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Hamiltonian function selection principle for generalized Hamiltonian modelling

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Abstract

Hamiltonian function is different, the internal structure relation given by generalized Hamiltonian model is different, and thus Hamiltonian function is core issue for generalized Hamiltonian modeling. In this paper, three principles of selecting Hamiltonian function are proposed, interface principle, energy principle and stability principle. For the class of system with single input and single output, applied methods of three principles are given. At last, the Hamiltonian modelling for nonlinear hydro turbine is taken as case to introduce its application.

Keywords: generalized Hamiltonian modeling; Hamiltonian function; interface principle; energy principle; stability principle

1. Introductions

The generalized Hamiltonian model, in contrast traditional differential equations model, their differential part is equivalent, but Hamiltonian function, energy flow and inner relative mechanism reflected by system matrix provide more dynamic information about object system. In recently, the generalized Hamiltonian theory could be used to describe practical system with energy dissipation in inner and energy exchanging with outside environment in [1,2].

However, in modelling theory for generalized Hamiltonian, the direct method is that Hamiltonian model is derived in mathematic deduce from basic dynamic characteristics of practical system. But, it is

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difficult and even impossible for most actual system, so the generalized Hamiltonian realization theory is developed, which transforms the differential equation into Hamiltonian model. Hamiltonian realization methods for many types physical system are developed, such as linear system [3], nonlinear system [4,5], time varying and time invariant [6,7], discrete system [8,9] and differential algebraic system [10,11].

In generalized Hamiltonian realization theory, the core issue is how to determine Hamiltonian function. Hamiltonian function is different, the structure and damping matrix of Hamiltonian model are different, accordingly, internal relation mechanisms given by them are different. If generalized Hamiltonian model doesn't correctly reflect system internal relation dynamic mechanism, it will lose all advantages relative to traditional differential equation model.

The main work of this paper is that proposes Hamiltonian function selection principles aiming at a class of Hamiltonian realization method, and takes Hamiltonian modelling of hydro turbine as case to introduce its application.

2. Generalized Hamiltonian realization basic theory

Consider the follow nonlinear system:

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}) + \boldsymbol{g}(\boldsymbol{x})u \tag{1}$$

Where $\boldsymbol{x} \in \boldsymbol{R}^n$ is the state vectors, u is the control action, $\boldsymbol{g}(\boldsymbol{x})$ is the input matrix.

If there are a proper coordinate card and a Hamiltonian $H(\boldsymbol{x})$, and makes system (1) to express as (2):

$$\begin{cases} \dot{\boldsymbol{x}} = \boldsymbol{T}(\boldsymbol{x})\dfrac{\partial H}{\partial \boldsymbol{x}} + \boldsymbol{g}(\boldsymbol{x})u \\ y = \boldsymbol{g}^T(\boldsymbol{x})\dfrac{\partial H}{\partial \boldsymbol{x}} \end{cases} \tag{2}$$

Then, system (1) has a Hamiltonian realization, $\boldsymbol{T}(\boldsymbol{x})$ is the system matrix, $\boldsymbol{y}$ is the nature output of Hamiltonian system.

Know very well, a matrix with constant rank can be uniquely decomposed as follow:

$$\boldsymbol{T}(\boldsymbol{x}) = \boldsymbol{J}(\boldsymbol{x}) + \boldsymbol{P}(\boldsymbol{x}) = \boldsymbol{J}(\boldsymbol{x}) + \boldsymbol{S}(\boldsymbol{x}) - \boldsymbol{R}(\boldsymbol{x}) \tag{3}$$

Where $\boldsymbol{J}(\boldsymbol{x})$ is anti-symmetric matrix, the natural damping matrix $\boldsymbol{R}(\boldsymbol{x})$ and $\boldsymbol{S}(\boldsymbol{x})$ are symmetric and semi positive definite matrix.

Then generalized Hamiltonian system (2) can be expressed as follow:

$$\begin{cases} \dot{\boldsymbol{x}} = [\boldsymbol{J}(\boldsymbol{x}) + \boldsymbol{S}(\boldsymbol{x}) - \boldsymbol{R}(\boldsymbol{x})]\dfrac{\partial H}{\partial \boldsymbol{x}} + \boldsymbol{g}(\boldsymbol{x})u \\ y = \boldsymbol{g}^T(\boldsymbol{x})\dfrac{\partial H}{\partial \boldsymbol{x}} \end{cases} \tag{4}$$

The Hamiltonian system energy change is:

$$\frac{\mathrm{d}H}{\mathrm{d}t} = -\frac{\partial^T H}{\partial \boldsymbol{x}}\boldsymbol{R}(\boldsymbol{x})\frac{\partial H}{\partial \boldsymbol{x}} + \frac{\partial^T H}{\partial \boldsymbol{x}}\boldsymbol{S}(\boldsymbol{x})\frac{\partial H}{\partial \boldsymbol{x}} + u^T y \tag{5}$$

The first item is the internal energy dissipation, the second item is the internal generate energy, the third item is the external supply energy. If $\boldsymbol{S}(\boldsymbol{x})=0$, then system (4) is called as the standard form of the port-controlled dissipation Hamiltonian model.

3. Hamiltonian function selection principles

3.1. Interface principle

For complex physical system modelling, the most frequently-used method is that decomposed it into multi subsystem to model respectively, and then connects to compose whole system model according to their physical connection and signal flow. Therefore, the input and output connection relation between the object subsystem and other subsystems should be considered while builds the subsystem Hamiltonian model.

Interface Principle： The nature output of Hamiltonian system should be similar to that of physical system in form, and then conveniently connects with other subsystem.

The complex system and multi input system can be usually decomposed into multi single input and single output subsystem. Thus, this paper only considers single input and single output system.

The nature output of generalized Hamiltonian system is:

$$y(\boldsymbol{x}) = \boldsymbol{g}^T(\boldsymbol{x})\frac{\partial H}{\partial \boldsymbol{x}} \tag{6}$$

The input channel matrix $\boldsymbol{g}(\boldsymbol{x})$ is column vector in single input system.

Theorem 1： If there is only a non-zero element in $\boldsymbol{g}(\boldsymbol{x})$, then Hamiltonian function reference form can be obtained from follow equation (7).

$$H(\boldsymbol{x}) = C\int \frac{y(\boldsymbol{x})}{g(x_i)}dx_i \tag{7}$$

The constant C is the difference between Hamiltonian system nature output and physical system output, which is used to ensure the positive definite.

Proof： Assumption the $g(x_i)$ is the non-zero element of column vector $\boldsymbol{g}(\boldsymbol{x})$, then (6) can be written as:

$$y(\boldsymbol{x}) = g(x_i)\frac{\partial H}{\partial x_i} \quad \rightarrow \quad \frac{\partial H}{\partial x_i} = \frac{y(\boldsymbol{x})}{g(x_i)}$$

Integral above equation about x_i, Hamiltonian function is derived. And multiply constant C to correct the Hamiltonian function, and ensure its positive definite, then (7) is derived.

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The Hamiltonian function derived in this way usually doesn't have energy dimension, thus is called as generalized energy.

3.2. Energy principle

Energy Principle： The Hamiltonian energy function should reflect constitute of system energy, and Hamiltonian energy flow should be accord with that of practical system.

The constitute of energy flow depend on $\boldsymbol{S}(\boldsymbol{x})$和 $\boldsymbol{R}(\boldsymbol{x})$ from (5), so decomposed $\boldsymbol{P}(\boldsymbol{x})$ is the key step while apply this principle.

In equation (3), while $\boldsymbol{P}(\boldsymbol{x})$ is decomposed into $\boldsymbol{R}(\boldsymbol{x})$ and $\boldsymbol{S}(\boldsymbol{x})$, the $\boldsymbol{R}(\mathbf{x})$ satisfies semi positive definite condition, that is $\boldsymbol{R}(\boldsymbol{x})\geq 0$. In this condition, system energy flow is:

$$\frac{\mathrm{d}H}{\mathrm{d}t} = -\frac{\partial^T H}{\partial \boldsymbol{x}}\boldsymbol{R}(\boldsymbol{x})\frac{\partial H}{\partial \boldsymbol{x}} + \frac{\partial^T H}{\partial \boldsymbol{x}}\boldsymbol{S}(\boldsymbol{x})\frac{\partial H}{\partial \boldsymbol{x}} + u^T y \leq \frac{\partial^T H}{\partial \boldsymbol{x}}\boldsymbol{S}(\boldsymbol{x})\frac{\partial^T H}{\partial \boldsymbol{x}} + u^T y \tag{8}$$

If the $\boldsymbol{S}(\boldsymbol{x})$ can be cancelled by using feedback equivalent method, then system become the dissipation system. So follow theorem 2 is given.

Theorem 2： If follow equivalent transformation exists:

$$\boldsymbol{S}(\boldsymbol{x})\frac{\partial H}{\partial \boldsymbol{x}} = \boldsymbol{J}'(\boldsymbol{x})\frac{\partial H}{\partial \boldsymbol{x}} + \boldsymbol{g}(\boldsymbol{x})u' \tag{9}$$

Then system (1) is dissipation, equation (4) can be transformed into generalized Hamiltonian model, its standard form is follow:

$$\begin{cases} \dot{\boldsymbol{x}} = [\bar{\boldsymbol{J}}(\boldsymbol{x}) - \boldsymbol{R}(\boldsymbol{x})]\dfrac{\partial H}{\partial \boldsymbol{x}} + \boldsymbol{g}(\boldsymbol{x})\bar{u} \\ y = \boldsymbol{g}^T(\boldsymbol{x})\dfrac{\partial H}{\partial \boldsymbol{x}} \end{cases} \tag{10}$$

Where $\bar{\boldsymbol{J}}(\boldsymbol{x}) = \boldsymbol{J}(\boldsymbol{x}) + \boldsymbol{J}'(\boldsymbol{x})$, $\bar{u} = u + u'$, $\boldsymbol{J}'(\boldsymbol{x})$ is anti-symmetric matrix, u' is equivalent control.

Proof: Substituting (9) into (8), system energy flow is:

$$\frac{\mathrm{d}H}{\mathrm{d}t} \le \frac{\partial^T H}{\partial \boldsymbol{x}}\boldsymbol{J}'(\boldsymbol{x})\frac{\partial H}{\partial \boldsymbol{x}} + \frac{\partial^T H}{\partial \boldsymbol{x}}\boldsymbol{g}(\boldsymbol{x})u' + u^T y = Cy^T u' + u^T y = (Cu' + u)y$$

That is, system energy change is less than input energy on its port, so system is dissipation.

Substituting (9) into (4) can derive (10).

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According to above analysis, follow remarks can be obtained:

Remark 1: From energy flow expression (8), decomposing $\boldsymbol{P}(\boldsymbol{x})$ should consider that items related with energy dissipation are classified into $\boldsymbol{R}(\boldsymbol{x})$, and ensure that $\boldsymbol{R}(x)$ is semi positive definite.

Remark 2: Equation (9) multiplies $\dfrac{\partial^T H}{\partial \boldsymbol{x}}$ in left, that is:

$$\frac{\partial^T H}{\partial \boldsymbol{x}}\boldsymbol{S}(\boldsymbol{x})\frac{\partial H}{\partial \boldsymbol{x}} = \frac{\partial^T H}{\partial \boldsymbol{x}}\boldsymbol{g}(\boldsymbol{x})u' = \frac{\partial H}{\partial x_i}g(x_i)u' = Cyu'$$

If system output $\dfrac{\partial H}{\partial x_i}g(x_i) \ne 0$, then the equivalent control u' can be derived as follow:

$$u' = \frac{1}{C}\frac{1}{y}\frac{\partial^T H}{\partial \boldsymbol{x}}\boldsymbol{S}(\boldsymbol{x})\frac{\partial H}{\partial \boldsymbol{x}} \tag{11}$$

Above (11) shows that internal generating energy is transformed into equivalent input control. This just is essence of Hamiltonian dissipation realization.

New control is:

$$\bar{u} = u + u' \tag{12}$$

Remark 3: Theorem 2 gives a decomposed method of $\boldsymbol{S}(\boldsymbol{x})$, in which the equivalent control is given by (11), and anti-symmetric matrix $\boldsymbol{J}'(\boldsymbol{x})$ is given by follow (13).

$$\boldsymbol{J}'(\boldsymbol{x}) = \frac{1}{\dfrac{\partial H}{\partial \boldsymbol{x}}\boldsymbol{g}^T(\boldsymbol{x})}\left\{\boldsymbol{S}(\boldsymbol{x})\frac{\partial H}{\partial \boldsymbol{x}}\boldsymbol{g}^T(\boldsymbol{x}) - \boldsymbol{g}(\boldsymbol{x})\frac{\partial^T H}{\partial \boldsymbol{x}}\boldsymbol{S}(\boldsymbol{x})\right\} \tag{13}$$

Proof: Equation (9) multiplies $\boldsymbol{g}^T(\boldsymbol{x})$ in right, and applies (8) can obtain follow relation:

$$\begin{aligned} \boldsymbol{J}'(\boldsymbol{x})\frac{\partial H}{\partial \boldsymbol{x}}\boldsymbol{g}^T(\boldsymbol{x}) &= \boldsymbol{S}(\boldsymbol{x})\frac{\partial H}{\partial \boldsymbol{x}}\boldsymbol{g}^T(\boldsymbol{x}) - \boldsymbol{g}(\boldsymbol{x})\frac{1}{C}\frac{1}{y}\frac{\partial^T H}{\partial \boldsymbol{x}}\boldsymbol{S}(\boldsymbol{x})\frac{\partial H}{\partial \boldsymbol{x}}\boldsymbol{g}^T(\boldsymbol{x}) \\ &= \boldsymbol{S}(\boldsymbol{x})\frac{\partial H}{\partial \boldsymbol{x}}\boldsymbol{g}^T(\boldsymbol{x}) - \boldsymbol{g}(\boldsymbol{x})\frac{\partial^T H}{\partial \boldsymbol{x}}\boldsymbol{S}(\boldsymbol{x}) \end{aligned}$$

Form above, $\boldsymbol{J}'(\boldsymbol{x})$ can be derived as (13).

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Following above energy principle decomposed, system energy flows are accord with that of physical in the generalized energy conception.

3.3. Stability principle

One of main application for generalized Hamiltonian theory is energy-based Lyapunov function method. In this application, Hamiltonian function usually needs to be modified, and makes it satisfy the positive definite of Hessian matrix, that is:

$$\frac{\partial^2 H}{\partial \boldsymbol{x}^2}(\boldsymbol{x}_*) - \frac{\partial^2 H_a}{\partial \boldsymbol{x}^2}(\boldsymbol{x}_*) > 0 \tag{14}$$

Where $H_a(\boldsymbol{x})$ is the additional energy function, that is usually injected energy by control, $H_d(\boldsymbol{x})$ is the modified energy function, $H_d(\boldsymbol{x}) = H(\boldsymbol{x}) + H_a(\boldsymbol{x})$, $\boldsymbol{x}_*$ is an equilibrium point.

The essence of Hessian matrix positive definite at given equilibrium can be explained as:

At given equilibrium point, first order partial derivative of Hamiltonian function $H_d(\boldsymbol{x})$ is zero, $\frac{\partial H_d}{\partial \boldsymbol{x}}(\boldsymbol{x}_*) = 0$. And if second order partial derivative is more than zero, $\frac{\partial^2 H_d}{\partial \boldsymbol{x}^2}(\boldsymbol{x}_*) > 0$, then $H_d(\boldsymbol{x})$ is the minimum at given equilibrium point. In this case, Hamiltonian function $H_d(\boldsymbol{x})$ is a Lyapunov function, so system is asymptotic stability.

From view of stability, determined Hamiltonian function by proper modifying should satisfy Hessian matrix positive definite condition at finally. Therefore, this condition can be used to judge rationality of selected $H(\boldsymbol{x})$. A weak condition is given to avoid from possibility difficult in later stability analysis as follow.

Stability Principle: At equilibrium point $\boldsymbol{x}_*$, the Hessian matrix of Hamiltonian function satisfies semi positive definite condition, that is：

$$\frac{\partial^2 H}{\partial \boldsymbol{x}^2}(\boldsymbol{x}_*) \geq 0 \tag{15}$$

If the stability condition doesn't satisfy, then Hamiltonian function need to be proper modified.

4. Example

4.1. Hydro turbine model with rigid water column

The hydro turbine model with rigid water column is taken as case to introduce application of above principles.

Hydro turbine differential equation model is:

$$\begin{cases} \dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}) + \boldsymbol{g}(\boldsymbol{x})u \\ y = A_t \dfrac{x_1^2}{x_2^2}(x_1 - q_{nl}) \end{cases} \tag{16}$$

$$\boldsymbol{f}(\boldsymbol{x}) = \begin{bmatrix} \dfrac{1}{T_w}\left(1 - f_p x_1^2 - \dfrac{x_1^2}{x_2^2}\right) \\ -\dfrac{1}{T_y}(x_2 - G_0) \end{bmatrix}, \quad \boldsymbol{g}(\boldsymbol{x}) = \begin{bmatrix} 0 \\ \dfrac{1}{T_y} \end{bmatrix}$$

Where $x_1=q$，$x_2=G$，q is the flow in per unit, q_{nl} is the no-load flow in per unit, G is the guide vane opening in per unit, G_0 is the initial of guide vane opening in per unit, T_w is water start time constant in second, f_p is the head loss coefficient, A_t is the gain of hydro turbine, T_y is the main servomotor time constant in second.

Follow expressions are basic equations of hydro turbine:

$$h = \frac{q^2}{G^2} \tag{17}$$

$$p_{\mathrm{m}} = A_{\mathrm{t}} h(q - q_{\mathrm{nl}}) \tag{18}$$

Where h is the head of hydro turbine in per unit, p_{m} is output power of hydro turbine in per unit.

The nature output of system is the output power of hydro turbine, that is $y=p_{\mathrm{m}}$.

4.2. Hamiltonian function selection

The input matrix $\boldsymbol{g}(\boldsymbol{x})$ satisfies condition of theorem 1, according to interface principle, Hamiltonian function is selected from (7) as follow:

$$H(\boldsymbol{x}) = C\int \frac{y(\boldsymbol{x})}{g(x_2)} dx_2 = -CT_y A_{\mathrm{t}} \frac{x_1^2}{x_2}(x_1 - q_{\mathrm{nl}}) \tag{19}$$

Let $C=-1$, then Hamiltonian function is:

$$H(\boldsymbol{x}) = T_y A_{\mathrm{t}} \frac{x_1^2}{x_2}(x_1 - q_{\mathrm{nl}}) \tag{20}$$

When the hydro turbine generating units connects with power system to operate, the flow x_1 is more than no-load flow q_{nl}, so Hamiltonian function is positive definite.

The output equation is:

$$y = \boldsymbol{g}(\boldsymbol{x})^T \frac{\partial H}{\partial \boldsymbol{x}} = -p_{\mathrm{m}} \tag{21}$$

The difference between nature output of Hamiltonian system y and actual output of physical system p_{m} is just a negative sign. One of generator input is the torque of hydro turbine m_{t}, $m_{\mathrm{t}}=-y/\omega$, ω is the angular speed of generator in per unit.

In traditional third order generator model, ω is approximately selected as 1, that is $\omega \approx 1$. It is clear that hydro turbine Hamiltonian model can expediently connect with generator Hamiltonian model.

Next, according to stability principle verifies the Hamiltonian function.

The Hessian matrix of Hamiltonian function is:

$$\frac{\partial^2 H}{\partial \boldsymbol{x}^2} = \begin{bmatrix} T_y A_t \dfrac{6x_1 - 2q_{nl}}{x_2} & -T_y A_t \dfrac{3x_1^2 - 2x_1 q_{nl}}{x_2^2} \\ -T_y A_t \dfrac{3x_1^2 - 2x_1 q_{nl}}{x_2^2} & 2T_y A_t \dfrac{x_1^2}{x_2^3}(x_1 - q_{nl}) \end{bmatrix} \tag{22}$$

Direct calculation above (22) can derive that the positive definite of Hessian matrix is $\boldsymbol{x}_1>4q_{\mathrm{nl}}/3$. Thus, selected Hamiltonian function accords with stability principle.

4.3. Orthogonal decomposition

In operating hydro turbine satisfies $\boldsymbol{x} \neq \boldsymbol{0}$. At any $\boldsymbol{x} \neq \boldsymbol{0}$, $\boldsymbol{f}(\boldsymbol{x})$ can be decomposed as follow.

$$\boldsymbol{f_{td}}(\boldsymbol{x}) = \boldsymbol{f}(\boldsymbol{x}) - \frac{\langle \boldsymbol{f}(\boldsymbol{x}), \nabla H \rangle}{\|\nabla H\|^2} \frac{\partial H}{\partial \boldsymbol{x}} = \begin{bmatrix} f_1 - Z\nabla_{x_1} H \\ f_2 - Z\nabla_{x_2} H \end{bmatrix} \tag{23}$$

Where $Z = \dfrac{\langle \boldsymbol{f}(\boldsymbol{x}), \nabla H \rangle}{\|\nabla H\|^2}$, $\langle \cdot,\cdot \rangle$ is the inner product operation, $\|\nabla H\|^2 = \left(\dfrac{\partial H}{\partial x_1}\right)^2 + \left(\dfrac{\partial H}{\partial x_2}\right)^2$.

Thus system (16) can be transformed as follow:

$$\dot{\boldsymbol{x}} = [\boldsymbol{J}(\boldsymbol{x}) + \boldsymbol{P}(\boldsymbol{x})]\frac{\partial H}{\partial \boldsymbol{x}} + \boldsymbol{g}(\boldsymbol{x})u \tag{24}$$

$$\boldsymbol{J}(\boldsymbol{x}) = \frac{1}{\|\nabla H\|^2}\left[\boldsymbol{f}_{td}(\boldsymbol{x})\nabla H^T - \nabla H \boldsymbol{f}_{td}{}^T(\boldsymbol{x})\right] = \frac{1}{\|\nabla H\|^2}\begin{bmatrix} 0 & f_1\nabla_{x2}H - f_2\nabla_{x1}H \\ -f_1\nabla_{x2}H + f_2\nabla_{x1}H & 0 \end{bmatrix} \tag{25}$$

$$\boldsymbol{P}(\boldsymbol{x}) = \frac{\langle \boldsymbol{f}(\boldsymbol{x}), \nabla H\rangle}{\|\nabla H\|^2}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} Z & 0 \\ 0 & Z \end{bmatrix} \tag{26}$$

4.4. Dissipative decomposition

$$Z = \frac{\langle \boldsymbol{f}(\boldsymbol{x}), \nabla H\rangle}{\|\nabla H\|^2} = \frac{\{f_1 \frac{\partial H}{\partial x_1} + f_2 \frac{\partial H}{\partial x_2}\}}{\|\nabla H\|^2} = \frac{1}{\|\nabla H\|^2}\{f_1 \frac{\partial H}{\partial x_1} + x_2 A_{\mathrm{t}} \frac{x_1^2}{x_2^2} x_1 - x_2 A_{\mathrm{t}} \frac{x_1^2}{x_2^2} q_{\mathrm{nl}} - G_0 A_{\mathrm{t}} \frac{x_1^2}{x_2^2}(x_1 - q_{\mathrm{nl}})\}$$

Selected:

$$r(\boldsymbol{x}) = \frac{1}{\|\nabla H\|^2}\{x_2 A_{\mathrm{t}} \frac{x_1^2}{x_2^2} q_{nl} + G_0 A_{\mathrm{t}} \frac{x_1^2}{x_2^2}(x_1 - q_{\mathrm{nl}})\} \tag{27}$$

$$s(\boldsymbol{x}) = \frac{1}{\|\nabla H\|^2}\{f_1 \frac{\partial H}{\partial x_1} + x_2 A_t \frac{x_1^2}{x_2^2} x_1\} \tag{28}$$

Then:

$$\boldsymbol{R}(\boldsymbol{x}) = \begin{bmatrix} r(\boldsymbol{x}) & 0 \\ 0 & r(\boldsymbol{x}) \end{bmatrix} \tag{29}$$

$$\boldsymbol{S}(\boldsymbol{x}) = \begin{bmatrix} s(\boldsymbol{x}) & 0 \\ 0 & s(\boldsymbol{x}) \end{bmatrix} \tag{30}$$

Energy dissipation:

$$\frac{\partial^T H}{\partial \boldsymbol{x}} \boldsymbol{R}(\boldsymbol{x}) \frac{\partial H}{\partial \boldsymbol{x}} = x_2 (A_{\mathrm{t}} h q_{nl} + \frac{G_0}{x_2} p_m) \tag{31}$$

$A_{\mathrm{t}} h q_{\mathrm{nl}}$ is no-load energy loss, the output of hydro turbine is taken as energy dissipation.
Internal generate energy:

$$\frac{\partial^T H}{\partial \boldsymbol{x}} \boldsymbol{S}(\boldsymbol{x}) \frac{\partial H}{\partial \boldsymbol{x}} = \frac{dx_1}{dt}\frac{\partial H}{\partial x_1} + x_2 A_{\mathrm{t}} h q \tag{32}$$

From view of system structure described in this paper, the hydraulic system is internal interface, it supplied energy $A_{\mathrm{t}}hq$ is internal energy as second item. The first item is inertia energy caused by flow changing.

Because of existing convert relation of T_y and A_{t}, above energy can explain as generalized energy. Thus, the Hamiltonian system energy flow accords with that of actual system in generalized energy.

4.5. Dissipative realization

From (11) and (12), the new control law is:

$$v = u + u' = u + \frac{1}{y}\frac{\partial^T H}{\partial \boldsymbol{x}} \boldsymbol{S}(\boldsymbol{x}) \frac{\partial H}{\partial \boldsymbol{x}} = u + \frac{1}{p_{\mathrm{m}}} s(\boldsymbol{x}) \|\nabla H\|^2 \tag{33}$$

Combining (12) and (25), the anti-symmetry $\boldsymbol{J}(\boldsymbol{x})$ is:

$$\bar{\boldsymbol{J}}(\boldsymbol{x}) = \boldsymbol{J}(\boldsymbol{x}) + \boldsymbol{J}'(\boldsymbol{x}) = \begin{bmatrix} 0 & C_T(\boldsymbol{x}) \\ -C_T(\boldsymbol{x}) & 0 \end{bmatrix} \tag{34}$$

Where $C_T(\boldsymbol{x}) = [f_1 + \frac{\partial H}{\partial x_1} r(\boldsymbol{x})] / \frac{\partial H}{\partial x_2}$.

The standard form of generalized Hamiltonian model is:

$$\begin{cases} \dot{\boldsymbol{x}} = [\bar{\boldsymbol{J}}(\boldsymbol{x}) - \boldsymbol{R}(\boldsymbol{x})] \frac{\partial H}{\partial \boldsymbol{x}} + \boldsymbol{g}(\boldsymbol{x}) \bar{u} \\ y = \boldsymbol{g}^T(\boldsymbol{x}) \frac{\partial H}{\partial \boldsymbol{x}} \end{cases} \tag{35}$$

5. Conclusions

The complexity of generalized Hamiltonian modelling originates from complex mathematic procedure and Hamiltonian function selection. Principles and method proposed in this paper give a kind of simple solving method. Although it is only suitable for single input and single output system, in which some issues need to be further studied, but proposed three principles of Hamiltonian function selection is universality.

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References

[1] Daizhan Cheng, Zairong Xi, Qiang Lu, et al. Geometrical structure and application of generalized controlled Hamiltonian systems. *Science China： Series E*, 2000, 30(4): 341-355.

[2] Van der Schaft, A J, *L_2-gain and passivity techniques in nonlinear control*. Berlin: Springer, 1996.

[3] J. Basto Goncalves．Realization Theory for Hamiltonian System[J]．SIAM J. Control and Optimization，1987，25(1)：63-73．

[4] P. E. Crouch, M. Irving, Dynamical realizations of homogeneous Hamiltonian systems, SIAM J. Control and Optimization, Vol. 24, No. 3, May 1986,374-395

[5] Yun Zeng, Lixiang Zhang, Tianmao Xu, et al, Hamiltonian model of nonlinear hydraulic turbine with elastic water column, *Journal of Drainage and Irrigation Machinery Engineering*, 2010, 28(6): 515-520.

[6]Wang Yuzhen, Cheng Daizhan, Hu Xiao-ming．Problems on time-varying port-controlled Hamiltonian systems: geometric structure and dissipative realization[J]．Automatica，2005，41(5)：717-723．

[7]Yuzhen Wang, Chunwen Li, Daizhan Cheng, Generalized Hamiltonian realization of time-invariant nonlinear systems, Automatica 39 (2003) 1437 – 1443

[8] Stefano Stramigioli，Cristian Secchi，Arjan J. van der Schaft，et al．Sampled Data Systems Passivity and Discrete Port-Hamiltonian Systems[J]．IEEE Transactions on Robotics，2005，21(4)：574-585．

[9] Goran Golo，Viswanath Talasila，Arjan J. van der Schaft，et al．Hamiltonian discretization of boundary control systems[J]．Automatica，2004，40(5)：757-771．

[10] LIU Yan-Hong, LI ChunWen, WANG Yu-Zhen, Decentralized Excitation Control of Multi-machine Multi-load Power

Systems Using Hamiltonian Function Method,Acta automatic Sinica,vol35,no7,2009,pp.919-925

[11] Jianyong Li, Yanhong Liu, Chunwen Li, On the Dissipative Hamiltonian realization of nonlinear differential algebraic systems, 2009 Second International Conference on Intelligent Computation Technology and Automation,577-580.