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# Application of the fracture mechanics and reliability methods to the fatigue problem

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## Abstract

The fatigue problem of high strength materials and complex mechanical systems subjected to cyclic loadings and corrosion environment is studied. It is assumed that the element of a system contains the initial cracks with the sizes expressed in the form of Weibull distribution. The kinetic equation of corrosion fatigue cracks growth, describing the cracks growth under Paris's law in the absence of the corrosion environment and purely corrosion growth of cracks in the absence of external stresses is developed. Taking into account that the reliability of the system as a whole depends on the reliability of individual elements and the type of their connection, the different types of connection, in particular, in series, parallel and with reservation are considered. For each system the reliability function and the corresponding fatigue strength criterion are formulated.

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## 1. Introduction

The most of modern technical systems are subjected to cyclic loadings with different frequencies. Such loadings lead to fatigue fractures, which in according to world scientific literature, are equal to 80-90% of all fractures (Manson (1965) and Schijve (2003)). Currently, due to numerous cases of operational failures in various fields of

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engineering practice occurring at low, but long-acting cyclic loads, new investigations in the field of very high cycle fatigue were carried out by Bathias (1999), Murakami et al. (1999) and Stanzl-Tschegg and Mayer (2001). These investigations show the need to revise the concept of the existence of infinite durability at stresses below the fatigue limit.

As it follows from numerous experiments by Beretta et al. (2010), Bayraktar et al. (2009) and Nitin et al (2011) carried out on different construction materials under the action of corrosion media, the fatigue curves don't have evident fatigue limit and there is no threshold value of stress intensity factor on the corrosion fatigue crack growth diagram. So there is no indexes on which the calculations on long time operation reliability of materials and structure elements can be done.

## 2. Formulation of corrosion-fatigue crack growth kinetic equation

The involvement of an aggressive environment leads to the intensification of the crack growth rate. We assume that the corrosion processes are controlled by kinetic equation corresponding to the reaction of decomposition of solid solution and stress intensity factor is responsible for the acceleration of corrosion processes (R. Arutyunyan and A. Denisova (2002)). As experiments show, there are different diagrams of corrosion fatigue crack growth, growth speed of which depends on, in particular, the load frequency. To describe these effects, it is suggested to express the kinetic equation of crack growth in the scale of effective time  $z = tf^\alpha = Nf^{\alpha-1}$  ( $f = N/t$ ) (A. Arutyunyan and R. Arutyunyan (2013)), where,  $z$  is effective time,  $f$  is the load frequency,  $t$  is real time,  $N$  is number of cycles,  $\alpha$  is constant.

Based on this thesis, the kinetic equation of corrosion crack growth is written in the following form

$$\frac{dl}{dz} = F(\Delta K)z^\beta, \quad (1)$$

where  $l$  is current crack length,  $\Delta K$  is stress intensity factor range for one cycle,  $F$  is some function of  $\Delta K$ ,  $\beta$  is constant.

Further the function  $F$  is taken as the power dependence

$$F(\Delta K) = K_1 (\Delta K)^m, \quad (2)$$

where  $K_1$ ,  $m$  are constants.

Introducing (2) into (1), and writing this equation through the number of loading cycles, when  $z = N$  ( $\alpha = 1$ ), we have

$$\frac{dl}{dN} = K_1 (\Delta K)^m N^\beta \quad (3)$$

When  $\beta = 0$  from equation (1) follows the Paris-Erdogan kinetic equation (Paris and Erdogan (1963)).

Taking  $\Delta K = \Delta\sigma\sqrt{\pi l}$  ( $\Delta\sigma = \sigma_{max} - \sigma_{min}$ ,  $\sigma_{max}$ ,  $\sigma_{min}$  are maximal and minimal stresses per cycle) and initial conditions  $N = 0$ ,  $l = l_0$ , we will receive the solution of equation (3)

$$l = \left[ \frac{2-m}{2(\beta+1)} (\Delta\sigma)^m \pi^{m/2} K_1 N^{\beta+1} + l_0^{\frac{2-m}{2}} \right]^{\frac{2}{2-m}} \quad (4)$$

The purely corrosion crack growth equation follows from (1), when  $F(\Delta K) = \text{const} = F_0$ ,  $\alpha = 0$  ( $z = t$ ). Solving the received equation, expressed in scale of real time, for the initial condition  $N = 0$ ,  $l = l_0$  we have

$$l = \frac{F_0}{\beta + 1} t^{\beta+1} + l_0 \quad (5)$$

On Fig. 1 the curves of fatigue crack growth for different values of stresses in according to formulas (4) and (5) are shown. The following values of coefficients are accepted:  $F_0 = 5 \cdot 10^{-20} [m] \cdot [\text{cycles}]^{-2}$ ,  $m = 4$ ,  $l_0 = 10^{-6} [m]$ ,  $\beta = 1$ ,  $K_I = 3 \cdot 10^{-15} [m]^{-1} \cdot [\text{cycles}]^{-2} \cdot [\text{MPa}]^{-4}$ .

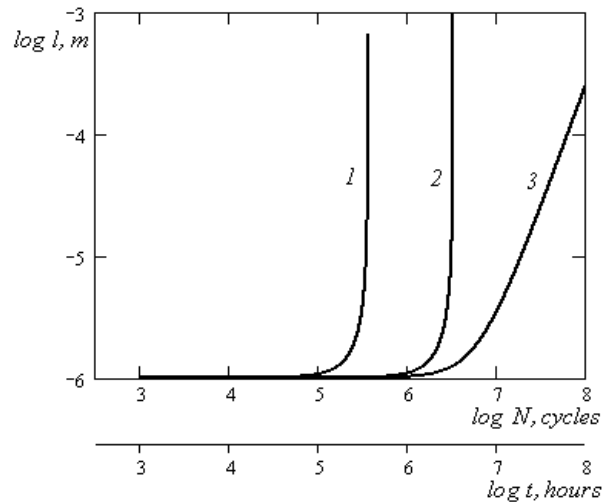


Fig. 1. Theoretical curves of fatigue crack growth according formula (4): for  $\Delta\sigma = 150 \dot{P} \dot{a}$  (curve 1), for  $\Delta\sigma = 50 \dot{P} \dot{a}$  (curve 2) and according formula (5) (curve 3).

As it follows from curve 3 (Fig. 1) the pure corrosion crack growth curve has the initial incubation and slow crack growth periods. When the stress and corrosion media are influenced together incubation period is followed by accelerated crack growth before the specimen fracture (curves 1, 2).

In the fracture mechanics the fracture condition is defined by the Griffith's (Griffith (1924)) or Irvin's (Irvin (1957)) criterions

$$l_* = \frac{2\gamma E}{\pi(\Delta\sigma)^2}, \quad l_* = \frac{2K_{Ic}^2}{\pi(\Delta\sigma)^2}, \quad (6)$$

where  $l_*$  is the critical value of crack length,  $\gamma$  is surface energy,  $E$  is modulus of elasticity,  $K_{Ic}$  is the fracture toughness.

It is assumed that the Griffith or Irvin relations (6) are valid for cyclic loading. In this case,  $\Delta\sigma$  be regarded as a stress amplitude  $\sigma_a = (\sigma_{max} - \sigma_{min})/2$  or maximum stress per cycle  $\sigma_{max}$ . As it is known, these values are used in the construction of fatigue curves – graphs that characterize the relationship between the maximum or amplitude stresses and the number of cycles to failure.

According to the Griffith or Irvin concept fracture won't happen if crack length is less than critical. Thus almost important case of slow growth of cracks of the sizes less critical under the influence of stress and the corrosion

environment isn't considered. In this case growth of a crack can be described by means of relation (4), and at the formulation of criterion of durability to use the Griffith or Irvin conditions (6). Taking into account these suppositions the strength criteria for a specimen (element of media) and complex mechanical systems are formulated. In the following calculations we will use the Griffith's fracture criterion.

**3. Fatigue strength criterion for a specimen (element of media)**

At first let's consider a fatigue strength criterion for a specimen (element of media) with some number of initial cracks which sizes are changes within  $l_0 \leq l \leq l_*$ . Considering the random distribution of sizes of initial cracks the reliability function can be set in the form of Weibull exponential distribution (Weibull (1951))

$$R_0(N) = \frac{e^{-\lambda l^\varphi(N)} - e^{-\lambda l_*^\varphi}}{e^{-\lambda l_0^\varphi} - e^{-\lambda l_*^\varphi}}, \tag{7}$$

where  $\lambda, \varphi$  are constants.

Setting the reliability level as  $R_0 = R_*$  and taking into account (4), from relation (7) we will receive the fatigue strength criterion for a specimen (element of media)

$$(\Delta\sigma)^m N^{\beta+1} = \frac{2(\beta+1)}{(2-m)\pi^{m/2} K_I} \left[ \left( -\frac{1}{\lambda} \ln(C) \right)^{\frac{2-m}{2\varphi}} - l_0^{\frac{2-m}{2}} \right], \tag{8}$$

where  $C = R_* \left( e^{-\lambda l_0^\varphi} - e^{-\lambda l_*^\varphi} \right) + e^{-\lambda l_*^\varphi}$ ,  $l_* = \frac{2\gamma E}{\pi(\Delta\sigma)^2}$ .

**4. Formulation of corrosion fatigue strength criterions of complex mechanical systems**

Further the probability criterions of corrosion fatigue strength of complex mechanical systems (Kapur and Lamberson (1977), Bolotin (1984) and R. Arutyunyan (1993, 2004), connected in series, parallel and with reservation, are formulated.

Let's the elements are interacted so that the failure of any element leads to a failure of the system. This type of connection is called in series. If the reliability indexes of all elements are equal to each other the reliability function of  $n$  elements connected in series is given as

$$R(N) = R_0^n(N) \tag{9}$$

Let's set the reliability level as  $R_0 = R_*$ . Taking into account (4) from relation (9) can be received the criterion of fatigue strength of system connected in series

$$(\Delta\sigma)^m N^{\beta+1} = \frac{2(\beta+1)}{(2-m)\pi^{m/2} K_I} \left( \left( -\frac{1}{\lambda} \ln(C_1) \right)^{\frac{2-m}{2\varphi}} - l_0^{\frac{2-m}{2}} \right) \tag{10}$$

$$\text{where } C_1 = R_*^{\frac{1}{n}} \left( e^{-\lambda l_0^{\varphi}} - e^{-\lambda l_*^{\varphi}} \right) + e^{-\lambda l_*^{\varphi}}.$$

To achieve a high level of reliability the reservation method is applied, in particular, parallel connection of  $k$  elements. For this system failure occur only if all the  $k$  elements are failed. The reliability function in this case is expressed in the following form

$$R(N) = I - [I - R_0(N)]^k \quad (11)$$

The criterion of corrosion fatigue strength for system with parallel connection will be received as

$$(\Delta\sigma)^m N^{\beta+1} = \frac{2(\beta+1)}{(2-m)\pi^{m/2} K_I} \left\{ \left[ -\frac{I}{\lambda} \ln(C_2) \right]^{\frac{2-m}{2\varphi}} - l_0^{\frac{2-m}{2}} \right\} \quad (12)$$

$$\text{where } C_2 = \left[ I - (I - R_*)^{\frac{1}{k}} \right] \left( e^{-\lambda l_0^{\varphi}} - e^{-\lambda l_*^{\varphi}} \right) + e^{-\lambda l_*^{\varphi}}.$$

In the case of general reservation with connected  $k$  parallel and  $n$  in series the reliability function and fatigue strength criterion are expressed in the following form

$$R(N) = I - [I - R_0^n(N)]^k \quad (13)$$

$$(\Delta\sigma)^m N^{\beta+1} = \frac{2(\beta+1)}{(2-m)\pi^{m/2} K_I} \left( \left( -\frac{I}{\lambda} \ln(C_3) \right)^{\frac{2-m}{2\varphi}} - l_0^{\frac{2-m}{2}} \right) \quad (14)$$

$$\text{where } C_3 = \left[ I - (I - R_*)^{\frac{1}{k}} \right]^{\frac{1}{n}} \left( e^{-\lambda l_0^{\varphi}} - e^{-\lambda l_*^{\varphi}} \right) + e^{-\lambda l_*^{\varphi}}.$$

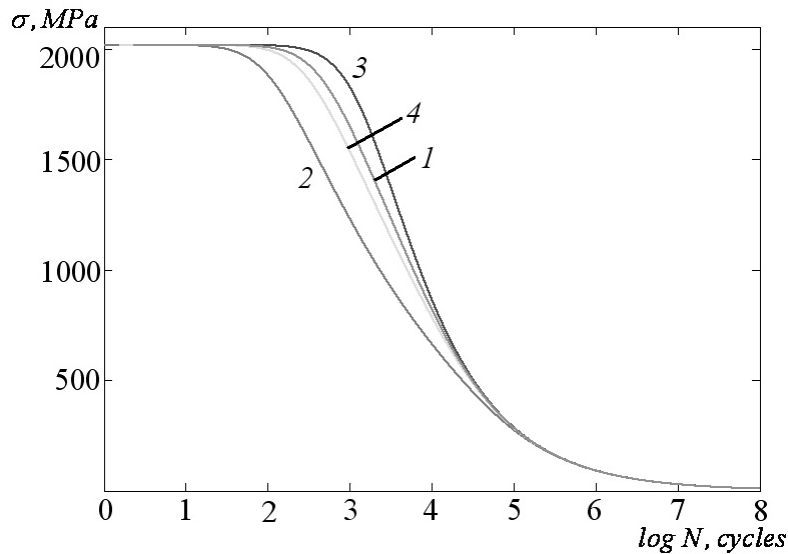


Fig. 2. Corrosion fatigue curves for  $\beta = 1$ : specimen (curve 1), connection of elements in series (curve 2), parallel connection of elements (curve 3) and general reservation of elements (curve 4).

Theoretical fatigue curves according formulas (8), (10), (12) and (14) are marked on Fig. 2 by numbers 1, 2, 3 and 4 correspondingly. The following values of coefficients were used when the calculations for this formulas were carried out:  $K_1 = 3 \cdot 10^{-15} [m]^{-1} \cdot [cycles]^{-2} \cdot [MPa]^{-4}$ ,  $m = 4$ ,  $l_0 = 10^{-3} m$ ,  $R_* = 0,8$ ,  $\beta = 1$ ,  $k = 10$ ,  $n = 20$ ,  $\varphi = 2$ ,  $C = 7,5 \cdot 10^{-11} [m]^{-1} \cdot [cycles]^{-2} \cdot [MPa]^{-4}$ ,  $\gamma = 0,15 J/m^2$ ,  $E = 10^5 MPa$ ,  $\lambda = 5 [m]^{-1}$ .

As it follows from Fig. 2 the durability of complex systems with reservation is higher then the system connected in series without reservation.

## 5. Conclusions

The kinetic equations of corrosion fatigue crack growth and criterions of fatigue strength for a specimen (element of media) with cracks and complex mechanical systems, connected in series, parallel and with reservation, are formulated.

The received criterions are expressed in the form of unified analytical relations and describe all parts of experimental fatigue curves. These relations are new and earlier in world scientific literature on fatigue problem were not considered.

It is shown that the durability of complex systems with reservation is greatly higher then the system connected in series. At that time the case of parallel reservation is the most reliable.

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