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A dynamical cosmological term from the Verlinde's maps

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Abstract

In this Letter it is proposed another generalization of the Verlinde's maps for the case $\Lambda \neq 0$. Thermodynamical arguments combined with this proposal conduce to a inverse square-law cosmological term behavior.

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Since the original introduction of the cosmological constant Λ by Einstein, aiming to keep *his universe* static, "the genie has been let out of the bottle and it is no longer easy to force it back in", paraphrasing Zel'dovich [1]. The cosmological constant has been an elusive problem during all these years of research. Meanwhile, its interpretation as a measure of the energy density of the vacuum has been an important issue that particle theorists have realized.

Usually on textbooks and also in some subareas within cosmology research Λ is considered as a constant parameter. However, recent developments in particle physics and inflationary theory have indicated that the cosmological term should be treated as a dynamical quantity [2].

On the other hand during the recent years many studies have been done about the Verlinde's maps [3]. These very interesting maps establish a relation between the Friedmann–Robertson–Walker (FRW) equations that control the cosmological expansion and the formulas that relate the energy and the entropy of a conformal field theory (CFT). In particular three amazing relations mapping the D-dimensional Friedmann equation

$$H^2 = \frac{16\pi G}{(D-1)(D-2)} \frac{E}{V} - \frac{1}{R^2} \tag{1}$$

into the Cardy's entropy formula [5]

$$S = 2\pi \sqrt{\frac{c}{6}\left(L_0 - \frac{c}{24}\right)}, \tag{2}$$

well known in two-dimensional CFT, have been set up. Those are

$$2\pi L_0 \Rightarrow \frac{2\pi}{D-1} E R,$$
$$2\pi \frac{c}{12} \Rightarrow (D-2)\frac{V}{4GR},$$
$$S \Rightarrow (D-2)\frac{HV}{4G}, \tag{3}$$

where L_0 is the zero-mode Virasoro operator, c the central charge, R the scale factor and $H = \dot{R}/R$ the Hubble parameter with the dot representing the time derivative.

The scenario considered in [3] was that of a closed radiation dominated FRW universe with a vanishing cosmological constant. Within this context Verlinde proposed that the Cardy formula for 2D CFT can be generalized to arbitrary spacetime dimensions. Such a generalized entropy formula is known as the Cardy–Verlinde formula.

Later soon, this result was studied and understood in several set ups [6–9]. Specifically in Ref. [7] the maps were generalized to two different classes of universes including cosmological constant: the de Sitter (dS) closed and the anti-de Sitter (AdS) flat. Both occupied by a universe-sized black hole. In

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the same way, working in the branes context, the CFT dominated universe has been described as a co-dimension one brane in the background of various kinds of (A)dS black holes. In such cases, when the brane crosses the black hole horizon, the entropy and temperature are expressed in terms of the Hubble constant and its time derivative. Such relations hold precisely when the holographic entropy bound is saturated.

Verlinde's proposal has inspired a considerable activity shedding further light on the various aspects of the Cardy–Verlinde formula. Nevertheless there is still no answer to the question about whether the merging of the CFT and FRW equations is a mere formal coincidence or, quoting Verlinde, whether this fact "*strongly indicates that both sets of these equations arise from a single underlying fundamental theory*".

Returning to the Verlinde's maps (3), we would like to stress that they are valid, without restrictions, for all times, at least formally. Nevertheless up to now the study of these maps has been restricted to the use of the machinery of entropy bounds, black holes, branes and the holographic principle [4]. Therefore our aim is to follow the spirit of the Verlinde's concern about the amazing relation between these two equations and what is behind. This direction was followed in [10], where some mathematical properties of the Friedmann's equation (1) that justified, at least in principle, the relation with the Cardy's formula (2) were explored. In this Letter we will show how it is possible to obtain a generalization of the Verlinde's maps for the case of the cosmological term different from zero just expressing Λ in terms of the vacuum energy density and this will lead us to find a dynamical behavior of the cosmological term.

In the presence of the cosmological term the Friedmann equation takes the form,

$$H^2 = \frac{16\pi G}{(D-1)(D-2)} \frac{E}{V} - \frac{1}{R^2} + \frac{\Lambda}{D-1}. \tag{4}$$

Now, in order to obtain the generalization of Verlinde's maps we will just deal with the first relationship, i.e.,

$$2\pi L_0 \Rightarrow \frac{2\pi}{D-1} ER. \tag{5}$$

Our proposal is to shift the value of the energy E adding the vacuum energy E_{vac}. In this manner, the energy $\mathcal{E}$ will be expressed as

$$\mathcal{E} = E + E_{\text{vac}}, \tag{6}$$

where

$$E_{\text{vac}} = \rho_{\text{vac}} V = \frac{(D-2)\Lambda}{16\pi G} V, \tag{7}$$

with ρ_{vac} representing the vacuum energy density. Therefore while the two last conditions of the Verlinde's maps (3) are preserved, the Virasoro operator L_0 is redefined now in terms of $\mathcal{E}$. Thus, the Verlinde's maps will take the form

$$\begin{aligned} &2\pi L_0 \Rightarrow \frac{2\pi}{D-1} \mathcal{E} R, \\ &2\pi \frac{c}{12} \Rightarrow (D-2) \frac{V}{4GR}, \\ &S \Rightarrow (D-2) \frac{HV}{4G}, \end{aligned} \tag{8}$$

and can be easily checked that the D-dimensional Friedmann (4) equation turns into the Cardy formula.

At this point it is important to stress that we considered the maps (8) valid for all times. Particularly we will think c as a c-function. This choice is based on some arguments given by Strominger in [11] where was conjectured that the cosmological evolution of an $(n+1)$-dimensional universe has a dual representation as the renormalization group (RG) flow between two conformal fixed points of a n-dimensional Euclidean field theory. The RG flow begins at a UV (ultraviolet) conformally invariant fixed point and ends at an IR (infrared) conformally invariant fixed point. Since late (early) times correspond to the UV (IR) fixed point then the RG flow corresponds to evolution back in time from the future to the past [11]. The proposed c-function was

$$c \sim \frac{1}{G_N |\frac{\dot{a}}{a}|^{n-1}}. \tag{9}$$

Using the Einstein equations can be showed that $\partial_t(\dot{a}/a) < 0$, provided that any matter in the spacetime satisfies the null energy condition. In other words, this guarantees that the c-function will always decrease in a contracting phase of the evolution or increase in an expanding phase [12]. Also, if the c-function can be evaluated on each slice of some foliation of the spacetime and the slice can be embedded in some de Sitter space the c-function (9) takes the form

$$c \sim \frac{1}{G_N \Lambda^{(n-1)/2}}. \tag{10}$$

In the original papers were considered spatially flat cosmological models with $k=0$ but their c-function was subsequently generalized [12,13] so as to apply to $k \neq 0$ models as well.

Returning to our main line we remember that in [3] was also obtained a universal Cardy entropy formula in terms of the energy E and the Casimir energy E_{C}. It should be also noted that in a CFT with large central charge the entropy and energy are not purely extensive. Being the volume finite, the energy of a CFT contains a non-extensive Casimir contribution proportional to the central charge c. Therefore, taking into account this fact, the total energy can be decomposed as,

$$E(S,V) = E_{\text{E}}(S,V) + \frac{1}{2} E_{\text{C}}(S,V), \tag{11}$$

where the first term is related to the purely extensive part of the energy and the second term represents the Casimir energy.

On the other hand it is known that conformal invariance implies that the product ER is independent of the volume and is only a function of the entropy S. Considering this fact and the behavior of the extensive and sub-extensive parts of the energy can be found that

$$\begin{aligned} &E_{\text{E}} = \frac{a}{4\pi R} S^{1+1/(D-1)}, \\ &E_{\text{C}} = \frac{b}{2\pi R} S^{1-1/(D-1)}, \end{aligned} \tag{12}$$

where a and b are a priori arbitrary positive coefficients. Using these expressions the entropy S takes the form

$$S = \frac{2\pi R}{\sqrt{ab}} \sqrt{E_{\text{C}}(2E - E_{\text{C}})}. \tag{13}$$

Obviously formula (2) can be recovered, ignoring the normalization, if we insert $ER = L_0$ and $E_C R = c/12$.

Extending the above mentioned reasoning to the case $\Lambda > 0$ it can be realized that it is much more natural to apply here the decomposition of the energy (11) since the Casimir energy is often invoked, via vacuum energy, as a decisive factor for explaining the cosmological term. So,

$$\mathcal{E} = E + \frac{1}{2} E_{\mathrm{C}}. \tag{14}$$

Therefore we arrive at the following relations

$$E_{\mathrm{E}} = E, \qquad E_{\mathrm{C}} = 2E_{\mathrm{vac}}. \tag{15}$$

Writing the last expression explicitly we get,

$$\frac{c}{12R} = \frac{2(D-2)\Lambda}{16\pi G} V. \tag{16}$$

Next we found, with the help of the second relation of the Verlinde's maps, that the cosmological term behaves as

$$\Lambda = \frac{1}{R^2}, \tag{17}$$

a decay law for Λ! (or vacuum energy density decay!). That is a remarkable result since among the decay laws of the cosmological term in function of the scale factor ($\Lambda = \alpha R^{-m}$ with α being a constant parameter), proposed in the literature, this case ($m = 2$) has been the one that has received most of the attention [2]. The inverse-square law dependence is supported by dimensional [14] and phenomenological arguments [15]. In our scenario this behavior has appeared (1) as a consequence of identifying the vacuum energy with half the Casimir energy and (2) as a consequence of the use of the relations between the cosmological quantities and conformal quantities through the Verlinde's maps. Here it is worth to note that if we introduce (17) in the second map of (8) the c-function (10) is obtained. In other words, the behavior found for the cosmological term is compatible with the c-function proposed in [12,13].

On the other hand on the basics of a rich body of astronomical observations [16,17] there is now convincing evidence that the recent Universe is dominated by an exotic dark energy density with negative pressure, responsible for the cosmic acceleration. The simplest candidate for dark energy is the cosmological constant. But, if general relativity is correct, cosmic acceleration implies that there must be a dark energy density which diminishes relatively slowly as the universe expands [18]. In this context it has been studied the possibility that the dark energy may decay [18–20]. Therefore the result obtained is also in agreement with the expectations from the dark energy side.

Thus, here we have gone one step ahead. Although we still do not know how to calculate Λ from first principles in this approach we have generated a genuinely R^{-2} varying Λ law.

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