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Near maximal atmospheric mixing in neutrino mass matrices with two vanishing minors

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ABSTRACT

In the flavor basis there are seven cases of two vanishing minors in the neutrino mass matrix which can accommodate the present neutrino oscillation data including the recent T2K data. It is found that two of these cases, namely B_5 and B_6 predict near maximal atmospheric neutrino mixing in the limit of large effective neutrino mass. This feature remains irrespective of the values of solar and reactor mixing angles. A non-zero reactor mixing angle is naturally accommodated in these textures.

1. Introduction

During the past decade there has been considerable experimental development in the determination of neutrino masses and mixings [1]. Recently, T2K experiment [2] has given unambiguous hints of a relatively large 1–3 mixing angle. In this light, it is natural to look for models which, naturally, accommodate a non-zero value of reactor mixing angle while the atmospheric mixing angle remains near its maximal value. Recently many papers have appeared which reproduce the relatively large value of the reactor mixing angle [3].

There are mainly two approaches to explain neutrino mixings:

(1) Mass independent textures [4] which lead to mixing matrices independent of the eigenvalues. The most celebrated example of this category is the tribimaximal (TBM) [5] scenario which has been derived from family symmetries and predicts a vanishing 1–3 mixing angle $\theta_{13}=0$, maximal 2–3 mixing angle $\theta_{23}=\pi/4$ and 1–2 mixing angle $\theta_{12}=\sin^{-1}(1/\sqrt{3})$. Non-zero θ_{13} can be accommodated in TBM and other similar models by considering deviations from symmetry.
(2) Mass dependent textures which induce relations between mixing matrix elements and mass eigenvalues. Such textures naturally accommodate a non-zero θ_{13}. Some examples of these are zero textures [6], vanishing minors [7,8], hybrid textures [9]. Zero textures have been particularly successful in explaining both the quark and the lepton masses and mixings.

In this work we identify a class of mass dependent textures which supplemented with the assumption of a large value of effective neutrino mass M_{ee} naturally predict near maximal θ_{23} and non-zero θ_{13}. Recently, it was shown by Grimus et al. [10] that near maximal atmospheric mixing is predicted for classes B3 and B4 of two zero textures supplemented with the assumption of quasidegeneracy. We consider two vanishing minors of the neutrino mass matrix in the flavor basis together with the assumption of large M_{ee}. This assumption is well motivated by the extensive search for this parameter in the ongoing experiments. We found that the two cases of two vanishing minors viz. B_5 and B_6 (Table 1) in the limit of large M_{ee} predict near maximal atmospheric mixing and this property holds irrespective of the values of solar and reactor mixing angles. The seesaw mechanism [11] is regarded as the prime candidate for understanding the scale of neutrino masses. In the framework of type-I seesaw mechanism, the effective Majorana mass matrix M_ν is given by

$$M_\nu = -M_D M_R^{-1} M_D^T \tag{1}$$

where M_D is the Dirac neutrino mass matrix and M_R is the right-handed Majorana mass matrix. In the framework of type-I seesaw mechanism M_ν is a quantity derived from M_D and M_R. Therefore, zeros of M_D and M_R have a deeper theoretical meaning. In a basis where M_D is diagonal, the zeros of M_R propagate as zero minors in M_ν.

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Table 1
Experimentally allowed classes of two zero minors, here C_{ij} denotes the zero minor corresponding to the (ij)th element of M_ν.

Class	Zero minors
A_1	$C_{3,3}$, $C_{3,2}$
A_2	$C_{2,2}$, $C_{3,2}$
B_3	$C_{3,3}$, $C_{3,1}$
B_4	$C_{2,2}$, $C_{2,1}$
B_5	$C_{3,3}$, $C_{1,2}$
B_6	$C_{2,2}$, $C_{1,3}$
D	$C_{3,3}$, $C_{2,2}$

2. Symmetry realization

In the basis where the charged lepton mass matrix is diagonal, there are fifteen possible two vanishing minors in M_ν. Out of these fifteen only seven patterns (Table 1) viz. A_1, A_2, B_3, B_4, B_5, B_6 and D can accommodate the neutrino oscillation data. Of all the allowed two zero minors in the neutrino mass matrix only three cases B_5, B_6 and D provide non-trivial zero minors, all other cases reduce to two zero textures when confronted with the neutrino oscillation data. We work in a basis where M_D is diagonal ($M_D = \mathrm{diag}(x, y, z)$), and the neutrino mixing arises solely from M_R. In this basis, a zero entry in M_R propagates as a vanishing minor in the effective neutrino mass matrix M_ν. Here, we focus on B_5 and B_6 class of vanishing minors. To obtain the classes B_5 and B_6 of neutrino mass matrices, we extend the Standard Model (SM) by adding three right-handed neutrino singlets ν_{R_i} and one scalar singlet χ. In order to enable the seesaw mechanism for suppressing the neutrino masses M_R must have the following structures for B_5 and B_6:

$$M_R(B_5) = \begin{pmatrix} a & 0 & c \\ 0 & d & e \\ c & e & 0 \end{pmatrix}, \qquad M_R(B_6) = \begin{pmatrix} a & b & 0 \\ b & 0 & e \\ 0 & e & f \end{pmatrix} \tag{2}$$

leading to the following effective neutrino mass matrices through the seesaw mechanism

$$M_\nu(B_5) = \frac{1}{c^2 d + ae^2}\begin{pmatrix} e^2x^2 & -cexy & cdxz \\ -cexy & c^2y^2 & aeyz \\ cdxz & aeyz & -adz^2 \end{pmatrix}, \qquad M_\nu(B_6) = \frac{1}{b^2 f + ae^2}\begin{pmatrix} e^2x^2 & bfxy & -bexz \\ bfxy & -afy^2 & aeyz \\ -bexz & aeyz & b^2z^2 \end{pmatrix}. \tag{3}$$

A general procedure for enforcing zero textures in arbitrary entries of the fermion mass matrices using abelian family symmetries has been outlined in [12]. The symmetry realization of all the allowed one zero and two zero textures was recently presented in [13]. For the symmetry realization of B_5 and B_6 textures of two zero minors we consider a small cyclic group Z_3 which corresponds to the minimal group since Z_2 leads to a non-diagonal charged lepton and Dirac neutrino mass matrix. Under Z_3 the SM Higgs doublet remains invariant and the leptonic fields are assumed to transform as:

$$\begin{aligned} &D_{L_1} \to D_{L_1}, \qquad l_{R1} \to l_{R1}, \qquad \nu_{R_1} \to \nu_{R_1}, \\ &D_{L_2} \to \omega D_{L_2}, \qquad l_{R2} \to \omega l_{R2}, \qquad \nu_{R_2} \to \omega \nu_{R_2}, \\ &D_{L_3} \to \omega^2 D_{L_3}, \qquad l_{R3} \to \omega^2 l_{R3}, \qquad \nu_{R_3} \to \omega^2 \nu_{R_3}, \end{aligned} \tag{4}$$

where $\omega = e^{i2\pi/3}$. Hence the bilinears $\overline{D}_{L_j} l_{R_k}$ and $\overline{D}_{L_j} \nu_{R_k}$, relevant for M_l and M_D transform as

$$\overline{D}_{L_j} l_{R_k} \sim \overline{D}_{L_j} \nu_{R_k} \sim \begin{pmatrix} 1 & \omega & \omega^2 \\ \omega^2 & 1 & \omega \\ \omega & \omega^2 & 1 \end{pmatrix}. \tag{5}$$

The SM Higgs doublet remains invariant under Z_3 leading to diagonal M_l and M_D. The bilinear $\nu_{R_j}\nu_{R_k}$ relevant for M_R transforms as

$$\nu_{R_j}\nu_{R_k} \sim \begin{pmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{pmatrix}. \tag{6}$$

We assume a scalar singlet χ transforming as $\chi \to \omega\chi$ for class B_5 and $\chi \to \omega^2\chi$ for class B_6 which leads to the following Z_3 invariant Yukawa Lagrangians for classes B_5 and B_6:

$$\begin{aligned} -\mathcal{L}_{(B_5)} = {} & Y^l_{11}\overline{D}_{L_1}\phi l_{R_1} + Y^l_{22}\overline{D}_{L_2}\phi l_{R_2} + Y^l_{33}\overline{D}_{L_3}\phi l_{R_3} + Y^D_{11}\overline{D}_{L_1}\tilde{\phi}\nu_{R_1} + Y^D_{22}\overline{D}_{L_2}\tilde{\phi}\nu_{R_2} \\ & + Y^D_{33}\overline{D}_{L_3}\tilde{\phi}\nu_{R_3} + \frac{Y^M_{13}}{2}\nu^T_{R_1}C^{-1}\nu_{R_3}\chi + \frac{Y^M_{22}}{2}\nu^T_{R_2}C^{-1}\nu_{R_2}\chi + \frac{M^M_{11}}{2}\nu^T_{R_1}C^{-1}\nu_{R_1} + \frac{M^M_{23}}{2}\nu^T_{R_2}C^{-1}\nu_{R_3} + \mathrm{H.c.} \end{aligned} \tag{7}$$

$$\begin{aligned} -\mathcal{L}_{(B_6)} = {} & Y^l_{11}\overline{D}_{L_1}\phi l_{R_1} + Y^l_{22}\overline{D}_{L_2}\phi l_{R_2} + Y^l_{33}\overline{D}_{L_3}\phi l_{R_3} + Y^D_{11}\overline{D}_{L_1}\tilde{\phi}\nu_{R_1} + Y^D_{22}\overline{D}_{L_2}\tilde{\phi}\nu_{R_2} \\ & + Y^D_{33}\overline{D}_{L_3}\tilde{\phi}\nu_{R_3} + \frac{Y^M_{12}}{2}\nu^T_{R_1}C^{-1}\nu_{R_2}\chi + \frac{Y^M_{33}}{2}\nu^T_{R_3}C^{-1}\nu_{R_3}\chi + \frac{M^M_{11}}{2}\nu^T_{R_1}C^{-1}\nu_{R_1} + \frac{M^M_{23}}{2}\nu^T_{R_2}C^{-1}\nu_{R_3} + \mathrm{H.c.} \end{aligned} \tag{8}$$

where $\tilde{\phi} = i\tau_2\phi^*$. Next, we show how a large effective neutrino mass can arise in such a model. We note that M_R contains two types of mass terms viz. (1) Bare mass term which does not need a scalar singlet and is invariant by itself. (2) Terms arising from Yukawa couplings to χ. The scale of latter is restricted by the scale of Z_3 breaking while there is no such restriction on the bare mass term which can have a higher mass scale. It can be seen from Eq. (3) that the ee and $\mu\tau$ entries of M_ν have contributions to their numerators from ee and $\mu\tau$ entries of M_R which arise from the bare mass term. We assume the mass eigenvalues of M_D to have same order of magnitude which leads to a large value of ee and $\mu\tau$ entries of M_ν while the other elements of M_ν are suppressed, thus, leading to a large value of M_{ee}. Since these textures are realized at the seesaw scale, the Renormalization Group (RG) evolution of the parameters of M_ν from the seesaw scale to the electroweak scale needs to be taken into account. It is well known that the RG effects are most prominent for the quasidegenerate mass spectrum which is precisely the case here due to the assumption of large M_{ee}. However, it is also known that zero minors in M_ν, at a given energy scale, remain zero at any other energy scale at the one loop level [7]. This is because the matrices at any two energy scales μ_1 and μ_2 are related by $M_\nu(\mu_1) = I M_\nu(\mu_2) I$, where I is diagonal, positive and non-singular. The operation of diagonal matrices from left and right on M_ν does not alter the zero minors of M_ν leading to zero minors in M_ν at any other scale.

3. Formalism

We reconstruct the neutrino mass matrix in the flavor basis assuming neutrinos to be Majorana particles. In this basis, a complex symmetric neutrino mass matrix can be diagonalized by a unitary matrix V as

$$M_\nu = V M_\nu^{\text{diag}} V^T \tag{9}$$

where $M_\nu^{\text{diag}} = \text{diag}(m_1, m_2, m_3)$. The matrix M_ν can be parameterized in terms of three neutrino masses (m_1, m_2, m_3), three neutrino mixing angles $(\theta_{12}, \theta_{23}, \theta_{13})$ and the Dirac-type CP-violating phase δ. The two additional phases α and β appear if neutrinos are Majorana particles. The matrix

$$V = UP \tag{10}$$

where [14]

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \tag{11}$$

with $s_{ij} = \sin\theta_{ij}$ and $c_{ij} = \cos\theta_{ij}$ and

$$P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha} & 0 \\ 0 & 0 & e^{i(\beta+\delta)} \end{pmatrix}$$

is the diagonal phase matrix with the two Majorana-type CP-violating phases α, β and Dirac-type CP-violating phase δ. The matrix V is called the neutrino mixing matrix or the Pontecorvo–Maki–Nakagawa–Sakata (PMNS) matrix [15]. Using Eqs. (9) and (10), the neutrino mass matrix can be written as

$$M_\nu = UPM_\nu^{\text{diag}}P^TU^T. \tag{12}$$

The CP violation in neutrino oscillation experiments can be described through a rephasing invariant quantity, J_{CP} [16] with $J_{CP} = \text{Im}(U_{e1}U_{\mu2}U_{e2}^*U_{\mu1}^*)$. In the above parametrization, J_{CP} is given by

$$J_{CP} = s_{12}s_{23}s_{13}c_{12}c_{23}c_{13}^2\sin\delta. \tag{13}$$

The simultaneous existence of two vanishing minors in the neutrino mass matrix implies

$$M_{\nu(pq)}M_{\nu(rs)} - M_{\nu(tu)}M_{\nu(vw)} = 0, \tag{14}$$

$$M_{\nu(p'q')}M_{\nu(r's')} - M_{\nu(t'u')}M_{\nu(v'w')} = 0. \tag{15}$$

These two conditions yield two complex equations viz.

$$\sum_{l,k=1}^{3}(V_{pl}V_{ql}V_{rk}V_{sk} - V_{tl}V_{ul}V_{vk}V_{wk})m_lm_k = 0, \tag{16}$$

$$\sum_{l,k=1}^{3}(V_{p'l}V_{q'l}V_{r'k}V_{s'k} - V_{t'l}V_{u'l}V_{v'k}V_{w'k})m_lm_k = 0. \tag{17}$$

The above equations can be rewritten as

$$m_1m_2A_3e^{2i\alpha} + m_2m_3A_1e^{2i(\alpha+\beta+\delta)} + m_3m_1A_2e^{2i(\beta+\delta)} = 0, \tag{18}$$

$$m_1m_2B_3e^{2i\alpha} + m_2m_3B_1e^{2i(\alpha+\beta+\delta)} + m_3m_1B_2e^{2i(\beta+\delta)} = 0, \tag{19}$$

where

$$A_h = (U_{pl}U_{ql}U_{rk}U_{sk} - U_{tl}U_{ul}U_{vk}U_{wk}) + (l \leftrightarrow k),$$
$$B_h = (U_{p'l}U_{q'l}U_{r'k}U_{s'k} - U_{t'l}U_{u'l}U_{v'k}U_{w'k}) + (l \leftrightarrow k), \tag{20}$$

with (h, l, k) as the cyclic permutation of $(1, 2, 3)$. These two complex equations (18) and (19) involve nine physical parameters m_1, m_2, m_3, θ_{12}, θ_{23}, θ_{13} and three CP-violating phases α, β and δ. The masses m_2 and m_3 can be calculated from the mass-squared differences Δm^2_{12} and $|\Delta m^2_{23}|$ using the relations

$$m_2 = \sqrt{m_1^2 + \Delta m^2_{12}}, \qquad m_3 = \sqrt{m_2^2 + |\Delta m^2_{23}|} \tag{21}$$

where $m_2 > m_3$ for Inverted Spectrum (IS) and $m_2 < m_3$ for Normal Spectrum (NS). Using the experimental inputs of the two mass-squared differences and the three mixing angles we can constrain the other parameters. Thus, in the two complex equations (18) and (19) we are left with four unknown parameters m_1, α, β and δ which are, obviously, correlated. Simultaneously solving Eqs. (18) and (19) for the two mass ratios, we obtain

$$\frac{m_1}{m_2}e^{-2i\alpha} = \frac{A_3B_1 - A_1B_3}{A_2B_3 - A_3B_2} \tag{22}$$

and

$$\frac{m_1}{m_3}e^{-2i\beta} = \frac{A_2B_1 - A_1B_2}{A_3B_2 - A_2B_3}e^{2i\delta}. \tag{23}$$

The mass ratios for class B_5 to first order in s_{13} are given by

$$\frac{m_1}{m_2}e^{-2i\alpha} \approx 1 + \frac{s_{13}s_{23}(c^2_{23}e^{-i\delta} + s^2_{23}e^{i\delta})}{c_{12}c^3_{23}s_{12}} \tag{24}$$

and

$$\frac{m_1}{m_3}e^{-2i\beta} \approx -\frac{s^2_{23}e^{2i\delta}}{c^2_{23}} - \frac{c_{12}s_{13}s^3_{23}(c^2_{23}e^{-i\delta} + s^2_{23}e^{i\delta})e^{2i\delta}}{c^5_{23}s_{12}}. \tag{25}$$

The mass ratios for class B_6 to first order in s_{13} are

$$\frac{m_1}{m_2}e^{-2i\alpha} \approx 1 - \frac{s_{13}c_{23}(c^2_{23}e^{i\delta} + s^2_{23}e^{-i\delta})}{c_{12}s^3_{23}s_{12}} \tag{26}$$

and

$$\frac{m_1}{m_3}e^{-2i\beta} \approx -\frac{c^2_{23}e^{2i\delta}}{s^2_{23}} + \frac{c_{12}s_{13}c^3_{23}(c^2_{23}e^{i\delta} + s^2_{23}e^{-i\delta})e^{2i\delta}}{s^5_{23}s_{12}}. \tag{27}$$

In the case of zero textures there exists a permutation symmetry between different patterns [17]. Similarly, there exists a permutation symmetry between patterns B_5 and B_6 of two zero minors which corresponds to the permutation in the 2–3 rows and 2–3 columns of M_ν. The corresponding permutation matrix is given by

$$P_{23} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}. \tag{28}$$

The right-handed Majorana mass matrix M_R for class B_6 can be obtained from M_R for class B_5 by the transformation

$$M_R^{B_6} = P_{23}M_R^{B_5}P_{23}^T \tag{29}$$

which after the seesaw gives

$$M_\nu^{B_6} = P_{23}M_\nu^{B_5}P_{23}^T. \tag{30}$$

This leads to the following relations between the parameters:

$$\theta^{B_6}_{12} = \theta^{B_5}_{12}, \qquad \theta^{B_6}_{13} = \theta^{B_5}_{13}, \qquad \theta^{B_6}_{23} = \frac{\pi}{2} - \theta^{B_5}_{23}, \qquad \delta^{B_6} = \delta^{B_5} - \pi. \tag{31}$$

The magnitude of the two mass ratios in Eqs. (22), (23), is given by

$$\rho = \left|\frac{m_1}{m_3}e^{-2i\beta}\right|, \tag{32}$$

$$\sigma = \left|\frac{m_1}{m_2}e^{-2i\alpha}\right| \tag{33}$$

while the CP-violating Majorana phases α and β are given by

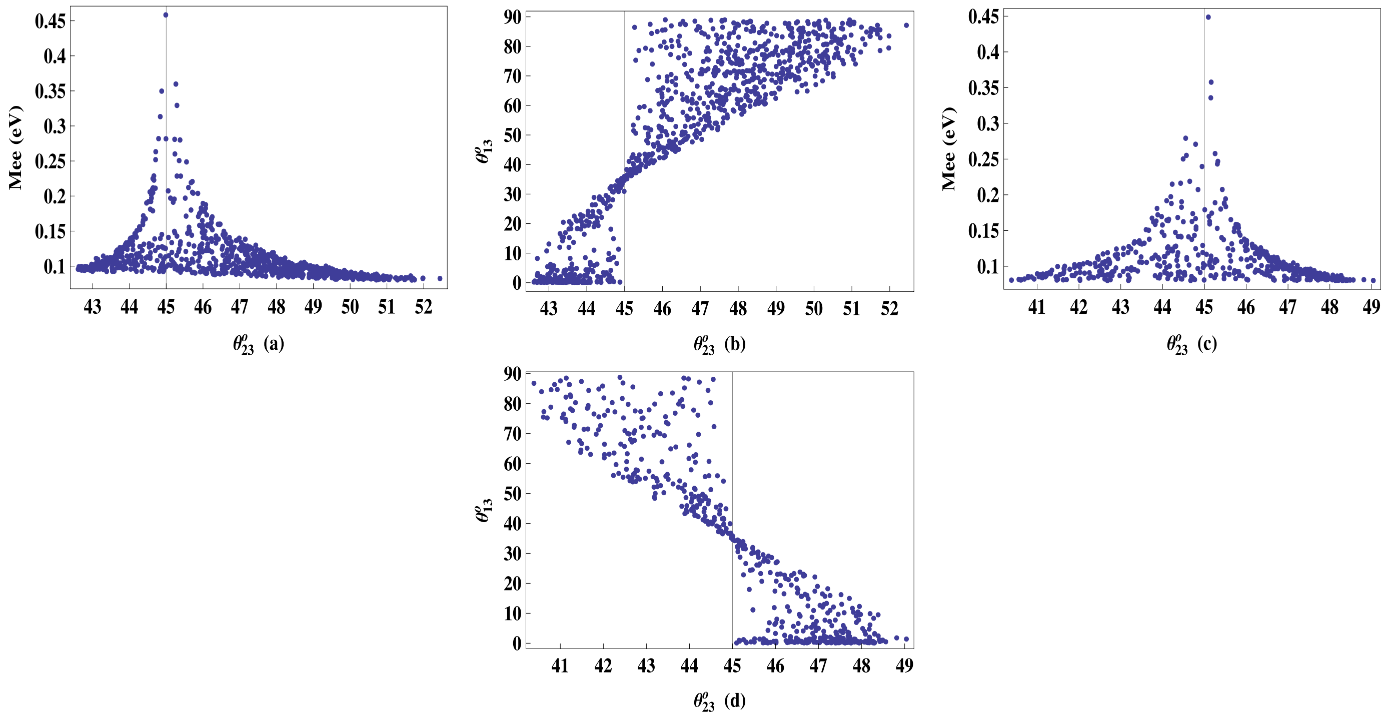

Fig. 1. Correlation plots for class B_5, plots (a), (b) correspond to Normal Spectrum (NS) and plots (c), (d) correspond to Inverted Spectrum (IS).

$$\alpha = -\frac{1}{2}\arg\left(\frac{A_3B_1 - A_1B_3}{A_2B_3 - A_3B_2}\right), \tag{34}$$

$$\beta = -\frac{1}{2}\arg\left(\frac{A_2B_1 - A_1B_2}{A_3B_2 - A_2B_3}e^{2i\delta}\right). \tag{35}$$

Since, Δm^2_{12} and $|\Delta m^2_{23}|$ are known experimentally, the values of mass ratios (ρ, σ) from Eqs. (32) and (33) can be used to calculate m_1. This can be done by inverting Eqs. (21) to obtain the two values of m_1 viz.

$$m_1 = \sigma\sqrt{\frac{\Delta m^2_{12}}{1-\sigma^2}}, \qquad m_1 = \rho\sqrt{\frac{\Delta m^2_{12} + |\Delta m^2_{23}|}{1-\rho^2}}. \tag{36}$$

4. Numerical analysis

The experimental constraints on neutrino parameters at 1, 2 and 3σ [18] are given below:

$$\Delta m^2_{12} = 7.58^{(+0.22,+0.41,+0.60)}_{(-0.26,-0.42,-0.59)} \times 10^{-5}\ \text{eV}^2, \qquad |\Delta m^2_{23}| = 2.35^{(+0.12,+0.22,+0.32)}_{(-0.09,-0.18,-0.29)} \times 10^{-3}\ \text{eV}^2,$$
$$\sin^2\theta_{12} = 0.312^{(+0.017,+0.035,+0.052)}_{(-0.016,-0.032,-0.047)}, \qquad \sin^2\theta_{23} = 0.42^{(+0.08,+0.18,+0.22)}_{(-0.03,-0.06,-0.08)}, \qquad \sin^2\theta_{13} = 0.025^{(+0.007,+0.016,+0.025)}_{(-0.007,-0.013,-0.020)}. \tag{37}$$

The observation of neutrinoless double beta (NDB) decay would signal lepton number violation and imply Majorana nature of neutrinos, for recent reviews see [19,20]. The effective Majorana mass of the electron neutrino M_{ee} which determines the rate of NDB decay is given by

$$M_{ee} = \left|m_1c^2_{12}c^2_{13} + m_2s^2_{12}c^2_{13}e^{2i\alpha} + m_3s^2_{13}e^{2i\beta}\right|. \tag{38}$$

Part of the Heidelberg–Moscow collaboration claimed a signal in NDB decay corresponding to $M_{ee} = (0.11\text{–}0.56)$ eV at 95% C.L. [21]. This claim was subsequently criticized in [22]. The results reported in [21] will be checked in the currently running and forthcoming NDB experiments. There are large number of projects such as CUORICINO [23], CUORE [24], GERDA [25], MAJORANA [26], SuperNEMO [27], EXO [28], GENIUS [29] which aim to achieve a sensitivity upto 0.01 eV for M_{ee}. In the present work, we take the upper limit of M_{ee} to be 0.5 eV [20]. We vary the oscillation parameters within their known experimental ranges. However, the Dirac-type CP-violating phase δ is varied within its full range. The two values of m_1 obtained from the mass ratios ρ and σ, respectively must be equal to within the errors of the oscillation parameters for the simultaneous existence of two vanishing minors in M_ν. The first step in the numerical analysis uses the information of the two known mass squared differences along with the constraint of two zero minors and large M_{ee} to get predictions for the mixing angles. It is found that both the classes B_5 and B_6 predict a near maximal atmospheric mixing angle while the other two mixing angles remain unconstrained. The atmospheric mixing angle θ_{23} moves towards $\pi/4$ with increasing M_{ee} as seen in Fig. 1 for class B_5 and Fig. 2 for class B_6. Thus classes B_5 and B_6 of two vanishing minors in M_ν naturally predict a near maximal atmospheric mixing angle.

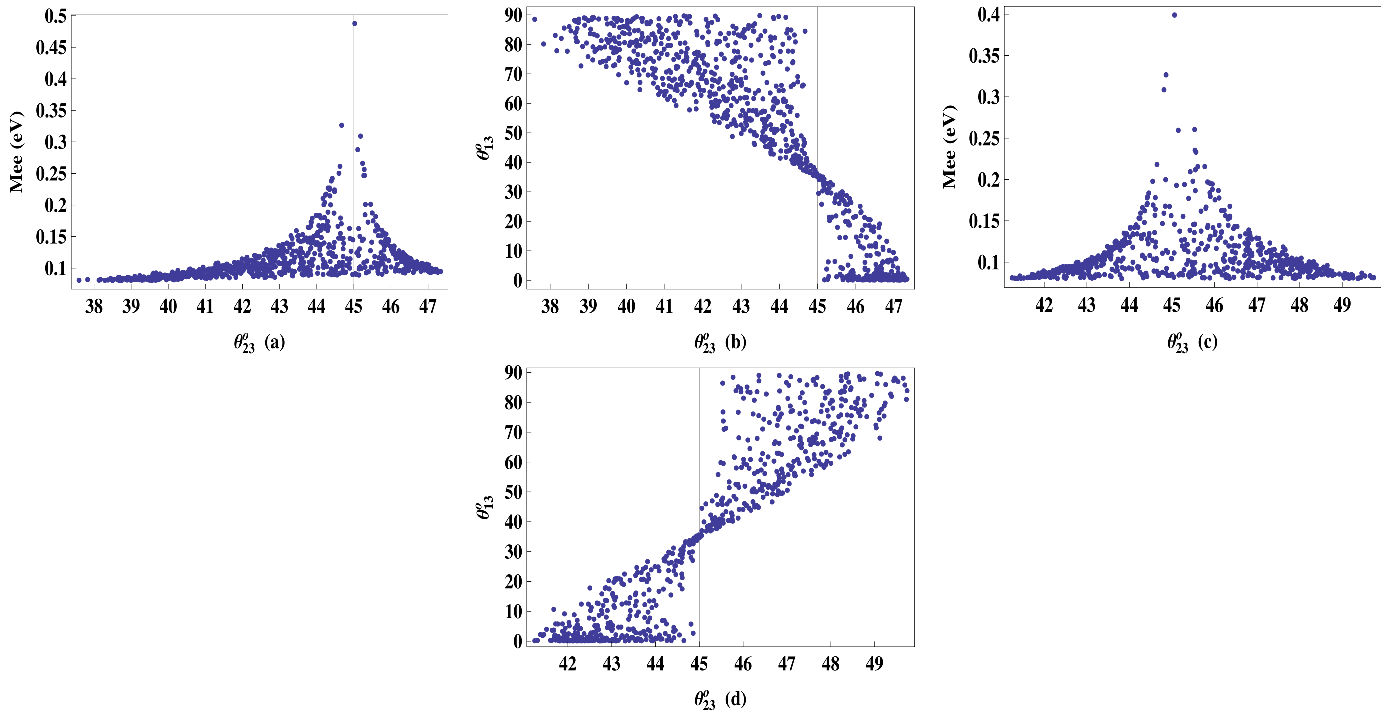

Fig. 2. Correlation plots for class B_6, plots (a), (b) correspond to NS and plots (c), (d) correspond IS.

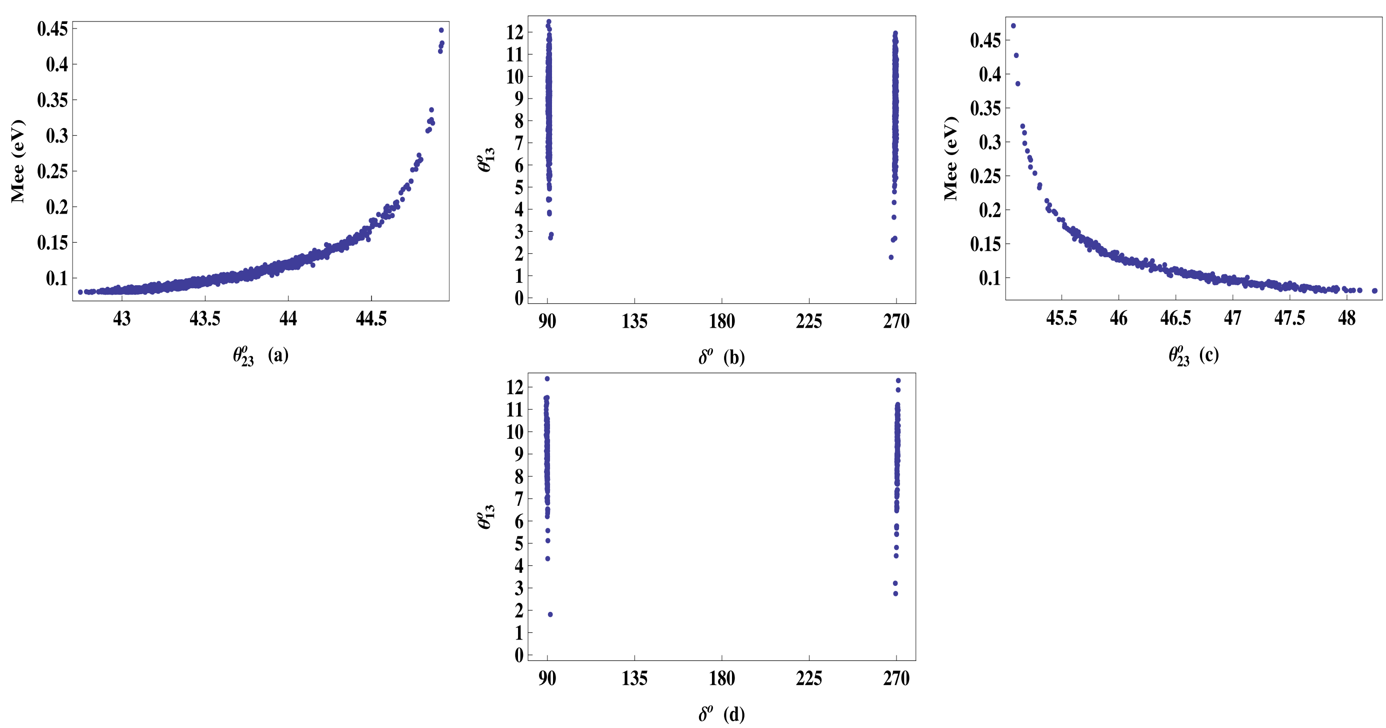

Fig. 3. Correlation plots for class B_5, plots (a), (b) correspond to NS and plots (c), (d) correspond IS.

The second step takes into account the experimental input on the three mixing angles including the recent T2K results on the reactor mixing angle. The results for θ_{23} are plotted in Fig. 3 for class B_5 and Fig. 4 for class B_6. Due to the relatively large value of θ_{13}, the Dirac-type CP violating phase is almost fixed near $\pi/2$ or $3\pi/2$ predicting almost maximal CP violation for these textures. Figs. 5 and 6 show the correlation between the two Majorana-type phases and M_{ee}: the phases α and β approach zero with increasing M_{ee}. As an example, we write the numerically estimated mass matrices for the pattern B_5, the matrices are obtained for the best fit values of Δm^2_{12}, $|\Delta m^2_{23}|$, θ_{12}, θ_{13} given in Eq. (37). For NS we have $M_{ee} = 0.1129$ eV, $\theta_{23} = 43.902°$, $\delta = 269.463°$ and for IS we have $M_{ee} = 0.1272$ eV, $\theta_{23} = 46.058°$, $\delta = 89.853°$

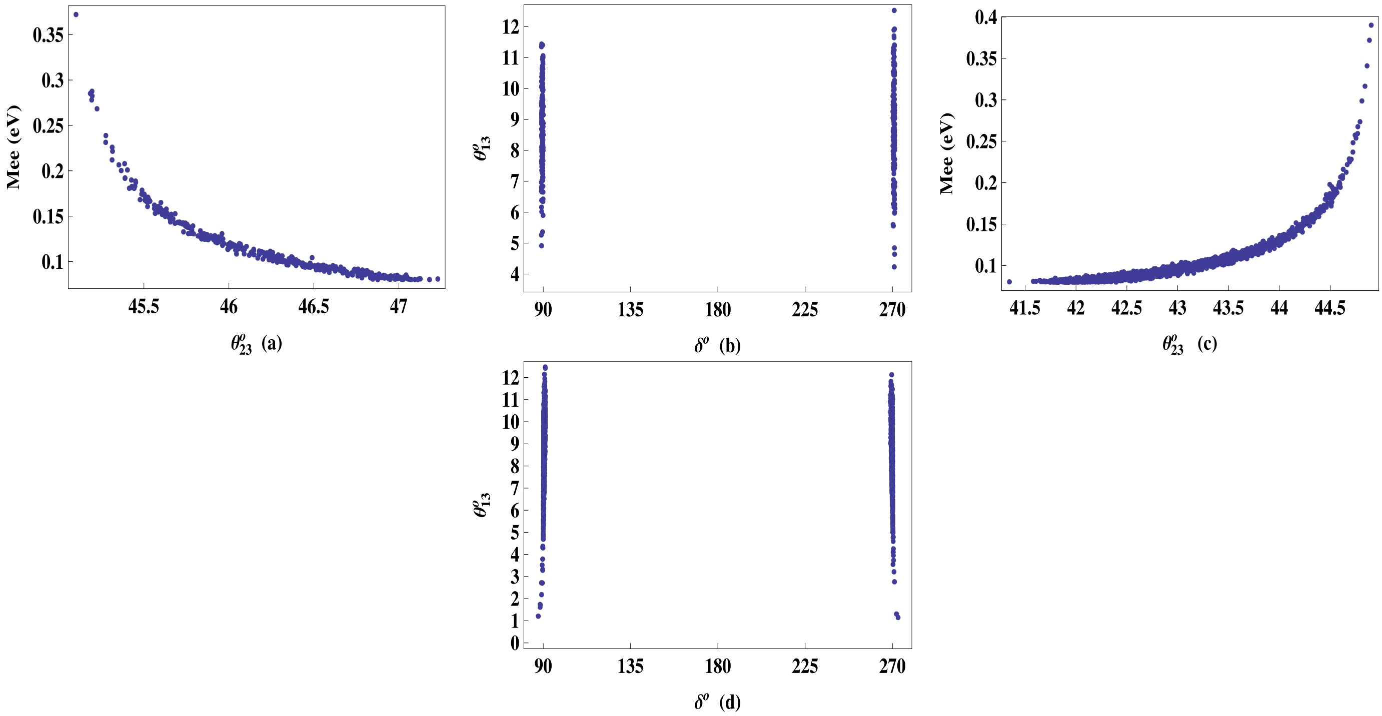

Fig. 4. Correlation plots for class B_6, plots (a), (b) correspond to NS and plots (c), (d) correspond IS.

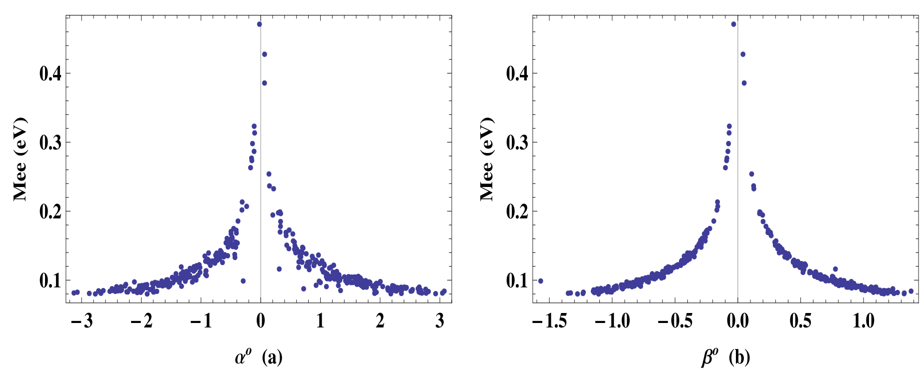

Fig. 5. Correlation plots of Majorana phases with M_{ee} for class B_5 (IS).

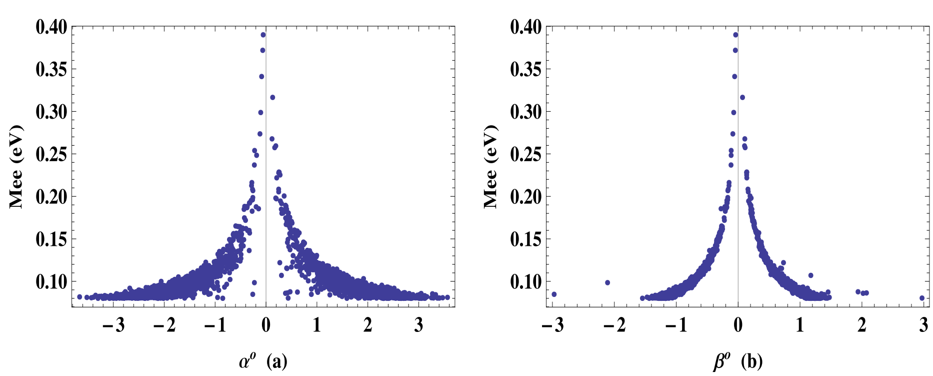

Fig. 6. Correlation plots of Majorana phases with M_{ee} for class B_6 (IS).

$$M_\nu^{B_5}(NS) = \begin{pmatrix} 0.112956 - 0.000905i & 0.000205 - 0.002058i & 0.000018 - 0.000166i \\ 0.000205 - 0.002058i & -0.000037 - 0.000008i & -0.117485 + 0.002109i \\ 0.000018 - 0.000166i & -0.117485 + 0.002109i & -0.009482 + 0.000114i \end{pmatrix}, \tag{39}$$

$$M_\nu^{B_5}(IS) = \begin{pmatrix} 0.127210 - 0.001176i & 0.000147 - 0.002268i & -0.000012 + 0.000177i \\ 0.000147 - 0.002268i & -0.000040 - 0.000006i & -0.122682 + 0.002478i \\ -0.000012 + 0.000177i & -0.122682 + 0.002478i & 0.009575 - 0.000154i \end{pmatrix}. \tag{40}$$

The numerical matrices for pattern B_6 can be obtained from above matrices with the operation of 2–3 permutation symmetry. The assumption of large M_{ee} is testable in the ongoing and forthcoming experiments [23–29] for NDB decay which will either confirm or rule out large M_{ee} in the next few years.

5. Summary

The recent results of the T2K experiment suggest a relatively large reactor mixing angle. Therefore, it is important to look for models naturally accommodating a non-zero value of reactor mixing angle while keeping the atmospheric mixing angle near maximal. In the present work, we studied the implications of classes B_5 and B_6 of two zero minors in M_ν for large effective neutrino mass. In the context of type-I seesaw mechanism, taking M_l and M_D to be diagonal, the zeros of M_R propagate as zero minors of M_ν and the origin of neutrino mixing is solely from M_R. We presented the symmetry realization of these patterns using a cyclic group Z_3. It was found that classes B_5 and B_6 predict a near maximal atmospheric mixing angle in the limit of large M_{ee}. Furthermore, this prediction is independent of the values of the reactor and the solar mixing angles. The assumption of large M_{ee} is testable in the ongoing experiments for NDB decay since the rate of NDB decay is proportional to M_{ee}. These experiments will either confirm or rule out a large value of M_{ee} in the next few years. The atmospheric mixing angle approaches $\pi/4$ with the increasing value of M_{ee}. A reactor mixing angle equal to zero is not allowed in these textures, thus, naturally accommodating a non-zero θ_{13} as suggested by the recent results of the T2K experiment. Due to the relatively large value of θ_{13} the Dirac-type CP violating phase is fixed near $\pi/2$ or $3\pi/2$ predicting almost maximal CP violation for these textures.

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References

[1] M.H. Ahn, et al., K2K Collaboration, Phys. Rev. Lett. 90 (2003) 041801, hep-ex/0212007;
Y. Fukuda, et al., Super-Kamiokande Collaboration, Phys. Rev. Lett. 81 (1998) 1562, hep-ex/9807003;
K. Eguchi, et al., KamLAND Collaboration, Phys. Rev. Lett. 90 (2003) 021802, hep-ex/0212021;
T. Araki, et al., KamLAND Collaboration, Phys. Rev. Lett. 94 (2005) 081801, hep-ex/0406035;
S. Abe, et al., KamLAND Collaboration Phys. Rev. Lett. 100 (2008) 221803, arXiv:0801.4589 [hep-ex];
C. Arpesella, et al., Borexino Collaboration, Phys. Lett. B 658 (2008) 101, arXiv:0708.2251 [astro-ph];
B.T. Cleveland, et al., Astrophys. J. B 496 (1998) 505;
J.N. Abdurashitov, et al., SAGE Collaboration, J. Exp. Theor. Phys. 95 (2002) 181, astro-ph/0204245;
W. Hampel, et al., GALLEX Collaboration, Phys. Lett. B 447 (1999) 127;
For recent reviews see e.g. C.F. King, Rep. Progr. Phys. 67 (2004) 107, hep-ph/0310204;
R.N. Mohapatra, A.Y. Smirnov, Ann. Rev. Nucl. Part. Sci. B 56 (2006) 569, hep-ph/0603118;
M.C. Gonzalez-Garcia, M. Maltoni, Phys. Rep. 460 (2008) 1, arXiv:0704.1800 [hep-ph];
T. Schwetz, M. Tortola, J.W.F. Valle, New J. Phys. 10 (2008) 113011, arXiv:0808.2016 [hep-ph].

[2] K. Abe, et al., T2K Collaboration, Phys. Rev. Lett. 107 (2011) 041801, arXiv:1106.2822 [hep-ex].

[3] Jun-Mou Chen, B. Wang, Xue-Qian Li, Phys. Rev. D 84 (2011) 073002, arXiv:1106.3133 [hep-ph];
Zhi-zhong Xing, arXiv:1106.3244 [hep-ph];
Ya-juan Zheng, Bo-Qiang Ma, arXiv:1106.4040 [hep-ph];
E. Ma, D. Wegman, Phys. Rev. Lett. 107 (2011) 061803, arXiv:1106.4269 [hep-ph];
Xiao-Gang He, A. Zee, Phys. Rev. D 84 (2011) 053004, arXiv:1106.4359 [hep-ph];
S. Zhou, Phys. Lett. B 704 (2011) 291, arXiv:1106.4808 [hep-ph];
T. Araki, Phys. Rev. D 84 (2011) 037301, arXiv:1106.5211 [hep-ph];
N. Haba, R. Takahashi, Phys. Lett. B 702 (2011) 388, arXiv:1106.5926 [hep-ph];
S. Morisi, K.M. Patel, E. Peinado, Phys. Rev. D 84 (2011) 053002, arXiv:1107.0696 [hep-ph];
W. Chao, Ya-juan Zheng, arXiv:1107.0738 [hep-ph];
H. Zhang, S. Zhou, arXiv:1107.1097 [hep-ph];
S. Dev, S. Gupta, R.R. Gautam, Phys. Lett. B 704 (2011) 527, arXiv:1107.1125 [hep-ph];
X. Chu, M. Dhen, T. Hambye, arXiv:1107.1589 [hep-ph];
P.S.B. Dev, R.N. Mohapatra, M. Severson, arXiv:1107.2378 [hep-ph];
R. de A. Toorop, F. Feruglio, C. Hagedron, Phys. Lett. B 703 (2011) 447, arXiv:1107.3486 [hep-ph];
S. Antusch, V. Maurer, arXiv:1107.3728 [hep-ph];
W. Rodejohann, H. Zhang, S. Zhou, arXiv:1107.3970 [hep-ph];
Y.H. Ahn, H.Y. Cheng, S. Oh, arXiv:1107.4549 [hep-ph];
S.F. King, C. Luhn, JHEP 1109 (2011) 042, arXiv:1107.5332 [hep-ph];
D. Marzocca, S.T. Petcov, A. Romanino, M. Spinarth, arXiv:1108.0614 [hep-ph];
S.F. Ge, D.A. Dicus, W.W. Repko, arXiv:1108.0964 [hep-ph];
S. Kumar, Phys. Rev. D 84 (2011) 077301, arXiv:1108.2137 [hep-ph];
F. Bazzocchi, arXiv:1108.2497 [hep-ph];
T. Araki, C.Q. Geng, JHEP 1109 (2011) 139, arXiv:1108.3175 [hep-ph];
S. Antusch, S.F. King, C. Luhn, M. Spinarth, arXiv:1108.4278 [hep-ph];
H. Fritzsch, Zhi-zhong Xing, S. Zhou, JHEP 1109 (2011) 083, arXiv:1108.4534 [hep-ph];

A. Rashed, A. Datta, arXiv:1109.2320 [hep-ph];
P.O. Ludl, S. Morisi, E. Peinado, arXiv:1109.3393 [hep-ph];
S. Verma, Nucl. Phys. B 854 (2012) 340, arXiv:1109.4228 [hep-ph].
[4] C.I. Low, R.R. Volkas, Phys. Rev. D 68 (2003) 033007, hep-ph/0305243;
C.S. Lam, Phys. Rev. D 74 (2006) 113004, hep-ph/0611017.
[5] P.F. Harrison, D.H. Perkins, W.G. Scott, Phys. Lett. B 530 (2002) 167, hep-ph/0202074;
P.F. Harrison, W.G. Scott, Phys. Lett. B 535 (2002) 163, hep-ph/0203209;
Zhi-zhong Xing, Phys. Lett. B 533 (2002) 85, hep-ph/0204049.
[6] Paul H. Frampton, Sheldon L. Glashow, Danny Marfatia, Phys. Lett. B 536 (2002) 79, hep-ph/0201008;
Zhi-zhong Xing, Phys. Lett. B 530 (2002) 159, hep-ph/0201151;
Bipin R. Desai, D.P. Roy, Alexander R. Vaucher, Mod. Phys. Lett. A 18 (2003) 1355, hep-ph/0209035;
S. Dev, Sanjeev Kumar, Surender Verma, Shivani Gupta, Nucl. Phys. B 784 (2007) 103, hep-ph/0611313;
S. Dev, Sanjeev Kumar, Surender Verma, Shivani Gupta, Phys. Rev. D 76 (2007) 013002, hep-ph/0612102.
[7] L. Lavoura, Phys. Lett. B 609 (2005) 317, hep-ph/0411232;
E.I. Lashin, N. Chamoun, Phys. Rev. D 78 (2008) 073002, arXiv:0708.2423 [hep-ph].
[8] E.I. Lashin, N. Chamoun, Phys. Rev. D 80 (2009) 093004, arXiv:0909.2669 [hep-ph];
S. Dev, Surender Verma, Shivani Gupta, R.R. Gautam, Phys. Rev. D 81 (2010) 053010, arXiv:1003.1006 [hep-ph];
S. Dev, Shivani Gupta, R.R. Gautam, Mod. Phys. Lett. A 26 (2011) 501, arXiv:1011.5587 [hep-ph].
[9] S. Kaneko, H. Sawanaka, M. Tanimoto, JHEP 0508 (2005) 073, hep-ph/0504074;
S. Dev, Surender Verma, Shivani Gupta, Phys. Lett. B 687 (2010) 53, arXiv:0909.3182 [hep-ph];
S. Dev, Shivani Gupta, R.R. Gautam, Phys. Rev. D 82 (2010) 073015, arXiv:1009.5501 [hep-ph].
[10] W. Grimus, P.O. Ludl, Phys. Lett. B 700 (2011) 356, arXiv:1104.4340 [hep-ph].
[11] P. Minkowski, Phys. Lett. B 67 (1977) 421;
T. Yanagida, in: O. Sawada, A. Sugamoto (Eds.), Proceedings of the Workshop on the Unified Theory and the Baryon Number in the Universe, KEK, Tsukuba, Japan, 1979, p. 95;
M. Gell-Mann, P. Ramond, R. Slansky, in: P. Van Nieuwenhuizen, D.Z. Freedman (Eds.), Complex Spinors and Unified Theories Supergravity, North-Holland, Amsterdam, 1979, p. 315;
R.N. Mohapatra, G. Senjanovic, Phys. Rev. Lett. 44 (1980) 912.
[12] W. Grimus, A.S. Joshipura, L. Lavoura, M. Tanimoto, Eur. Phys. J. C 36 (2004) 227, hep-ph/0405016.
[13] S. Dev, Shivani Gupta, R.R. Gautam, Phys. Lett. B 701 (2011) 605, arXiv:1106.3451 [hep-ph].
[14] G.L. Fogli, et al., Prog. Part. Nucl. Phys. 57 (2006) 742, hep-ph/0506083.
[15] B. Pontecorvo, Zh. Eksp. Teor. Fiz. (JETP) 33 (1957) 549;
B. Pontecorvo, Zh. Eksp. Teor. Fiz. (JETP) 34 (1958) 247;
B. Pontecorvo, Zh. Eksp. Teor. Fiz. (JETP) 53 (1967) 1717;
Z. Maki, M. Nakagawa, S. Sakata, Progr. Theoret. Phys. 28 (1962) 870.
[16] C. Jarlskog, Phys. Rev. Lett. 55 (1985) 1039.
[17] H. Fritzsch, Zhi-zhong Xing, S. Zhou, JHEP 1109 (2011) 083, arXiv:1108.4534 [hep-ph].
[18] G.L. Fogli, E. Lisi, A. Marrone, A. Palazzo, A.M. Rotunno, Phys. Rev. D 84 (2011) 53007, arXiv:1106.6028 [hep-ph].
[19] F.T. Avignone III, S.R. Elliott, J. Engel, Rev. Mod. Phys. 80 (2008) 481, arXiv:0708.1033 [nucl-ex];
S.M. Bilenky, Phys. Part. Nucl. 41 (2010) 690, arXiv:1001.1946 [hep-ph];
J.J. Gomez-Cadenas, J. Martin-Albo, M. Mezzetto, F. Monrabal, M. Sorel, arXiv:1109.5515 [hep-ex].
[20] W. Rodejohann, arXiv:1106.1334 [hep-ph].
[21] H.V. Klapdor- Kleingrothaus, A. Dietz, H.L. Harney, I.V. Krivosheina, Mod. Phys. Lett. A 16 (2001) 2409, hep-ph/0202018.
[22] F. Feruglio, A. Strumia, F. Vissani, Nucl. Phys. B 637 (2002) 345, hep-ph/0201291;
C.E. Aalseth, et al., Mod. Phys. Lett. A 17 (2002) 1475, hep-ph/0202018.
[23] C. Arnaboldi, et al., CUORICINO Collaboration, Phys. Lett. B 584 (2004) 260.
[24] C. Arnaboldi, et al., Nucl. Instrum. Methods Phys. Res. A 518 (2004) 775.
[25] I. Abt, et al., GERDA Collaboration, hep-ex/0404039.
[26] R. Gaitskell, et al., Majorana Collaboration, hep-ex/0311013.
[27] A.S. Barabash, NEMO Collaboration, Czech. J. Phys. 52 (2002) 567, nucl-ex/0203001.
[28] M. Danilov, et al., Phys. Lett. B 480 (2000) 12, hep-ex/0002003.
[29] H.V. Klapdor- Kleingrothaus, et al., Eur. Phys. J. A 12 (2001) 147, hep-ph/0103062.