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Inclusion of Yang–Mills fields in string corrected supergravity

S. Bellucci [a,*], D. O'Reilly [b]

[a] INFN-Laboratori Nazionali di Frascati, Via E. Fermi 40, 00044 Frascati, Italy
[b] Department of Mathematics, The Graduate School and University Center, 365 Fifth Avenue, New York, NY 10016-4309, USA

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ABSTRACT

We consistently incorporate Yang–Mills matter fields into string corrected (deformed) $D = 10$, $N = 1$ supergravity. We solve the Bianchi identities within the framework of the modified beta function favored constraints to second order in the string slope parameter γ also including the Yang–Mills fields. In the torsion, curvature and H sectors we find that a consistent solution is readily obtained with a Yang–Mills modified supercurrent A_{abc}. We find a solution in the F sector following our previously developed method.

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1. Introduction

String corrections to quantum field theories are believed to contribute Gauss–Bonnet terms to the action. This invariant has been studied in many different scenarios, for example Gauss–Bonnet modified cosmology [1]. These terms are introduced by hand into the quantum gravity models. However it is also known that Gauss–Bonnet terms occur naturally in the context of string corrected gravity [2]. The leading order corrections in terms of the string tension parameter, γ contain these invariants. Hence we are motivated by this and by other reasons to study string deformed supergravity, with $D = 10$, $N = 1$ as the low energy limit of string theory. Here we wish to include Yang–Mills fields in the non-minimal theory. As several of the terms are extremely lengthy we avoid writing them explicitly. In a future work we will explore the simplified bosonic case.

Some years ago a scenario was developed to construct a manifestly supersymmetric theory of string corrected supergravity [2–6]. This involved incorporating the Lorentz Chern–Simons superform, X^{LL} into the geometry of $D = 10$, $N = 1$ supergravity. It was initially successful at first order in γ [2]. It ran into difficulties and controversy at second order [7]. It was suggested that at second order the torsion $T_{\alpha\beta}{}^{g}$ should be modified to include the so-called X tensor [3]. A search for the X tensor ensued and a candidate was proposed in [8] which was shown to allow for the solution of the Bianchi identities in the H sector and also the torsion and curvature sectors.

In this Letter we show that a simple modification of the X tensor Ansatz allows also for the inclusion of matter fields in the H torsion and curvature sectors. We show that the assumption $F^{(2)}{}_{\alpha\beta} = 0$ does not allow for a solution. We propose a candidate that does allow for a solution. Prior to finding the X tensor also by way of an Ansatz, in [8], the search for a second order solution consisted of systematically studying the table of irreducible representations [9]. However the task proved formidable because of the number of unknown quantities. It was eventually shown that the form given in Eq. (33) of the first reference of [8] for $T^{(2)}{}_{\alpha\beta}{}^{g}$, satisfying the torsion identity at dimension one half, thereafter allowed ultimately for a consistent solution in the torsion and curvature sectors.

In Ref. [8], we found a solution to $D = 10$, $N = 1$ supergravity to second order in the string slope parameter with the modified tensor

$$G_{ADG} = H_{ADG} + \gamma Q_{ADG}. \tag{1}$$

Here we extend this as follows:

$$G_{ADG} = H_{ADG} + \gamma Q_{ADG} + \beta Y_{ADG}. \tag{2}$$

Here β is the Yang–Mills coupling constant. Matter fields will not have consequences for the appearance of the Gauss–Bonnet term however, but we wish to construct a complete model.

* Corresponding author.
E-mail addresses: bellucci@lnf.infn.it (S. Bellucci), doreilly@gc.cuny.edu, dunboyne@vzavenue.net (D. O'Reilly).

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2. The solution

The Bianchi identities in superspace are as follows:

$$\big[[\nabla_{[A},\nabla_B\},\nabla_{C)}\big\}=0. \tag{3}$$

Here we have extended the commutator in [8] to include the Yang–Mills field strengths F^I_{AB}

$$[\nabla_A,\nabla_B\}=T_{AB}{}^C\nabla_C+\frac{1}{2}R_{AB}{}^{de}M_{ed}+iF^I_{AB}t_I. \tag{4}$$

The t_I are the generators of the Yang–Mills gauge group. For notation convenience we drop the group index I. For notational brevity we also write

$$R_{ABde}=R^{(0)}{}_{ABde}+R^{(1)}{}_{ABde}+R^{(2)}{}_{ABde}+\cdots \tag{5}$$

and

$$T_{AD}{}^G=T^{(0)}{}_{AD}{}^G+T^{(1)}{}_{AD}{}^G+T^{(2)}{}_{AD}{}^G+\cdots. \tag{6}$$

Where the numerical superscript refers to the order of the quantity. A quantity which re-occurs is the following

$$\varOmega^{(1)}{}_{gef}=L^{(1)}{}_{gef}-\frac{1}{4}A^{(1)}{}_{gef} \tag{7}$$

and its spinor derivative which we denote simply as

$$\varOmega^{(1)}{}_{\gamma gef}=\nabla_\gamma\left[L^{(1)}{}_{gef}-\frac{1}{4}A^{(1)}{}_{gef}\right]. \tag{8}$$

We evaluate this in Appendix A. Ref. [2] began with the conventional constraints as follows:

$$\begin{aligned}&i\sigma_b{}^{\alpha\beta}T_{\alpha\beta}{}^a=16\delta^a{}_b,\qquad \sigma_{abcde}{}^{\alpha\beta}T_{\alpha\beta}{}^a=0,\qquad \sigma_a{}^{\alpha\beta}T_{\alpha\beta}{}^a=0,\\ &T_{\alpha[bc]}=0,\qquad T_{abc}=\frac{1}{6}T_{[abc]},\qquad T_{a\alpha}{}^\beta=\frac{1}{48}\sigma_{a\alpha\lambda}\sigma^{pqr\beta\lambda}A_{pqr}.\end{aligned} \tag{9}$$

Using these constraints then led to the first order solution. That is $G^{(1)}{}_{ABC}$, $T^{(1)}{}_{AB}{}^C$, and $R^{(1)}{}_{ABde}$ were found, along with additional modified constraints. In [8] we found that no modification to the condition $H_{\alpha\beta\gamma}=0$ was necessary. We continue with this assumption.

We have the H sector Bianchi identities with Yang–Mills fields as follows [2]:

$$\frac{1}{6}\nabla_{(\alpha|}H_{|\beta\gamma\delta)}-\frac{1}{4}T_{(\alpha\beta|}{}^MH_{M|\gamma\delta)}=-\frac{\gamma}{4}R_{(\alpha\beta|ef}R_{|\gamma\delta)}{}^{ef}-\frac{\beta}{4}F_{(\alpha\beta|}{}^IF_{|\gamma\delta)}{}^I, \tag{10}$$

$$\frac{1}{2}\nabla_{(\alpha|}H_{|\beta\gamma)d}-\nabla_dH_{\alpha\beta\gamma}-\frac{1}{2}T_{(\alpha\beta|}{}^MH_{M|\gamma)d}+\frac{1}{2}T_{d(\alpha|}{}^MH_{M|\beta\gamma)}=-\gamma R_{(\alpha\beta|ef}R_{|\gamma)d}{}^{ef}-\beta F_{(\alpha\beta|}{}^IF_{|\gamma)d}{}^I, \tag{11}$$

$$\begin{aligned}&\nabla_{(\alpha|}H_{\beta)cd}+\nabla_{[c|}H_{|d]\alpha\beta}-T_{\alpha\beta}{}^MH_{Mcd}-T_{cd}{}^MH_{M\alpha\beta}-T_{(\alpha|[c|}{}^MH_{M|d]|\beta)}\\ &\quad=-\gamma\big[2R_{\alpha\beta ef}R_{cd}{}^{ef}+R_{(\alpha|[c|ef}R_{|d]\beta)}{}^{ef}\big]-\beta\big[2F_{\alpha\beta}{}^IF_{cd}{}^I+F_{(\alpha|[c|}{}^IF_{|d]\beta)}{}^I\big].\end{aligned} \tag{12}$$

We must also solve the torsions and curvatures

$$T_{(\alpha\beta|}{}^\lambda T_{|\gamma)\lambda}{}^d-T_{(\alpha\beta|}{}^gT_{|\gamma)g}{}^d-\nabla_{(\alpha|}T_{\beta\gamma)}{}^d=0, \tag{13}$$

$$T_{(\alpha\beta|}{}^\lambda T_{|\gamma)\lambda}{}^\delta-T_{(\alpha\beta|}{}^gT_{|\gamma)g}{}^\delta-\nabla_{(\alpha|}T_{|\beta\gamma)}{}^\delta-\frac{1}{4}R^{(2)}{}_{(\alpha\beta|de}\sigma^{de}{}_{|\gamma)}{}^\delta=0, \tag{14}$$

$$T_{(\alpha\beta|}{}^\lambda R_{|\gamma)\lambda de}-T_{(\alpha\beta|}{}^\lambda R_{|\gamma)\lambda de}-\nabla_{(\alpha|}R_{|\beta\gamma)de}=0. \tag{15}$$

To this list we now add the equations arising form the F sector

$$\nabla_{[A|}F_{|BC)}-T^M{}_{[AB|}F_{M|C)}=0. \tag{16}$$

In our previous work the supercurrent $A^{(1)}{}_{gef}$ was given by

$$A^{(1)}{}_{gef}=+i\gamma\sigma_{gef\epsilon\tau}T^{mn\epsilon}T_{mn}{}^\tau. \tag{17}$$

To begin with we modify $A^{(1)}{}_{gef}$ to include matter fields as in [2].

$$A^{(1)}{}_{gef}\longrightarrow+i\sigma_{gef\epsilon\tau}\big[\gamma T^{mn\epsilon}T_{mn}{}^\tau+\beta\lambda^\epsilon\lambda^\tau\big]=A^{(\gamma)}{}_{gef}+A^{(\beta)}{}_{gef}. \tag{18}$$

Here we use the notation where the superscript γ or β is self evident. This has the effect of splitting the Bianchi identities however we still have to be careful of cross terms. In order to be cautious we will examine all contributions in the H sector, in particular that for $H^{(2)}{}_{\alpha\beta d}$, for thoroughness (see Appendix A). In all of the Bianchi identities we encounter the spinor derivative $\nabla_\alpha A^{(1)}{}_{abc}$. We now show that a key equation which led to the previous second order modified beta function favored (βFF) solution is still valid but with the modified $A^{(1)}{}_{abc}$. The spinor derivative of $A^{(1)}{}_{abc}$ will now contain the contributions due to the spinor derivative of λ^τ at zeroth order. We have the following modifications from [2]:

$$\nabla_\gamma T^{(0)}{}_{ef}{}^{\delta} = -\frac{1}{4}\sigma^{mn}{}_{\gamma}{}^{\delta} R^{(0)}{}_{efmn} + T^{(0)}{}_{ef}{}^{\lambda} T^{(0)}{}_{\gamma\lambda}{}^{\delta}, \tag{19}$$

$$\nabla_\gamma \lambda^{(0)\delta} = -\frac{1}{4}\sigma^{mn}{}_{\gamma}{}^{\delta} F_{mn} + \lambda^{\lambda} T^{(0)}{}_{\gamma\lambda}{}^{\delta}. \tag{20}$$

The fundamental equation which enable a solution to be found with higher order βFF constraints, now with the modified supercurrent, $A^{(1)}{}_{pqr}$ is still given by

$$T^{(0)}{}_{(\alpha\beta|}{}^{\lambda}\sigma^{pqref}{}_{|\gamma)\lambda} A^{(1)}{}_{pqr} H^{(0)}{}_{def} - \sigma^{pqref}{}_{(\alpha\beta|} H^{(0)}{}_{def} \nabla_{|\gamma)} A^{(1)}{}_{pqr} = -24\sigma^{g}{}_{(\alpha\beta|} H^{(0)}{}_{d}{}^{ef}\left[\Omega^{(1)}{}_{|\gamma)gef}\right]. \tag{21}$$

In the H sector we will have the following quantities

$$H_{ABC} = H^{(\gamma\gamma)}{}_{ABC} + H^{(\beta\beta)}{}_{ABC} + H^{(\beta\gamma)}{}_{ABC}. \tag{22}$$

We will also adopt the modified torsion

$$T^{(2)}{}_{\alpha\beta}{}^{g} = -\frac{i\gamma}{6}\sigma^{pqref}{}_{\alpha\beta} H^{(0)d}{}_{ef} A^{(1)}{}_{pqr}. \tag{23}$$

We then find no change in the form of the H sector results.

$$\bar{H}^{(2)}{}_{\alpha\beta d} = H^{(\gamma\beta)}{}_{\alpha\beta d} = \sigma_{\alpha\beta}{}^{g}\left[-i\gamma H^{(0)}{}_{def} A^{(\beta)}{}_{g}{}^{ef}\right] - \sigma^{pqref}{}_{\alpha\beta}\left[\frac{i\gamma}{12} H^{(0)}{}_{def} A^{(\beta)}{}_{pqr}\right]. \tag{24}$$

Eq. (21) contains the second order contribution of the spinor derivative $\nabla_\gamma H_{\alpha\beta g}$. Term by term we notice no order β^2 contributions occur. We also seek terms of the form $O(\gamma\beta)$ and note that none appear other than those resulting from the modification to A_{abc}. We also find $H^{(2)}{}_{g\gamma d}$ as in [8], but we write it in a different way

$$\begin{aligned}
+\frac{i}{2}\sigma_{(\alpha\beta|}{}^{g} H^{(2)}{}_{g|\gamma)d} = {} & -\frac{\gamma}{12}\sigma^{\gamma|}{}_{(\alpha\beta|}\sigma^{pqr}{}_{g}{}^{f}{}_{|\gamma)\lambda} A^{(1)}{}_{pqr} T_{df}{}^{\lambda} + i\sigma^{g}{}_{(\alpha\beta|}\Big\{4\gamma\left[\nabla_{|\gamma)} H^{(0)}{}_{d}{}^{ef}\right] L^{(1)}{}_{gef} + 4\gamma H^{(0)}{}_{d}{}^{ef}\nabla_{|\gamma)} L^{(1)}{}_{gef} \\
& - \frac{\gamma}{2}\left[\nabla_{|\gamma)} H^{(0)}{}_{d}{}^{ef}\right] A^{(1)}{}_{gef} - \frac{\gamma}{2} H^{(0)}{}_{d}{}^{ef}\nabla_{|\gamma)} A^{(1)}{}_{gef} - 2\gamma H^{(0)}{}_{g}{}^{ef} R^{(1)}{}_{|\gamma)def} - 2\gamma L^{(1)}{}_{g}{}^{ef} R^{(0)}{}_{|\gamma)def} \\
& + \frac{\gamma}{4} A^{(1)}{}_{gef} R^{(0)}{}_{|\gamma)def} + 2\gamma\nabla_{|\gamma)}\left[H^{(0)}{}_{d}{}^{ef} H^{(0)}{}_{gef}\right]^{\text{Order}(1)}\Big\}.
\end{aligned} \tag{25}$$

The symmetrized result can be extracted as in [2]. We list it in Appendix A. We also believe that it can be simplified further. We also have, along with (33) of the first reference of [8],

$$i\sigma^{g}{}_{(\alpha\beta|} T^{(2)}{}_{|\gamma)g}{}^{\delta} = +2i\gamma\sigma^{g}{}_{(\alpha\beta|} T^{(0)ef\delta}\Omega^{(1)}{}_{|\gamma)gef}, \tag{26}$$

$$+i\sigma^{g}{}_{(\alpha\beta|} T^{(2)}{}_{|\gamma)gd} = +4i\gamma\sigma^{g}{}_{(\alpha\beta|} H^{(0)}{}_{d}{}^{ef}\left[\Omega^{(1)}{}_{|\gamma)gef}\right] + \frac{\gamma}{6}\sigma^{g}{}_{(\alpha\beta|}\sigma^{pqre}{}_{g|\gamma)\phi} A^{(1)}{}_{pqr} T^{(0)}{}_{de}{}^{\phi}, \tag{27}$$

$$R_{\alpha\beta de} = -\frac{i\gamma}{12}\sigma^{pqref}{}_{\alpha\beta} A^{(1)}{}_{pqr} R_{efde}. \tag{28}$$

3. *F* sector Bianchi identities

We have seen that in the H and torsion sectors the required minimal Bianchi identities have been fully satisfied by simply modifying the supercurrent as in Eq. (28). The fundamental identity (21) is then used to solve in each case. This is the identity that enables such solutions to be obtained in the modified β function favored set of constraints. Here we show that it can also be used to solve in the F sector coupled with an Ansatz. To begin with we consider the following Bianchi identities:

$$T_{(\alpha\beta|}{}^{\lambda} F_{|\gamma)\lambda} - T_{(\alpha\beta|}{}^{g} F_{|\gamma)g} - \nabla_{(\alpha|} F_{|\beta\gamma)} = 0, \tag{29}$$

$$\nabla_\alpha F_{ab} - \nabla_{[a|} F_{\alpha|b]} - T_{\alpha[a|}{}^{M} F_{M|b]} - T_{ab}{}^{M} F_{M\alpha} = 0. \tag{30}$$

We have from [2] to first order,

$$F^{(0)}{}_{\alpha\beta} = F^{(1)}{}_{\alpha\beta} = F^{(1)}{}_{\alpha d} = 0, \tag{31}$$

$$F^{(0)}{}_{\alpha d} = -i\sigma_{d\alpha\phi}\lambda^{\phi}, \tag{32}$$

$$\nabla_\alpha F^{(0)}{}_{ef} = i\sigma_{[e|\alpha\phi}\nabla_{|f]}\lambda^{\phi} - 2i\sigma^{g}{}_{\alpha\phi} H^{(0)}{}_{gef}. \tag{33}$$

Eq. (29) is not satisfied by $F^{(2)}{}_{\alpha\beta} = 0$. We propose the following candidate and show that it works.

$$F^{(2)}{}_{\alpha\beta} = -\frac{i\gamma}{12}\sigma^{pqref}{}_{\alpha\beta} A^{(1)}{}_{pqr} F_{ef}. \tag{34}$$

At second order equation (29) becomes

$$T^{(0)}{}_{(\alpha\beta|}{}^{\lambda} F^{(2)}{}_{|\gamma)\lambda} - \nabla_{(\alpha|} F^{(2)}{}_{|\beta\gamma)} - i\sigma^{g}{}_{(\alpha\beta|} F^{(0)}{}_{|\gamma)g} + i\frac{\gamma}{6}\sigma^{pqref}{}_{(\alpha\beta|} A^{(1)}{}_{pqr} H^{(0)g}{}_{ef}\left[-i\sigma_{g|\gamma)\phi}\lambda^{\phi}\right] = 0 \tag{35}$$

or, more clearly

$$T^{(0)}{}_{(\alpha\beta|}{}^{\lambda}\left[-\frac{i\gamma}{12}\sigma^{pqref}{}_{|\gamma)\lambda}F_{ef}A^{(1)}{}_{pqr}\right]+\frac{i\gamma}{12}\sigma^{pqref}{}_{(\alpha\beta|}F_{ef}\nabla_{|\gamma)}A^{(1)}{}_{pqr}-\frac{i\gamma}{12}\sigma^{pqref}{}_{(\alpha\beta|}A^{(1)}{}_{pqr}\nabla_{|\gamma)}F_{ef}$$
$$-i\sigma^{g}{}_{(\alpha\beta|}F^{(0)}{}_{|\gamma)g}+i\frac{\gamma}{6}\sigma^{pqref}{}_{(\alpha\beta|}A^{(1)}{}_{pqr}H^{(0)g}{}_{ef}\left[-i\sigma_{g|\gamma)\phi}\lambda^{\phi}\right]=0. \tag{36}$$

Using (21) generates two solvable terms and we also set up a cancellation. We also must use

$$\sigma^{pqref}{}_{(\alpha\beta|}\sigma_{e|\gamma)\phi}=-\sigma^{pqref}{}_{\phi(\alpha|}\sigma_{e|\beta\gamma)}. \tag{37}$$

Hence, we obtain

$$-i\sigma^{g}{}_{(\alpha\beta|}F^{(2)}{}_{|\gamma)g}+2i\gamma\sigma^{g}{}_{(\alpha\beta|}\Omega^{(1)}{}_{|\gamma)gef}F^{(0)ef}-\frac{\gamma}{6}\sigma^{g}{}_{(\alpha\beta|}\sigma^{pqrgf}{}_{|\gamma)\phi}A^{(1)}{}_{pqr}\nabla_{f}\lambda^{\phi}+\frac{i\gamma}{12}\sigma^{pqref}{}_{(\alpha\beta|}A^{(1)}{}_{pqr}\left[-2i\sigma^{g}{}_{|\gamma)\phi}H^{(0)}{}_{gef}\right]$$
$$-i\frac{\gamma}{6}\sigma^{pqref}{}_{(\alpha\beta|}A^{(1)}{}_{pqr}H^{(0)g}{}_{ef}\left[-i\sigma_{g|\gamma)\phi}\lambda^{\phi}\right]=0. \tag{38}$$

The last two terms conveniently cancel. We find

$$i\sigma^{g}{}_{(\alpha\beta|}F^{(2)}{}_{|\gamma)g}=+2i\gamma\sigma^{g}_{(\alpha\beta|}\Omega^{(1)}{}_{|\gamma)gef}F^{(0)ef}-\frac{\gamma}{6}\sigma^{g}{}_{(\alpha\beta|}\sigma^{pqrgf}{}_{|\gamma)\phi}A^{(1)}{}_{pqr}\nabla_{f}\lambda^{\phi} \tag{39}$$

or

$$F^{(2)}{}_{\gamma g}=+2\gamma\Omega^{(1)}{}_{\gamma gef}F^{(0)ef}+i\frac{\gamma}{6}\sigma^{pqr}{}_{g}{}^{f}{}_{\gamma\phi}A^{(1)}{}_{pqr}\nabla_{f}\lambda^{\phi}. \tag{40}$$

Finally for completeness, we consider the derivatives, $\nabla_{\alpha}F^{(2)}{}_{bc}$ and $\nabla_{(\alpha|}F^{(2)}{}_{|\beta)g}$. We have

$$\nabla_{(\alpha|}F_{|\beta)d}-T_{\alpha\beta}{}^{g}F_{gd}-T_{\alpha\beta}{}^{\lambda}F_{\lambda d}+T_{(\alpha|d}{}^{g}F_{g|\beta)}+T_{(\alpha|d}{}^{\lambda}F_{\lambda|\beta)}=0 \tag{41}$$

and

$$\nabla_{\alpha}F_{bc}=\nabla_{[b|}F_{\alpha|c]}+T_{\alpha[b|}{}^{g}F_{g|c]}+T_{\alpha[b|}{}^{\lambda}F_{\lambda|c]}-T_{bc}{}^{g}F_{g\alpha}+T_{bc}{}^{\lambda}F_{\lambda\alpha}. \tag{42}$$

Substitution of (31), (32), (33), (34) and (40), as well as other results quoted in [2] and

$$T^{(2)}{}_{\alpha\beta}{}^{\lambda}=-\frac{i\gamma}{12}\sigma^{pqref}{}_{\alpha\beta}A^{(1)}{}_{pqr}T^{(0)\lambda}{}_{ef} \tag{43}$$

give, respectively, for (41) and (42),

$$-i\sigma_{d(\alpha|\phi}\nabla_{|\beta)}\lambda^{\phi}|^{(2)}+2\gamma\nabla_{(\alpha|}\Omega^{(1)}{}_{|\beta)def}F^{ef}+4i\gamma\Omega^{(1)}{}_{(\alpha|d}{}^{ef}\sigma_{e|\beta)\phi}\nabla_{f}\lambda^{\phi}-4i\gamma\Omega^{(1)}{}_{(\alpha|d}{}^{ef}\sigma_{g|\beta)\phi}H^{(0)g}{}_{ef}$$
$$+\frac{i\gamma}{6}\sigma^{pqr}{}_{d}{}^{f}{}_{(\alpha|\phi}\left(\nabla_{|\beta)}A^{(1)}{}_{pqr}\right)\left[\nabla_{f}\lambda^{\phi}\right]+\frac{i\gamma}{6}\sigma^{pqr}{}_{d}{}^{f}{}_{(\alpha|\phi}A^{(1)}{}_{pqr}\left[\nabla_{|\beta)}\nabla_{f}\lambda^{\phi}\right]-i\sigma^{g}{}_{\alpha\beta}F^{(2)}{}_{gd}+\frac{\gamma}{12}\sigma^{pqref}{}_{\alpha\beta}\sigma_{d\lambda\phi}A^{(1)}{}_{pqr}T_{ef}{}^{\lambda}\lambda^{\phi}$$
$$-2\gamma T^{(0)}{}_{\alpha\beta}{}^{\lambda}\Omega^{(1)}{}_{\lambda def}F^{(0)ef}-i\frac{\gamma}{6}T^{(0)}{}_{\alpha\beta}{}^{\lambda}\sigma^{pqr}{}_{d}{}^{f}{}_{\lambda f}A^{(1)}{}_{pqr}\nabla_{f}\lambda^{\phi}-i\sigma_{g(\alpha|\phi}\lambda^{\phi}T^{(2)}{}_{|\beta)d}{}^{g}=0, \tag{44}$$

$$\nabla_{\alpha}F_{bc}=2\gamma\nabla_{[b|}\Omega^{(1)}{}_{\alpha|c]ef}F^{ef}-i\frac{\gamma}{6}\sigma^{pqr}{}_{[b|}{}^{f}{}_{\alpha\phi}A^{(1)}{}_{pqr}\nabla_{|c]}\nabla_{f}\lambda^{\phi}-i\frac{\gamma}{6}\sigma^{pqr}{}_{[b|}{}^{f}{}_{\alpha\phi}\left[\nabla_{|c]}A^{(1)}{}_{pqr}\right]\nabla_{f}\lambda^{\phi}$$
$$+T^{(2)}{}_{\alpha[b|}{}^{g}F^{(0)}{}_{g|c]}-iT^{(2)}{}_{\alpha[b|}{}^{\lambda}\sigma_{|b]\lambda\phi}\lambda^{\phi}+2\gamma T^{(0)g}{}_{bc}\Omega_{\alpha gef}F^{ef}+i\frac{\gamma}{6}\sigma^{pqr}{}_{g}{}^{f}{}_{\lambda\phi}A^{(1)}{}_{pqr}T^{(2)}{}_{bc}{}^{g}\nabla_{f}\lambda^{f}-i\gamma_{g\alpha\phi}\lambda^{\phi}T^{(2)}{}_{bc}{}^{g}$$
$$-i\frac{\gamma}{12}\sigma^{pqref}{}_{\lambda\alpha}A^{(1)}{}_{pqr}F^{(0)}{}_{ef}T^{(0)}{}_{bc}{}^{\lambda}. \tag{45}$$

4. Modified commutator

Finally, we note how the commutator (4) is modified. We have

$$[\nabla_{\alpha},\nabla_{\beta}\}=-\frac{i\gamma}{12}\sigma^{pqref}{}_{\alpha\beta}A^{(1)}{}_{pqr}\left[T_{ef}{}^{\gamma}\nabla_{\gamma}+2H^{(0)}{}_{ef}{}^{g}\nabla_{g}+\frac{1}{2}R^{(0)}{}_{ef}{}^{mn}M_{mn}+iF^{(0)}{}_{ef}{}^{I}t_{I}\right]. \tag{46}$$

But

$$2H^{(0)}{}_{ef}{}^{g}=-T^{(0)}{}_{ef}{}^{g}, \tag{47}$$

and

$$R^{(0)}{}_{ef}{}^{mn}M_{mn}=-\frac{1}{8}R^{(0)}{}_{ef\lambda}{}^{\delta}\sigma^{mn}{}_{\delta}{}^{\lambda} \tag{48}$$

so we get the modified geometry proportional to σ^{5} as follows:

$$[\nabla_{\alpha},\nabla_{\beta}\}|^{(2)}=-\frac{i\gamma}{12}\sigma^{pqref}{}_{\alpha\beta}A^{(1)}{}_{pqr}\left[T^{(0)}{}_{ef}{}^{\gamma}\nabla_{\gamma}-T^{(0)}{}_{ef}{}^{g}\nabla_{g}-\frac{1}{16}R^{(0)}{}_{ef\lambda}{}^{\delta}\sigma^{mn}{}_{\delta}{}^{\lambda}M_{mn}-iF^{(0)}{}_{ef}{}^{I}t_{I}\right]. \tag{49}$$

A point conceptually important can be stressed here by observing that the latter expression agrees in its structure with the conjecture in Ref. [5] for supersymmetric Yang–Mills couplings.

5. Conclusion

The geometrical methods currently known as deformations [3] and the constraints often referred to as beta function favored constraints [10–12] allowed for the determination of the most general higher derivative Yang–Mills action to the third order, which is globally supersymmetric and Lorentz covariant in $D=10$ spacetime (see e.g. [13]), a result which is important for topologically nontrivial gauge configurations of the vector field, such as, for instance, in the case of compactified string theories on manifolds with topologically nontrivial properties. Building upon our previous works on the subject, we have been able here to provide a consistent inclusion of Yang–Mills matter fields into string corrected (deformed) $D=10$, $N=1$ supergravity. Our solution to the Bianchi identities, obtained within the framework of the modified beta function favored constraints, holds to the second order in the string slope parameter γ and includes also the Yang–Mills fields. We obtained as well a consistent solution in the torsion, curvature and H sectors with a Yang–Mills modified supercurrent A_{abc}. Following a technique we developed in earlier papers, we also found a solution in the F sector and gave an explicit formula for the modification induced in the commutator expression.

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Appendix A

For the sake of extra caution as the Bianchi identity for example for $H_{\alpha bd}$ is long, we can write out the full version as follows:

$$
\begin{aligned}
&\frac{1}{2}\nabla_{(\alpha|}H_{|\beta\gamma)d}{}^{(\gamma\gamma;\beta\beta;\gamma\beta)} - \nabla_d H_{\alpha\beta\gamma}{}^{\mathrm{Order}(2)} - \frac{1}{2}T^{(0)}{}_{(\alpha\beta|}{}^{\lambda}H^{(\gamma\gamma)}{}_{\lambda|\gamma)d} - \frac{1}{2}T^{(0)}{}_{(\alpha\beta|}{}^{\lambda}H^{(\gamma\beta)}{}_{\lambda|\gamma)d} - \frac{1}{2}T^{(0)}{}_{(\alpha\beta|}{}^{\lambda}H^{(\beta\beta)}{}_{\lambda|\gamma)d}\\
&\quad - \frac{1}{2}T^{(\gamma)}{}_{(\alpha\beta|}{}^{\lambda}H^{(\beta)}{}_{\lambda|\gamma)d} - \frac{1}{2}T^{(\gamma)}{}_{(\alpha\beta|}{}^{\lambda}H^{(\gamma)}{}_{\lambda|\gamma)d} - \frac{1}{2}T^{(\beta)}{}_{(\alpha\beta|}{}^{\lambda}H^{(\beta)}{}_{\lambda|\gamma)d} - \frac{1}{2}T^{(\beta)}{}_{(\alpha\beta|}{}^{\lambda}H^{(\gamma)}{}_{\lambda|\gamma)d} - \frac{1}{2}T^{(\gamma\gamma)}{}_{(\alpha\beta|}{}^{\lambda}H^{(0)}{}_{\lambda|\gamma)d}\\
&\quad - \frac{1}{2}T^{(\gamma\beta)}{}_{(\alpha\beta|}{}^{\lambda}H^{(0)}{}_{\lambda|\gamma)d} - \frac{1}{2}T^{(\beta\beta)}{}_{(\alpha\beta|}{}^{\lambda}H^{(0)}{}_{\lambda|\gamma)d} - \frac{1}{2}T^{(0)}{}_{(\alpha\beta|}{}^{g}H^{(\gamma\gamma)}{}_{g|\gamma)d} - \frac{1}{2}T^{(0)}{}_{(\alpha\beta|}{}^{g}H^{(\gamma\beta)}{}_{g|\gamma)d} - \frac{1}{2}T^{(0)}{}_{(\alpha\beta|}{}^{g}H^{(\beta\beta)}{}_{g|\gamma)d}\\
&\quad - \frac{1}{2}T^{(\gamma)}{}_{(\alpha\beta|}{}^{g}H^{(\beta)}{}_{g|\gamma)d} - \frac{1}{2}T^{(\beta)}{}_{(\alpha\beta|}{}^{g}H^{(\gamma)}{}_{g|\gamma)d} + \frac{1}{2}T^{(\gamma\gamma)}{}_{d(\alpha|}{}^{g}H^{(0)}{}_{g|\beta\gamma)} + \frac{1}{2}T^{(\gamma\beta)}{}_{d(\alpha|}{}^{g}H^{(0)}{}_{g|\beta\gamma)} + \frac{1}{2}T^{(\beta\beta)}{}_{d(\alpha|}{}^{g}H^{(0)}{}_{g|\beta\gamma)}\\
&\quad + \frac{1}{2}T^{(\gamma)}{}_{d(\alpha|}{}^{g}H^{(\gamma)}{}_{g|\beta\gamma)} + \frac{1}{2}T^{(\gamma)}{}_{d(\alpha|}{}^{g}H^{(\beta)}{}_{g|\beta\gamma)} + \frac{1}{2}T^{(\beta)}{}_{d(\alpha|}{}^{g}H^{(\gamma)}{}_{g|\beta\gamma)} + \frac{1}{2}T^{(\beta)}{}_{d(\alpha|}{}^{g}H^{(\beta)}{}_{g|\beta\gamma)} + \frac{1}{2}T^{(0)}{}_{d(\alpha|}{}^{g}H^{(\gamma\gamma)}{}_{g|\beta\gamma)}\\
&\quad + \frac{1}{2}T^{(0)}{}_{d(\alpha|}{}^{g}H^{(\gamma\beta)}{}_{g|\beta\gamma)} + \frac{1}{2}T^{(0)}{}_{d(\alpha|}{}^{g}H^{(\beta\beta)}{}_{g|\beta\gamma)}\\
&\quad + \gamma\big[R^{(\gamma)}{}_{(\alpha\beta|ef}R^{(0)}{}_{|\gamma)d}{}^{ef} + R^{(\beta)}{}_{(\alpha\beta|ef}R^{(0)}{}_{|\gamma)d}{}^{ef} + R^{(0)}{}_{(\alpha\beta|ef}R^{(\beta)}{}_{|\gamma)d}{}^{ef} + R^{(0)}{}_{(\alpha\beta|ef}R^{(\gamma)}{}_{|\gamma)d}{}^{ef}\big]\\
&\quad + \beta\big[F^{(\gamma)}{}_{(\alpha\beta|}F^{(0)}{}_{|\gamma)d} + F^{(\beta)}{}_{(\alpha\beta|}F^{(0)}{}_{|\gamma)d} + F^{(0)}{}_{(\alpha\beta|}F^{(\beta)}{}_{|\gamma)d} + F^{(0)}{}_{(\alpha\beta|}F^{(\gamma)}{}_{|\gamma)d}\big] = 0.
\end{aligned}
\tag{A.1}
$$

Applying the first order constraints and allowing H_{ABC} terms to drop out leaves the $\bar{H}_{ABC}$ contributions.

$$
\begin{aligned}
&\frac{1}{2}\nabla_{(\alpha|}H_{|\beta\gamma)d}{}^{(\beta\beta;\gamma\beta)} - \frac{1}{2}T^{(0)}{}_{(\alpha\beta|}{}^{\lambda}H^{(\gamma\beta)}{}_{\lambda|\gamma)d} - \frac{1}{2}T^{(0)}{}_{(\alpha\beta|}{}^{\lambda}H^{(\beta\beta)}{}_{\lambda|\gamma)d} - \frac{1}{2}T^{(\gamma\beta)}{}_{(\alpha\beta|}{}^{\lambda}H^{(0)}{}_{\lambda|\gamma)d} - \frac{1}{2}T^{(\gamma\beta)}{}_{(\alpha\beta|}{}^{\lambda}H^{(0)}{}_{\lambda|\gamma)d}\\
&\quad - \frac{1}{2}T^{(0)}{}_{(\alpha\beta|}{}^{g}H^{(\gamma\beta)}{}_{g|\gamma)d} - \frac{1}{2}T^{(0)}{}_{(\alpha\beta|}{}^{g}H^{(\beta\beta)}{}_{g|\gamma)d} + \frac{1}{2}T^{(\gamma\beta)}{}_{d(\alpha|}{}^{g}H^{(0)}{}_{g|\beta\gamma)} + \frac{1}{2}T^{(\beta\beta)}{}_{d(\alpha|}{}^{g}H^{(0)}{}_{g|\beta\gamma)}\\
&\quad + \frac{1}{2}T^{(\gamma\gamma)}{}_{d(\alpha|}{}^{g}H^{(0)}{}_{g|\beta\gamma)} + \gamma\big[R^{(\beta)}{}_{(\alpha\beta|ef}R^{(0)}{}_{|\gamma)d}{}^{ef} + R^{(0)}{}_{(\alpha\beta|ef}R^{(\beta)}{}_{|\gamma)d}{}^{ef}\big] = 0.
\end{aligned}
\tag{A.2}
$$

We therefore find as before that no order β^2 terms exist. Also there is no need to make modifications other than the adjustment of A_{abc} in the H sector.

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