_k f(X_k, U_k, \tau_k; t_0, t_f) \tag{18}
\]

where \( \omega_k \) is the Gauss weights.

The integral term in the cost function can also be approximated with a Gauss quadrature:

\[
J = \Phi(X_0, t_0, X_t, t_f) + \frac{t_f - t_0}{2} \sum_{k=1}^{N} \omega_k g(X_k, U_k, \tau_k; t_0, t_f) \tag{19}
\]

Considering the boundary constraint,

\[
\phi(X_0, t_0, X_t, t_f) = 0 \tag{20}
\]

path constraint,

\[
C(X_k, U_k, \tau_k; t_0, t_f) \leq 0, \quad (k = 1, 2, \ldots, N) \tag{21}
\]

cost function of Eq. (19), and constraints of Eq. (18), the GPM transforms the optimization problem to a non-linear programming problem.

5.3. Glide trajectory optimization

By using each set condition of the orthogonal table as the initial condition, glide trajectory optimization is performed to obtain the evaluation indexes. Each condition has two groups of optimization results: maximum downrange and maximum cross range. The performance index of maximum downrange is denoted by

\[
J = -DR(\bar{E}_t) = -\int_{t_0}^{t_f} V \cos \gamma \cos \Delta \psi \, d\bar{E} \tag{22}
\]

The performance index of maximum cross range yields

\[
J = -CR(\bar{E}_t) = -\int_{t_0}^{t_f} V \cos \gamma \sin \Delta \psi \, d\bar{E} \tag{23}
\]

where \( \Delta \psi = \psi - \psi_0 \). In glide trajectory optimization, the constraints have to be considered:

(1) State constraints

\[
\begin{bmatrix} 0 \ m \\ 0 \ \degree \end{bmatrix} \leq \begin{bmatrix} H \\ \lambda \end{bmatrix} \leq \begin{bmatrix} 200 \ km \\ 180 \degree \end{bmatrix}
\]

(2) Control constraints

\[
\begin{bmatrix} 10 \degree \\ -80 \degree \end{bmatrix} \leq \begin{bmatrix} \alpha \\ \sigma \end{bmatrix} \leq \begin{bmatrix} 20 \degree \\ 80 \degree \end{bmatrix}
\]

(3) Path constraints

\[
\begin{bmatrix} n = \sqrt{L^2 + D^2} \\ q = \frac{1}{2} \rho V^2 \\ \dot{Q} = C_d p_o^{0.5} V^{1.15} \end{bmatrix} \leq \begin{bmatrix} n_{\text{max}} \\ q_{\text{max}} \\ \dot{Q}_{\text{max}} \end{bmatrix} = \begin{bmatrix} 5 \ \text{kPa} \\ 600 \ \text{kW/m}^2 \end{bmatrix}
\]

(4) Terminal constraints

\[
\begin{bmatrix} H_t \\ V_f \end{bmatrix} = \begin{bmatrix} 30 \ km \\ 1800 \ m/s \end{bmatrix} \leq \begin{bmatrix} 0 \degree \end{bmatrix}
\]

The dynamic equations in Eqs. (3)–(7) have five states. In the optimization, \( V_{f_0}, H_{f_0} \) and \( \gamma_0 \) are provided by the orthogonal array. \( \delta_0, \phi_0 \), and \( \psi_0 \) are assigned to zero. The General Pseudospectral Optimal Control Software (GPOPS) is used to fulfill the optimization, and the results are shown in Table 3. \( L_{\text{down}} \) and \( L_{\text{cross}} \) indicate the optimization results of the maximum down range and maximum cross range, respectively.
6. Index synthesis and AHP

The benefit index and cost index are considered for the evaluation of the optimal IGCs. However, we cannot draw an evaluation conclusion by simply contrasting each index. Therefore, we have to synthesize the indexes into an integrated indicator. Considering the big difference in magnitude among the indexes, normalization of the indexes is necessary.

6.1. Index normalization

The normalization method of the quantification index is as follows: the upper and lower value of the performance index is mapped to 100 and 0, respectively. A normalized function that maps the index value to a real number between 100 and 0 is defined. The normalized function is denoted by \( F(x): y_R \rightarrow y_N \in [0, 100] \).

A rising half-trapezoid function is used to normalize the benefit index:

\[
y_N = \begin{cases} 
100(y_R - a) & a < y_R \leq b \\
100 - \frac{b - a}{a} y_R & y_R > b \\
0 & y_R \leq a
\end{cases} 
\]  

(24)

The index value \( y_R \) is normalized by Eq. (24). \( y_R \) linearly increases between lower value \( a \) and upper value \( b \) (see Fig. 3(a), subscript “nor” indicate the normalized value).

The normalization of maximum downrange \( L_{D_{\text{max}}} \) and maximum cross range \( L_{C_{\text{max}}} \) is suitable for this model.

A decline half-trapezoid function is used to normalize the cost index (see Fig. 3(b)):

\[
y_N = \begin{cases} 
100(b - y_R) & a < y_R \leq b \\
100 - \frac{b - a}{a} y_R & y_R > b \\
0 & y_R \leq a
\end{cases} 
\]  

(25)

6.2. AHP

Saaty\(^{14}\) developed the AHP in 1980. The AHP is a popular and widely used method for multi-criterion decision-making. The

Table 3  Optimization results of orthogonal table.

<table>
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<tr>
<th>Experiment</th>
<th>( L_{D_{\text{max}}} ) (km)</th>
<th>( L_{C_{\text{max}}} ) (km)</th>
<th>( \dot{Q}_{\text{max}} ) (kW/m²)</th>
<th>( n_{\text{max}} )</th>
<th>( L_{D_{\text{max}}} )</th>
<th>( L_{C_{\text{max}}} )</th>
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AHP helps state the decision by considering the decision in the context of a hierarchy: goals are at the top, criteria are at the second level, sub-criteria are at various lower levels, and alternatives are at the bottom of the hierarchy. Based on a nine-item verbal/numerical judgment scale, the decision maker conducts pairwise comparisons of elements at each level of the hierarchy. Each element at a particular hierarchy level is compared with other elements at that level to determine the preferred or more important elements.15,16 A numerical weight or priority is derived for each element of the hierarchy. In the final step of the process, numerical priorities are calculated for each decision alternative. The procedure for using the AHP can be summarized as follows:

(1) Model the problem as a hierarchy that contains the decision goal, alternatives for reaching the goal, and criteria for evaluating the alternatives.

(2) Establish priorities among the elements of the hierarchy by making a series of judgments on the basis of the pairwise comparisons of the elements.

(3) Synthesize the judgments to yield a set of overall priorities for the hierarchy.

(4) Verify the consistency of the judgments.

(5) Make a final decision based on the results.

The hierarchy for the multi-criteria analysis of the optimal IGCs is shown in Fig. 2. The hierarchy contains 25 alternatives, one of which is the optimization results of each set of IGCs. Considering that maneuverability is more important than cost, the judgment matrix of the criteria level is provided by Table 4. \( C_{D}\) and \( C_{C}\) denote the cost of maximum downrange and the cost of maximum cross range, respectively.

Verifying the consistency of the judgment matrix is necessary. If the consistency is valid, the derived weights can fully reflect the relative importance of the criteria; otherwise, the judgment matrix has to adjust until the consistency test is verified. The consistency test can be fulfilled by calculating the consistency index:

\[
CI = \frac{\lambda_{\text{max}} - n}{n - 1}
\]

where \( \lambda_{\text{max}} \) is the biggest eigenvalue of the judgment matrix, and \( n \) the rank. Finding \( CI = 0 \) is easy because \( \lambda_{\text{max}} = 4 \).

The random consistency index yields:

\[
CR = \frac{CI}{RI}
\]

In Eq. (27), RI is the average random consistency scale. The RI values are shown in Table 5.15 Looking up the Table 5, \( RI = 0.89 \), and then CR is equal to zero. When CR is less than 0.1, the consistency of the judgment matrix is acceptable. Hence, the eigenvector corresponding to \( \lambda_{\text{max}} \) of the judgment matrix can be regarded as the weight vector:

\[
w = [0.3750 \quad 0.3750 \quad 0.1250 \quad 0.1250]
\]

Considering that the heat flux is more important than the other constraints and that the final time is relatively trivial, the judgment matrix of the sub-criteria level is given by Table 6.

The biggest eigenvalue is \( \lambda_{\text{max}} = 3 \), \( CI = 0 \), \( CR = 0 \). Thus, the judgment matrix of the sub-criteria level passes the consistency test; thus, the weight vector is expressed as follows:

\[
w_2 = [0.25 \quad 0.25 \quad 0.50]
\]

Table 3 shows the 25 groups of optimization results. The upper and lower values of the indexes are shown in Table 7. The normalized index values are listed in Table 8. \( C_{\text{nor}} \) indicates the normalization cost of the maximum range by AHP. By applying the AHP and weight vector \( w_1 \) and \( w_2 \), the integrated indicator SI is obtained for each experiment set. The optimal IGCs can be obtained by analyzing the SI. The range analysis result and variance analysis result are shown in Tables 9 and 10, respectively. In the tables, \( k_i (i = 1, 2, \ldots, 5) \) means level average, \( R \) means factor range, DOF means degree of freedom, SS means sum of squares, MS means mean square. Regression analysis is used to achieve the predicted optimal IGC,
The results of the range analysis (Table 9) and variance analysis (Table 10) show that velocity is highly significant to the SI, whereas height, path angle and azimuth angle are not significant to the SI. The main effect in Fig. 4 validates the results. That is, initial glide velocity plays an important role in the maneuverability of the HGV. Initial glide velocity is the main mean for improving the maneuverability. When the total thrust impulse of the booster rocket is certain, maneuverability can be improved more effectively by increasing the velocity rather than the height. The range analysis result shows that there are some differences between the predicted optimal IGCs and optimal IGCs derived from range analysis. From the Fig. 5, we can find the optimal results of $V_0$, $H_0$, $\gamma_0$, $\psi_0$ are 7000 m/s for velocity, 80 km for height, 0° for path angle, and 105° for azimuth angle.

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a slightly higher score for the predicted optimal IGCs. However, the conclusions reached under these two IGCs are similar. The optimization results of the predicted optimal IGCs are shown in Fig. 6.

Table 12 reveals that the optimal initial glide height is 85.1 km. If the initial altitude is too high, a wide oscillation of the glide trajectory and a severe peak value of the constraints such as heat flux and dynamic pressure will occur. However, if the altitude is low, the vehicle will rapidly drop into the dense atmosphere, thus dramatically increasing drag and significantly reducing range.

From the optimization results (see Table 3), it can be seen that the 90° for azimuth angle brings out maximum downrange and minimum cross-range due to the planet rotation. It can be concluded that a large azimuth deviation from the east will produce a long cross-range and a short downrange.

On the basis of the above analysis, the optimal initial glide altitude should be approximately 80–90 km, path angle should be around 0°, and initial gliding speed should have a large value for the HGV to complete a long-range flight. In addition, the larger the deviation of the initial azimuth angle from the east is, the longer cross-range is and the shorter downrange is.

8. Conclusions

HGVs have high maneuverability and can be used for long-range missions. The IGCs play a significant role in the unpowered flight of HGVs. We fully analyze the optimal IGC and conclude that the altitude should not be too high or low and should be around 80–90 km. The path angle should be around 0°, and the initial gliding speed should have a large value. The optimal azimuth angle should be 90° to achieve the maximum downrange and minimum cross-range.
path angle should be around 0°. Besides, azimuth angle derivation from the east should increase cross-range, and speed improves maneuverability. Although the analysis is based on the CAV, the conclusion can be universally applied to HGVs.

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References


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