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Higher-dimensional supersymmetry as an origin of the three families for quarks and leptons

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Abstract

In a $(0,1)$ supersymmetric (SUSY) six-dimensional gauge theory, a gauge fermion gives rise to box anomalies. These anomalies are completely canceled by assuming a vector multiplet of $(1,1)$ SUSY. With a $\mathbf{T}^2/\mathbf{Z}_3$ orbifold compactification of the extra two-dimensional space, the theory provides three chiral multiplets and three equivalent fixed points. We regard them as the origin of the three families of quarks and leptons. Quasi anarchy structure in the SU(5)-$\mathbf{5}^*$ sector and hence the bilarge mixing in the neutrino oscillation are explained quite naturally in this framework. We also discuss a family symmetry as a remnant of the higher-dimensional R symmetry.

1. Introduction

The triplicate family structure of quarks and leptons is one of the most mysterious features in particle physics, and it is expected to be an important indication of more fundamental physics beyond the standard model. In the four-dimensional spacetime all gauge anomalies are canceled within one family and there is no necessity to introduce other two families in nature. However, the anomaly cancellation in higher-dimensional theories imposes further nontrivial conditions on the theories, and it sometimes requires multiplication of massless particles.

In this Letter, we show that the triplicate family structure arises naturally from a six-dimensional supersymmetric (SUSY) gauge theory. We assume a SUSY SO(10) gauge theory and put the vector multiplet of SO(10) in the six-dimensional bulk. The anomalies in the six-dimensional spacetime are completely canceled out by introducing the $(1,1)$ SUSY vector multiplet in the bulk. We adopt a $\mathbf{T}^2/\mathbf{Z}_3$ orbifold to reduce the theory to a four-dimensional theory with $\mathcal{N}=1$ SUSY. We find that the SO(10) is broken down to $\mathrm{SU}(5)\times\mathrm{U}(1)_5$ and three families of $\mathbf{10}$'s of the SU(5) remain massless after the orbifolding. The $[\mathrm{SU}(5)]^3$ anomalies are localized at the three independent fixed points, which are canceled by introducing one $\mathbf{5}^*$ at the each fixed point (i.e., three $\mathbf{5}^*$'s). The three massless $\mathbf{10}$'s come from three SO(10)-*adjoint* $\mathcal{N}=1$ chiral multiplets Σ, Σ' and Σ'' in the $(1,1)$

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SUSY vector multiplet. Thus, we consider that the $(1,1)$ SUSY in the six-dimensional spacetime is the origin of three families of quarks and leptons.[1]

We have an $\mathrm{SU}(2)_{\mathbf{4}_+}$ family symmetry which is a subgroup of the R symmetry of the $(1,1)$ SUSY. The SU(5)-**10**'s transform as $\mathbf{1}+\mathbf{2}$ under this $\mathrm{SU}(2)_{\mathbf{4}_+}$ symmetry while the SU(5)-$\mathbf{5}^*$'s are singlets. The breaking of the $\mathrm{SU}(2)_{\mathbf{4}_+}$ symmetry leads to the observed mass hierarchies of the quarks and leptons and the CKM mixing angles. The bilarge mixing in the neutrino sector [1] is naturally explained, since the $\mathbf{5}^*$ are all equivalent to each other [2].

2. A SUSY SO(10) gauge theory in the six-dimensional spacetime

Let us consider a SUSY SO(10) gauge theory in the six-dimensional spacetime. The vector multiplet of the $(0,1)$ SUSY gives rise to irreducible box anomalies [3,4], and these anomalies must be canceled by introducing suitable SO(10)-charged hyper multiplets in the bulk.[2] The simplest and the most beautiful way to do this is to introduce an $(1,1)$ SUSY vector multiplet as a whole. Once we assume the $(1,1)$ SUSY in the six-dimensional spacetime, then not only the irreducible box anomalies but also reducible anomalies vanish. It is not necessary to resort to the Green–Schwartz mechanism [5,6] or to introducing extra particles. There is no global anomaly either [7,8]. If one takes a torus ($\mathbf{T}^2$) compactification of the extra two space dimensions x_4 and x_5, we obtain a Lagrangian for the Kaluza–Klein zero modes which has $\mathcal{N}=4$ SUSY in the four-dimensional spacetime. The Kaluza–Klein zero modes form an $\mathcal{N}=4$ SO(10) vector multiplet, $\mathcal{W}_\alpha$, Σ, Σ' and Σ'', where the $\mathcal{W}_\alpha$ is the field strength tensor ($\mathcal{N}=1$ vector multiplet) and the rests are SO(10)-*adjoint* $\mathcal{N}=1$ chiral multiplets. The two scalar components of the Σ are the two independent polarization modes of the six-dimensional SO(10) vector field in the fourth and fifth dimensions.

The R symmetry of the $(1,1)$ SUSY in the six-dimensional spacetime is $\mathrm{SU}(2)_{\mathbf{4}_-}\times\mathrm{SU}(2)_{\mathbf{4}_+}$.[3] These $\mathrm{SU}(2)_{\mathbf{4}_-}$ and $\mathrm{SU}(2)_{\mathbf{4}_+}$ are R symmetries that are associated to the six-dimensional SUSY charges,

$$\mathcal{Q}^{(6)}_{\mathbf{4}_-}=\begin{pmatrix} & -\mathcal{Q}^{(4)2}_\alpha \\ -\overline{\mathcal{Q}}^{(4)\dot\alpha}_1 & \end{pmatrix} \quad \text{and}$$
$$\mathcal{Q}^{(6)}_{\mathbf{4}_+}=\begin{pmatrix} \mathcal{Q}^{(4)4}_\alpha & \\ & \overline{\mathcal{Q}}^{(4)\dot\alpha}_3 \end{pmatrix}, \tag{1}$$

which belong to $\mathbf{4}_-$ and $\mathbf{4}_+$ spinor representations of the $\mathrm{SO}(5,1)$, respectively. After a Kaluza–Klein reduction, the six-dimensional SUSY charges are decomposed into four four-dimensional SUSY charges, $\mathcal{Q}^{(4)1,2,3,4}_\alpha$, and not only the $\mathrm{SU}(2)_{\mathbf{4}_-}\times\mathrm{SU}(2)_{\mathbf{4}_+}$ R symmetry but the rotational symmetry $\mathrm{SO}(2)_{45}$ of the extra space dimensions is also regarded as a subgroup of the $\mathrm{SU}(4)_\mathrm{R}$ R symmetry of the $\mathcal{N}=4$ SUSY in four-dimensional spacetime (i.e., $\mathrm{SO}(2)_{45}\times\mathrm{SU}(2)_{\mathbf{4}_-}\times\mathrm{SU}(2)_{\mathbf{4}_+}\subset\mathrm{SU}(4)_\mathrm{R}$). The four (four-dimensional) SUSY charges $\mathcal{Q}^{(4)1,2,3,4}_\alpha$ transform as a **4** representation of the $\mathrm{SU}(4)_\mathrm{R}$, four fermions in the four chiral multiplets (of Kaluza–Klein zero modes), $\mathcal{W}_\alpha$, Σ, Σ' and Σ'', as a $\mathbf{4}^*$ representation, and six real scalars in the Σ, Σ' and Σ'' as a $\mathbf{4}^*\wedge\mathbf{4}^*\simeq\mathbf{6}$, respectively. In particular, we note for the later purpose that the $\mathrm{SU}(2)_{\mathbf{4}_+}$ commutes with the $\mathcal{N}=1$ SUSY transformation generated by $\mathcal{Q}^{(4)1}$ and that the two $\mathcal{N}=1$ chiral multiplets Σ' and Σ'' form a doublet of this $\mathrm{SU}(2)_{\mathbf{4}_+}$ symmetry (see, e.g., [9] or the appendix).

Now we compactify the $(1,1)$ SUSY SO(10) gauge theory on the $\mathbf{T}^2/\mathbf{Z}_3\langle\sigma\rangle$ geometry rather than on the $\mathbf{T}^2$ (see Fig. 1) to obtain a four-dimensional theory with only $\mathcal{N}=1$ SUSY.[4] The generator σ rotates the

[1] Bosonic components of these three Σ, Σ' and Σ'' multiplets may be regarded as six independent polarizations (45, 67 and 89 directions) of a ten-dimensional vector field, since the $(1,1)$ SUSY vector multiplet in the six dimensions can be obtained through a compactification of $\mathcal{N}=1$ vector multiplet of the ten-dimensional spacetime. In this sense, the triplicate family structure may originate from the extra six space dimensions.

[2] It is pointed out in Ref. [3] that the anomaly is finite and unique despite the nonrenormalizability of the theory. T.Y. thanks M. Shifman for raising this issue at "Peccei's Fest".

[3] For conventions adopted in this Letter, see [9] or the appendix of this Letter.

[4] It is argued in Ref. [10] that the number of family can be constrained by considering the gauge anomalies in six-dimensional field theories. However, "the number of family in the six-dimensional spacetime" which they discuss has, in principle, no direct connection to the number of family as they admit by themselves. It is only after giving a definite way and/or principle to obtain chiral fermions in the four-dimensional spacetime that the use of higher-dimensional spacetime makes sense in discussing the number of

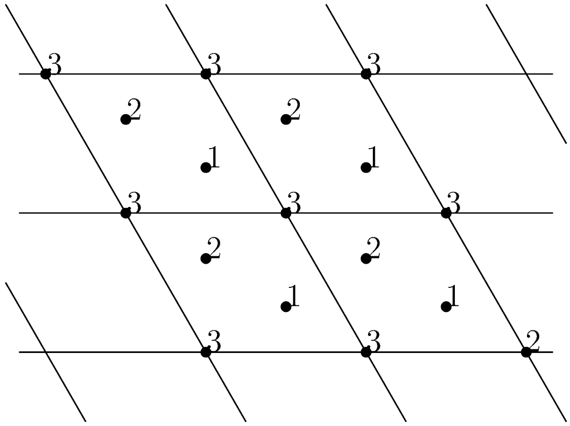

Fig. 1. A picture of the $\mathbf{T}^2/\mathbf{Z}_3\langle\sigma\rangle$ geometry is given. Unit cell of the $\mathbf{T}^2$ torus is described by parallel lines and three fixed points labeled by 1, 2, 3 are given on it. One can see that all three fixed points are equivalent to each other.

extra two-dimensional space by angle $-(2/3)2\pi$:

$$\sigma : (x_4 + ix_5) \mapsto \omega^{-2}(x_4 + ix_5), \tag{2}$$

where $\omega \equiv e^{2\pi i/3}$. Some of Kaluza–Klein zero modes are always projected out from the Hilbert space of the theory on the orbifold $\mathbf{T}^2/\mathbf{Z}_3\langle\sigma\rangle$. We take, here, the orbifold projection conditions as follows [11];

$$\mathcal{W}_\alpha = \gamma_\sigma \mathcal{W}_\alpha \gamma_\sigma^{-1}, \tag{3}$$

$$\Sigma = \omega^{-2}\gamma_\sigma \Sigma \gamma_\sigma^{-1}, \tag{4}$$

$$\Sigma' = \omega\gamma_\sigma \Sigma' \gamma_\sigma^{-1}, \tag{5}$$

$$\Sigma'' = \omega\gamma_\sigma \Sigma'' \gamma_\sigma^{-1}, \tag{6}$$

where we have taken an $\mathrm{SU}(4)_\mathrm{R}$-twist as $\mathrm{diag}(1, \omega^2, \omega^{-1}, \omega^{-1}) \in \mathrm{SU}(4)_\mathrm{R}$. The gauge-twisting matrix γ_σ associated with the generator σ is

$$\gamma_\sigma = \mathrm{diag}\big(\overbrace{\omega, \ldots, \omega}^{5}, \overbrace{\omega^{-1}, \ldots, \omega^{-1}}^{5}\big) \in \mathrm{SO}(10) \tag{7}$$

in the Cartan-diagonal base. Note that the $\mathrm{SO}(2)_{45}$ rotation, $\mathrm{diag}(\omega, \omega, \omega^{-1}, \omega^{-1}) \in \mathrm{SU}(4)_\mathrm{R}$, given in Eq. (2) is accompanied by a twist of the internal symmetry, $\mathrm{diag}(\omega^{-1}, \omega, 1, 1) \in \mathrm{SU}(2)_{\mathbf{4}_-}$, so that the combined $\mathrm{SU}(4)_\mathrm{R}$-twist, $\mathrm{diag}(1, \omega^2, \omega^{-1}, \omega^{-1})$, belongs to an SU(3) subgroup of the $\mathrm{SU}(4)_\mathrm{R}$; the $\mathcal{N} = 1$ SUSY survives when and only when the $\mathrm{SU}(4)_\mathrm{R}$-twist belongs to the SU(3) subgroup of which the $\mathcal{Q}_\alpha^{(4)1}$ is singlet [11,12].

The SO(10) gauge symmetry is now broken down to $\mathrm{SU}(5) \times \mathrm{U}(1)_5$, and the massless particles remaining in the Hilbert space are the $\mathrm{SU}(5) \times U(1)_5$ $\mathcal{N} = 1$ vector multiplets and three SU(5)-**10** $\mathcal{N} = 1$ chiral multiplets. The vector multiplets arise from the $\mathcal{W}_\alpha$ and the three **10**'s from Σ, Σ' and Σ''. We identify these three **10**'s with those of quarks and leptons.[5] Therefore, the origin of the three families is the (1, 1) SUSY in the six-dimensional spacetime (i.e., $\mathcal{N} = 4$ SUSY in the four-dimensional spacetime).

Because there are three families of **10**'s in the bulk, we have $[\mathrm{SU}(5)]^3$ anomaly. Such an anomaly localizes only at fixed points of the orbifold [17]. In the present $\mathbf{T}^2/\mathbf{Z}_3\langle\sigma\rangle$ orbifold the anomaly distributes at the three fixed points with the same amount since they are all equivalent to each other[6] (see Fig. 1). Therefore, the simplest way to cancel these anomalies is to introduce a $\mathbf{5}^*$ at each fixed point (i.e., three $\mathbf{5}^*$'s as a whole). We identify these $\mathbf{5}^*$'s with the three families of $\mathbf{5}^*$'s of quarks and leptons. The charges of these $\mathbf{5}^*$'s under the surviving $\mathrm{U}(1)_5$ gauge symmetry is still arbitrary, and hence the mixed anomalies $\mathrm{U}(1)_5 \cdot [\mathrm{SU}(5)]^2$ does not vanish in general. However, if the $\mathrm{U}(1)_5$ charge of the $\mathbf{5}^*$'s are (-3) times those of the **10**'s, then, this

family and family structure. In fact, one obtains any number of families in their approach.

5 A coset space $\mathrm{SO}(10)/(\mathrm{SU}(5) \times \mathrm{U}(1)_5)$ contains SU(5)-**10**. Possible connection between this fact and the origin of the SU(5)-**10** of quarks and leptons was pointed out long time ago in [14,15]. Ref. [16] also obtains three **10**'s from SO(11) ten-dimensional $\mathcal{N} = 1$ vector multiplet. They considered the Type I string theory on a $\mathbf{T}^6/\mathbf{Z}_3$ orientifold, and three families of $\mathrm{SU}(5) - \mathbf{5}^*$ also survives the orbifold projection there. However, this model is not acceptable as a realistic model because too rapid proton decay is inevitable through dimension-four operators.

6 We can also confirm this distribution by an explicit calculation. The distribution function is given by

$$A(\mathbf{y}) = 3\sum_{\mathbf{k}} \left(\left|\psi^{\mathbf{Z}_3}_{\mathbf{10},\mathbf{k}}(\mathbf{y})\right|^2 - \left|\psi^{\mathbf{Z}_3}_{\mathbf{10}^*,\mathbf{k}}(\mathbf{y})\right|^2\right), \tag{8}$$

where $\mathbf{k}$ runs for all Kaluza–Klein momenta, $\mathbf{y}$ is the coordinate of the torus $\mathbf{T}^2$ and the wave functions are

$$\psi^{\mathbf{Z}_3}_{\mathbf{10}^{(*)},\mathbf{k}} = \frac{1}{3\sqrt{V}} \sum_{n=0}^{2} \gamma^n_{\mathbf{10}^{(*)}} e^{i\mathbf{k}\cdot(\sigma^n \cdot \mathbf{y})}, \tag{9}$$

where $\gamma_{\mathbf{10}} = 1$ and $\gamma_{\mathbf{10}^*} = \omega^{-1}$, and V is the volume of the torus $\mathbf{T}^2$.

anomaly is automatically canceled at each fixed point. Other mixed anomalies $\mathrm{U}(1)_5 \cdot [\text{gravity}]^2$ and $[\mathrm{U}(1)_5]^3$ can be canceled simultaneously by introducing a right-handed neutrino at each fixed point. This extra $\mathrm{U}(1)_5$ gauge symmetry, which is usually called as the fiveness, is a linear combination of the $\mathrm{U}(1)_\mathrm{Y}$ and $\mathrm{U}(1)_{\mathrm{B-L}}$, and the small neutrino masses are naturally obtained by the see-saw mechanism [18] when the $\mathrm{U}(1)_{\mathrm{B-L}}$ is spontaneously broken. Even if the $\mathrm{U}(1)_5$ charge of the $\mathbf{5}^*$'s does not satisfy the above relation, all three anomalies discussed above can be canceled by invoking a generalized Green–Schwarz mechanism [5,13] at each fixed point.

If one starts with an E_6 vector multiplet of the $(1,1)$ SUSY in the six-dimensional spacetime, then three families of SO(10)-**16** survives the orbifold projection on the $\mathbf{T}^2/\mathbf{Z}_3$ [14,15]. In this case, there is no $[\mathrm{SO}(10)]^3$ anomaly at each fixed point.

3. Discussion

We discuss, in this section, phenomenological consequences of the R symmetry $\mathrm{SO}(2)_{45} \times \mathrm{SU}(2)_{\mathbf{4}_-} \times \mathrm{SU}(2)_{\mathbf{4}_+}$. Since the three families of **10**'s originate from a single vector multiplet of the $(1,1)$ SUSY, some part of the R symmetry becomes a inter-family symmetry and some part becomes a low-energy R symmetry of the four-dimensional $\mathcal{N}=1$ SUSY.

3.1. $\mathrm{SU}(2)_{\mathbf{4}_+}$ *family symmetry*

The orbifold projection conditions Eqs. (3)–(6) do not violate the $\mathrm{SU}(2)_{\mathbf{4}_+}$ symmetry, and hence we assume this symmetry to be preserved even in the theory on the $\mathbf{T}^2/\mathbf{Z}_3$ orbifold. This $\mathrm{SU}(2)_{\mathbf{4}_+}$ symmetry is a pure family symmetry in the sense that the Grassmann coordinates of the $\mathcal{N}=1$ superspace do not rotate under this symmetry. The three families of **10**'s transform as $\mathbf{1}+\mathbf{2}$ (which we denote as $\mathbf{10}_3 + \mathbf{10}_a|_{a=1,2}$) under this SU(2) family symmetry. On the other hand, the $\mathbf{5}^*$'s are all singlets of the family symmetry $\mathrm{SU}(2)_{\mathbf{4}_+}$, since the $\mathbf{5}^*$'s localize at different fixed points while the $\mathrm{SU}(2)_{\mathbf{4}_+}$ is an internal symmetry and it does not exchange fields on separated points of spacetime. The anarchy structure in the neutrino sector [2] is naturally expected in this framework, since there is no distinction between the three $\mathbf{5}^*$'s.

We introduce Higgs multiplets, $H(\mathbf{5})$ and $\overline{H}(\mathbf{5}^*)$, at one of the fixed points. Given this situation, we have to consider the orbifold geometry whose length scale is of order of the fundamental scale, because otherwise the Yukawa couplings would be highly suppressed for the two families of $\mathbf{5}^*$'s which do not localize at the same fixed point of the Higgs multiplets, and the resulting mass spectra of the down-type quarks and charged leptons would be unrealistic. Therefore, the three fixed points should be close to each other, and no suppression of interaction is expected between fields that localize at different fixed points. Thus, there is no essential difference between the three $\mathbf{5}^*$'s, and the quasi anarchy structure is still expected in the $\mathbf{5}^*$'s.

As long as the $\mathrm{SU}(2)_{\mathbf{4}_+}$ family symmetry is unbroken, only one family of quarks and charged leptons acquire their masses:

$$W = y\mathbf{10}_3\mathbf{10}_3 H(\mathbf{5}) + y_i \mathbf{5}_i^* \cdot \mathbf{10}_3 \cdot \overline{H}(\mathbf{5}^*), \quad i = 1,2,3. \tag{10}$$

On the other hand, Majorana neutrino masses are allowed by the family symmetry for all three families:

$$W = \frac{\kappa_{ij}}{M_R} \mathbf{5}_i^* H(\mathbf{5}) \mathbf{5}_j^* H(\mathbf{5}), \quad i,j = 1,2,3. \tag{11}$$

Coefficients y, y_i and κ_{ij} are expected to be of order 1. The above result explains why the masses are large for the quarks and charged leptons in the third family and why the mixings among families are large in the neutrino sector [1].

Now we have to introduce small breaking of the $\mathrm{SU}(2)_{\mathbf{4}_+}$ symmetry so that the first and the second families of quarks and leptons are able to obtain their non-zero masses. Let us suppose that the $\mathrm{SU}(2)_{\mathbf{4}_+}$ breaking is implemented through two doublets,

$$\phi^a = \begin{pmatrix} 0 \\ \epsilon \end{pmatrix}, \qquad \tilde{\phi}^a = \begin{pmatrix} \epsilon' \\ \epsilon'' \end{pmatrix}, \tag{12}$$

where we assume $\epsilon', \epsilon'' \lesssim \mathcal{O}(\epsilon)$. Then, the mass matrices are roughly given by

$$m_u \sim \begin{pmatrix} \epsilon'^2 & \epsilon\epsilon' & \epsilon' \\ \epsilon\epsilon' & \epsilon^2 & \epsilon \\ \epsilon' & \epsilon & 1 \end{pmatrix}, \qquad m_{d,e} \sim \begin{pmatrix} \epsilon' & \epsilon & 1 \\ \epsilon' & \epsilon & 1 \\ \epsilon' & \epsilon & 1 \end{pmatrix},$$
$$m_\nu \sim \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}, \tag{13}$$

which are obtained from additional superpotential,

$$W = \mathbf{10}_3(\mathbf{10}\cdot\Phi)H + (\mathbf{10}\cdot\Phi)(\mathbf{10}\cdot\Phi)H + y'_i\mathbf{5}^*_i\cdot(\mathbf{10}\cdot\Phi)\cdot\overline{H}, \tag{14}$$

where Φ represents ϕ and $\tilde{\phi}$. In particular, the empirical relation $m_s/m_b \sim m_\mu/m_\tau \sim \epsilon$, $|V_{cb}| \sim \epsilon$ and $m_c/m_t \sim \epsilon^2$ is obtained. The bilarge mixing in the neutrino sector is also a preferable consequence of the anarchy structure of the $\mathbf{5}^*$'s. If one assumes $\epsilon' \sim \epsilon^2$, then the mass matrices are similar to that in the Froggatt–Nielsen framework [19], and their phenomenological success is discussed in various references [20,21]. Note that we have neglected the effects of the GUT breaking. One can also understand the violation of the SU(5) GUT relation in the masses of the first and the second families, if one takes account of contributions that involve GUT breaking vacuum-expectation values.

3.2. Low-energy R symmetry

It is clear that the $\mathrm{SO}(2)_{45}\times\mathrm{SU}(2)_{\mathbf{4}_-}$ subgroup is not preserved in the orbifold projection conditions Eqs. (3)–(6). Only the $\mathrm{SO}(2)_{45}$ and the Cartan part $\mathrm{U}(1)_{\mathbf{4}_-}$ of the $\mathrm{SU}(2)_{\mathbf{4}_-}$ can be preserved. A suitable linear combination of these $\mathrm{SO}(2)_{45}$ and the $\mathrm{U}(1)_{\mathbf{4}_-}$ yields a U(1) R symmetry[7] which might be relevant in the low-energy physics. This U(1) R symmetry is specified so that the three families of $\mathbf{10}$'s have the same charge: 2/3. We assume that the other independent linear combination of the two U(1) symmetries is broken down (otherwise necessary Yukawa couplings would be forbidden). The charge assignment of the U(1) R symmetry is not determined for the three $\mathbf{5}^*$'s or Higgs multiplets, since their origin is not clear. However, those charges are fixed by phenomenological requirements that the U(1) R symmetry allows up-type and down/charged-lepton-type Yukawa couplings and Majorana neutrino masses: that is, 1/3 for $\mathbf{5}^*$, 2/3 for $H(\mathbf{5})$ and 1 for $\overline{H}(\mathbf{5}^*)$. Then, as a consequence, the notorious dimension-four proton decay operators $W = \mathbf{5}^*\cdot\mathbf{10}\cdot\mathbf{5}^*$ and an enormous mass-term for the Higgs multiplets are forbidden.[8]

Anisotropy of the orbifold geometry $\mathbf{T}^2/\mathbf{Z}_3$, however, might lead to further breaking of this low-energy R symmetry;[9] only a discrete subgroup of the $\mathrm{SO}(2)_{45}$ symmetry is left unbroken. We show, in the following, that even if there are operators that violate continuous U(1) R symmetry due to this anisotropy, the U(1) R symmetry is preserved by[10] mod charge 2. This is because when the $\mathbf{10}$'s transform as $\mathbf{10}(\theta) \to e^{-i\alpha 2/3}\mathbf{10}(e^{i\alpha}\theta)$, the U(1) R symmetry,

$$\begin{aligned}&\mathrm{diag}(e^{i\alpha/3}, e^{i\alpha/3}, e^{-i\alpha/3}, e^{-i\alpha/3})\\ &\quad\times\mathrm{diag}(e^{i\alpha 2/3}, e^{-i\alpha 2/3}, 1, 1)\\ &\qquad\in \mathrm{SO}(2)_{45}\times\mathrm{SU}(2)_{\mathbf{4}_-}\subset\mathrm{SU}(4)_{\mathrm{R}},\end{aligned}$$

rotates the extra-dimensional space $\mathbf{T}^2$ by $(x_4 + ix_5) \to e^{-i\alpha 2/3}(x_4 + ix_5)$. The orbifold geometry we consider has a discrete rotational symmetry by angle $2\pi/3$, and this means that the R symmetry is preserved for $\alpha \in \pi\mathbf{Z}$. The R symmetry preserved by mod 2 is sufficient to forbid the dimension-four proton decay operators and the Higgs mass term, since the R charges of these operators are

[7] Here, a term "R symmetry" is used in its narrow sense: a symmetry that rotates the Grassmann coordinates of the $\mathcal{N}=1$ SUSY.

[8] The color-triplet components of the $H(\mathbf{5})$ and $\overline{H}(\mathbf{5}^*)$ can receive the GUT-breaking mass term keeping this U(1) R symmetry [22].

[9] If the $\mathrm{SU}(2)_{\mathbf{4}_+}$-symmetry breakings given in Eq. (12) were charged also under this symmetry, then, the symmetry would be broken further. However, we assume that the $\mathrm{SU}(2)_{\mathbf{4}_+}$-symmetry breaking is not charged either under the $\mathrm{SU}(2)_{\mathbf{4}_-}$ or under the $\mathrm{SO}(2)_{45}$. This assumption does not lead to any apparent contradictions, since we do not know the origin of the breaking Eq. (12).

[10] There are two ways in describing a discrete subgroup of a U(1) symmetry: one is the "$\mathbf{Z}_N$ subgroup" and the other is the "U(1) mod charge Q". The former is used when one can take all U(1) charges to be integers, while the latter is used when there is a canonical normalization of the U(1) charges. $N = Q$ when all charges are integers in their canonical normalization, but it is not always the case. In the present case, the U(1) R symmetry have a canonical normalization (i.e., the $\mathcal{N}=1$ Grassmann coordinates have charge 1 and a superpotential has charge 2), and some fields have fractional charges in this normalization. Thus, we take the latter description "U(1) R symmetry mod charge Q" in the text. If one would rescale the R charges by multiplying 3 to make all the charges integral (then the superpotential has R charge 6), then "the U(1) R mod charge 2" could be referred to as "$\mathbf{Z}_6$ R symmetry". Proton decay operators $\mathbf{5}^*\cdot\mathbf{10}\cdot\mathbf{5}^*$ and $\mathbf{5}^*\mathbf{101010}$ have R charge 4 and 7, respectively, and the doublet-Higgs mass term 5 in this normalization.

$4/3 \neq 2 \pmod 2$ and $5/3 \neq 2 \pmod 2$, respectively.

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Appendix A. SUSY and R Symmetry in the six-dimensional spacetime

A.1. Chirality

Spinor representations of the SO$(5, 1)$ is $\mathbf{4}_+ \oplus \mathbf{4}_-$. Both $\mathbf{4}_+$ and $\mathbf{4}_-$ representations have four complex-valued components. These four-component spinors are regarded as Dirac spinors of the SO$(3, 1)$, or in other words, are comprised of $(1/2, 0)$-representation (left-handed) and $(0, 1/2)$-representation (right-handed) two-component Weyl spinors. Although $\mathbf{4}_\pm$ representations are also sometimes referred to as left and right spinors, we do not use these terminologies in order not to make confusion with the same 'left- and right-' for spinors of SO$(3, 1)$.

Complex conjugate of the $\mathbf{4}_+$ spinor is isomorphic to itself, and so is the $\mathbf{4}_-$. That is,

$$\mathbf{4}_+^* \simeq \mathbf{4}_+, \qquad \mathbf{4}_-^* \simeq \mathbf{4}_-. \tag{A.1}$$

This is in contrast to the fact that the complex conjugate of the $(1/2, 0)$-representation is the $(0, 1/2)$-representation and vice versa in the four-dimensional spacetime. Therefore, in the six-dimensional spacetime, spinor fields and their complex conjugates (in other words, fermionic states and their CPT conjugates) belong to the same spinor representation of the SO$(5, 1)$ (i.e., $\mathbf{4}_+$ or $\mathbf{4}_-$), *that is,* they have a definite chirality.

Mass partners of $\mathbf{4}_+$-spinor fields(states) are $\mathbf{4}_-$ and vice versa. This is also in contrast with the fact that the mass partner of a $(1/2, 0)$-spinor is again an $(1/2, 0)$-spinor in the four-dimensional spacetime. In particular, box anomalies from a $\mathbf{4}_-$-state are opposite to those of a $\mathbf{4}_+$-state with otherwise the same representation.

A.2. SUSY and R symmetry

Each SUSY charge in the six-dimensional spacetime, which is a spinor of the SO$(5, 1)$, has definite chirality. This is why the SUSY of the six-dimensional field theory is characterized by a pair of non-negative integers $(\mathcal{N}_+, \mathcal{N}_-)$; $\mathcal{N}_+$ and $\mathcal{N}_-$ are the number of SUSY charges that transform as $\mathbf{4}_+$ and $\mathbf{4}_-$, respectively. In gauge theories with $(0, 1)$ SUSY, the SUSY charge and fermions in hyper multiplets are $\mathbf{4}_-$-spinors while the parameter of the SUSY transformation and fermions in vector multiplets are $\mathbf{4}_+$-spinors. In gauge theories with $(1, 1)$ SUSY there are SUSY charges in $\mathbf{4}_+$ and $\mathbf{4}_-$, transformation parameters in $\mathbf{4}_-$ and $\mathbf{4}_+$, and gauge fermions in $\mathbf{4}_-$ and $\mathbf{4}_+$. In particular, all box anomalies are canceled within a single vector multiplet in $(1, 1)$ SUSY gauge theories, since vector multiplets contain both $\mathbf{4}_+$ and $\mathbf{4}_-$ gauge fermions.

One can think of an SU(2) transformation that exchanges a SUSY charge and its complex conjugate, since the complex conjugate belong to the same spinor representation as the original one.[11] This internal symmetry that acts on SUSY charges is an R symmetry of the six-dimensional SUSY theories. There are two independent SU(2) transformations in the $(1, 1)$ SUSY theories; one (which we denote as SU$(2)_{\mathbf{4}_+}$) acts on the SUSY charge in $\mathbf{4}_+$-spinor, and the other (SU$(2)_{\mathbf{4}_-}$) on the SUSY charges in $\mathbf{4}_-$-spinor. Thus, the R symmetry of the $(1, 1)$ SUSY theories is SU$(2)_{\mathbf{4}_-} \times$ SU$(2)_{\mathbf{4}_+}$.

A.3. Kaluza–Klein reduction

Let us take a toroidal compactification and consider an effective theory of Kaluza–Klein zero-modes. $(0, 1)$ SUSY gauge theories become $\mathcal{N} = 2$ SUSY theories, since the SUSY charge $\mathcal{Q}^{(6)}_{\mathbf{4}_-}$ of six-dimensional theories consists of two independent SUSY charges $\mathcal{Q}^{(4)1}_\alpha$ and $\mathcal{Q}^{(4)2}_\alpha$ of four-dimensional theories. $(1, 1)$-SUSY theories become $\mathcal{N} = 4$ SUSY theories with four SUSY charges $\mathcal{Q}^{(4)1\cdots4}_\alpha$. Gauge fermions also decompose into four Weyl spinors $\chi_{\alpha,1\cdots4}$; the $\mathbf{4}_-$ gauge

[11] For more detail, see Section 3 and the appendix of [9].

fermion into the $\chi_{1,2}$ and the $\mathbf{4}_+$ gauge fermion into the $\chi_{3,4}$. The $\mathcal{Q}^{(4)1,2}$ and $\chi_{1,2}$ are doublets of the $\mathrm{SU}(2)_{\mathbf{4}_-}$ R symmetry, and $\mathcal{Q}^{(4)3,4}$ and $\chi_{3,4}$ are doublets of the $\mathrm{SU}(2)_{\mathbf{4}_+}$.

Rotational symmetry of the two extra-dimensional space $\mathrm{SO}(2)_{45}$ that was originally a subgroup of the Lorentz symmetry $\mathrm{SO}(5,1)$ is now regarded as an internal symmetry. This $\mathrm{SO}(2)_{45}$ is also an R symmetry, since the SUSY charges transform non-trivially under the $\mathrm{SO}(5,1)$ and hence under the $\mathrm{SO}(2)_{45}$.

It is useful to invoke SU(4) notation in summarizing relation between various R symmetries ($\mathrm{SU}(2)_{\mathbf{4}_-} \times \mathrm{SU}(2)_{\mathbf{4}_+}$ and the $\mathrm{SO}(2)_{45}$) and their action on various fields. SU(4) is the maximal R symmetry of the $\mathcal{N}=4$ SUSY gauge theories on the four-dimensional spacetime and the above R symmetries are regarded as subgroups of the $\mathrm{SU}(4)_\mathrm{R}$ symmetry. SUSY generators $\mathcal{Q}_\alpha^{(4)a}$ ($a = 1, \ldots, 4$) are $\mathbf{4}$ of the $\mathrm{SU}(4)_\mathrm{R}$ symmetry, SUSY transformation parameters and gauge fermions $\chi_{\alpha,a}$ are $\mathbf{4}^*$, and 4×4 2nd rank antisymmetric tensor $(\mathbf{4}^* \wedge \mathbf{4}^*)$ φ_{ab} with a reality condition $\varphi_{ab}\epsilon^{abcd}/2 = \varphi^{*cd}$ are six scalars of the $\mathcal{N}=4$ multiplet, among which $\varphi_{12} \propto (A_4 + iA_5)$, the two independent polarizations of the vector field in the extra two space directions. The $\mathrm{SU}(2)_{\mathbf{4}_-} \times \mathrm{SU}(2)_{\mathbf{4}_+}$ R symmetry is included in the $\mathrm{SU}(4)_\mathrm{R}$ as

$$\begin{pmatrix} \mathrm{SU}(2)_{\mathbf{4}_-} & \\ & \mathrm{SU}(2)_{\mathbf{4}_+} \end{pmatrix} \subset \mathrm{SU}(4)_\mathrm{R} \tag{A.2}$$

in the 4×4 fundamental representation. The $\mathrm{SO}(2)_{45}$ subgroup is included in the $\mathrm{SU}(4)_\mathrm{R}$ as

$$\mathrm{diag}\big(e^{i\alpha}, e^{i\alpha}, e^{-i\alpha}, e^{-i\alpha}\big) \subset \mathrm{SU}(4)_\mathrm{R}, \tag{A.3}$$

which corresponds to the rotation of the 4th and 5th plane by

$$\begin{aligned} &(x_4 + ix_5) \to e^{-2i\alpha}(x_4 + ix_5), \\ &(A_4 + iA_5) \to e^{-2i\alpha}(A_4 + iA_5). \end{aligned} \tag{A.4}$$

$\mathcal{N}=1$ SUSY generated by $\mathcal{Q}^{(4)1}$ survives the orbifolding, as long as the $\mathrm{SU}(4)_\mathrm{R}$-twist (see the text) of the orbifold projection is included in the lower right SU(3) subgroup of the $\mathrm{SU}(4)_\mathrm{R}$. Under this $\mathcal{N}=1$ SUSY, χ_1 is the gaugino, and the χ_2 is the SUSY partner of the $\varphi_{12} \propto (A_4 + iA_5)$. It will be clear that the $\mathrm{SU}(2)_{\mathbf{4}_+}$ now acts like an ordinary (non-R) symmetry in the four-dimensional effective theory with $\mathcal{N}=1$ SUSY, since it keeps the χ_1 invariant.

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