Available online at www.sciencedirect.com
ScienceDirect
Physics Letters B 659 (2008) 891–893
PHYSICS LETTERS B
www.elsevier.com/locate/physletb

The cosmological constant is probably zero, and a proof is possibly right

Zhong Chao Wu

Department of Physics, Zhejiang University of Technology, Hangzhou 310032, China

Received 21 September 2007; received in revised form 9 November 2007; accepted 11 December 2007

Available online 28 December 2007

Editor: M. Cvetič

Abstract

Hawking proposed that the cosmological constant is probably zero in quantum cosmology. Duff claimed that Hawking's proof is invalidated. Using the right configuration for the wave function of the universe, we provide a complete proof.

PACS: 04.60.-m; 98.80.Qc; 98.80.-k

Keywords: Cosmological constant; Quantum cosmology; Euclidean quantum gravity

The largest discrepancy between theoretical calculations and observations in the history of physics might be the value of the cosmological constant. In order to resolve this, Hawking proposed in quantum cosmology that "the apparent cosmological constant is not necessarily zero but that zero is by far the most probable value" [1].

The contribution to the cosmological constant comes from the ground states of all matter fields and "the bare cosmological constant". It is also known that a rank-3 antisymmetric tensor gauge field $A_{\nu\rho\sigma}$ could contribute to the cosmological constant [2]. $A_{\nu\rho\sigma}$ arises naturally in the $N = 8$ supergravity in four dimensions. Hawking showed that when the total cosmological constant becomes very small, the Euclidean action is most negative, and the probability is therefore highest [1].

The relative creation probability of the universe is [3]

$$P \approx \exp(-I), \tag{1}$$

where I is the Euclidean action of the seed instanton. The action takes the form

$$I = -\int_M \left(\frac{1}{16\pi}(R - 2\Lambda_0) - \frac{1}{48} F^{\mu\nu\rho\sigma} F_{\mu\nu\rho\sigma} \right), \tag{2}$$

where the Planckian unit is used, R is the scalar curvature, Λ_0 represents the all contributions of "the bare cosmological constant" and matter fields apart from $A_{\nu\rho\sigma}$ [1], and F is the field strength of $A_{\nu\rho\sigma}$

$$F_{\mu\nu\rho\sigma} = \partial_{[\mu} A_{\nu\rho\sigma]}. \tag{3}$$

The gauge potential has the following gauge freedom

$$A_{\nu\rho\sigma} \to A_{\nu\rho\sigma} + \partial_{[\nu} \lambda_{\rho\sigma]}. \tag{4}$$

Hawking argued that for the seed S_4 instanton the solution to the gauge field equation

$$F^{\mu\nu\rho\sigma}{}_{;\sigma} = 0 \tag{5}$$

should take the form

$$\sqrt{g} F^{\mu\nu\rho\sigma} = \kappa \epsilon^{\mu\nu\rho\sigma}, \tag{6}$$

where κ is an arbitrary constant. In this model the S_4 instanton will evolve into the universe with the de Sitter spacetime metric.

One can see from (2) that the F^2 term in the action behaves like an effective cosmological constant

$$\Lambda_{\text{eff}} = 4\pi\kappa^2 \tag{7}$$

and the total cosmological constant is

$$\Lambda_{\text{total}} = \Lambda_0 + \Lambda_{\text{eff}}. \tag{8}$$

The radius of S_4 is $(3/\Lambda_{\text{total}})^{1/2}$, and the action is $-3\pi/\Lambda_{\text{total}}$, here it is assumed that Λ_{total} is positive. The action is the negative of entropy of the created de Sitter spacetime. From (1),

E-mail address: zcwu2007@yahoo.com.

0370-2693
doi:10.1016/j.physletb.2007.12.019

it follows that the most probable configuration will be those with very small values of Λ_{total}, and nature will automatically select the right value of κ for this [1].

However, Duff pointed out that substituting a field configuration into the action and varying it is not equivalent to substituting the configuration into the field equations [4]. He explicitly showed that for the configuration (6), the Einstein equation is

$$G_{\mu\nu} = 4\pi\kappa^2 g_{\mu\nu} - \Lambda_0 g_{\mu\nu}. \tag{9}$$

This implies that from the field equation the total cosmological constant must be $\Lambda_0 - 4\pi\kappa^2$, instead of $\Lambda_0 + 4\pi\kappa^2$! Apparently, from observing the evolution of the universe, the cosmological constant should take this value.

To recover the right effective cosmological constant in the action, Aurelia, Nicolai and Townsend added a total divergence term to (2) [2]

$$I_{\text{div}} = -\int_M dx^4 \frac{1}{24}\kappa\epsilon^{\mu\nu\rho\sigma} F_{\mu\nu\rho\sigma}. \tag{10}$$

But in this case the value of κ, i.e., Λ_{eff} is fixed, in contradiction to the Hawking mechanism. Thus, this prescription does not work for the cosmological constant issue.

Therefore, Duff claimed that "this invalidates Hawking's proof that the cosmological constant is probably zero" [4].

The motivation of this Letter is to resolve this dilemma.

The probability expression (1) is derived from the wave function of the universe [3], and the equator of the instanton and the other fields at the equator are the configuration of the wave function. For the action (2) one implicitly chooses 3-metric h_{mn} of the equator and $A_{\nu\rho\sigma}$ on it as the configuration. Indeed, to derive the gauge field equation from the action (2), one has to fix the value $A_{\nu\rho\sigma}$ at the boundary, i.e., the equator in our case. In other words, if one simply uses the action (2) in the no-boundary path integral, then the configuration of the wave function of the universe should be $(h_{mn}, A_{\nu\rho\sigma})$.

In deriving the probability formula, one joins the south hemisphere of the instanton and its time reversal, the north hemisphere, at the equator. There is no way to get a regular $A_{\nu\rho\sigma}$ for the whole S_4 in one piece. Instead, one can choose the gauge with the regularity condition at the south hemisphere for $A_{\nu\rho\sigma}$. The value at the north hemisphere is obtained similarly via a sign change under the time reversal. This results in a discontinuity across the equator. Once the gauge is fixed, one is not allowed to smooth it by a gauge transform (4). Therefore $A_{\nu\rho\sigma}$ is not a right representation due to the discontinuity across the equator, since deriving the probability (1) from the two wave functions (for the north and south hemispheres) one needs the same configuration from the two sides of the equator. On the other hand, $F^{\mu\nu\rho\sigma}$ is a right representation due to the continuity there.

One can always Fourier transform a wave function from one representation to its conjugate in quantum theory in the Lorentzian regime. In the Euclidean regime, at the *WKB* level, this kind of transform is equivalent to a Legendre transform of the instanton action. The Legendre term is the summation of products of the canonically conjugate variables at the boundary. For our model, the term reads

$$I_{\text{Legendre}} = -\int_{\Sigma_{S+N}} dS_\mu \frac{1}{6} A_{\nu\rho\sigma} F^{\mu\nu\rho\sigma}, \tag{11}$$

where Σ_{S+N} denotes the two equator boundaries for both the south and north hemispheres.

Adding this term, the action (2) must be revised into

$$I = -\int_M \left(\frac{1}{16\pi}(R - 2\Lambda_0) - \frac{1}{48} F^{\mu\nu\rho\sigma} F_{\mu\nu\rho\sigma}\right) - \int_{\Sigma_{S+N}} dS_\mu \frac{1}{6} A_{\nu\rho\sigma} F^{\mu\nu\rho\sigma}. \tag{12}$$

Apparently, varying the action (12) will result in the same field equation (9) under the condition that $F^{\mu\nu\rho\sigma}$ is fixed at the boundary. For our case, the boundary is Σ_{S+N}.

For the instanton, using the gauge field equation (5), one can readily convert the Legendre term into the following form

$$I_{\text{Legendre}} = -\int_M \frac{1}{24} F^{\mu\nu\rho\sigma} F_{\mu\nu\rho\sigma} \tag{13}$$

and then the action (12) becomes

$$I = -\int_M \left(\frac{1}{16\pi}(R - 2\Lambda_0) + \frac{1}{48} F^{\mu\nu\rho\sigma} F_{\mu\nu\rho\sigma}\right) = -\int_M \left(\frac{1}{16\pi}(R - 2\Lambda_0) + \frac{1}{2}\kappa^2\right). \tag{14}$$

From the action (14) one can see that the F^2 term behaves as an effective cosmological constant $-4\pi\kappa^2$ after substituting the gauge field configuration of the instanton, which is the same as what appears in the field equation (9). Therefore, as far as the cosmological constant is concerned, Duff's dilemma has been dispelled.

It is worth emphasizing that (12) and (14) are equivalent for the instanton, or more accurately, the solution to the gauge field equation. They are not equivalent for the more general case, since we have used the gauge field equation (5) in deriving (14) from (12).

After substituting the configuration, one is not allowed to consider the gauge field as a variable again. However, considering the F^2 term in the action (14) as a constant, one can still vary the action with respect to the rest of the variables and this results in the Einstein equation with a total cosmological constant which is equivalent to (9), of course. Everything is consistent here.

In the above argument, it is assumed that Λ_{total}, i.e., R is positive. The Euclidean action is obtained via the analytic continuation from the Lorentzian action. There is a sign ambiguity in action (2) due to the continuation of the factor $\sqrt{-g}$ from the Lorentzian action. The term associated with R in the Euclidean action must be negative, so that the primordial fluctuation will take the ground state allowed by the Heisenberg uncertainty principle [5]. By the same argument, if Λ_{total} or R is negative, then the Euclidean action should take the negative of the expressions (2), (12) and (14), and the above argument remains intact. For both cases, the Euclidean action can be written as $-3\pi/|\Lambda_{\text{total}}|$, and the probability would exponentially increase no matter in which direction the value of Λ_{total} approaches zero. For the case with negative Λ_{total}, the instanton is also S_4 but with a negative definite metric signature, and the created universe is described by anti-de Sitter spacetime AdS_4.

One may wonder why the choice of representation in quantum cosmology is so crucial. It is well known that one can equally use any representation in quantum theory. For that case one is working in the Lorentzian spacetime, while the quantum creation scenario occurs in the Euclidean spacetime with imaginary time. In the Euclidean regime, the no-boundary path integral with a wrong representation does not make much sense.

Duff also said "...casts doubt on similar attempts based on maximizing the exponential of minus the Euclidean action" [4]. I believe this statement can only be applied when one chooses the irrelevant configuration or representation in the no-boundary path integral.

One can interpret the path integral as the partition function in gravitational thermodynamics [6], the right representation corresponds to the microcanonical ensemble. This is very useful in dealing with the problems of black hole with distinct surface gravities or temperatures.

The representation of the wave function of the universe has been previously discussed in the scenario of primordial black hole creation [7] and spacetime dimensionality [8] in quantum cosmology.

Acknowledgements

This work is supported by NSFC No. 10703005.

References

[1] S.W. Hawking, Phys. Lett. B 134 (1984) 403.
[2] M.J. Duff, P. van Nieuwenhuizen, Phys. Lett. B 94 (1980) 179;
A. Aurilia, H. Nicolai, P.K. Townsend, Nucl. Phys. B 176 (1980) 509.
[3] J.B. Hartle, S.W. Hawking, Phys. Rev. D 28 (1983) 2960.
[4] M. Duff, Phys. Lett. B 226 (1989) 36.
[5] J.J. Halliwell, S.W. Hawking, Phys. Rev. D 31 (1985) 1777.
[6] G.W. Gibbons, S.W. Hawking, Phys. Rev. D 15 (1977) 2752.
[7] S.W. Hawking, S.F. Ross, Phys. Rev. D 52 (1995) 5865;
R.B. Mann, S.F. Ross, Phys. Rev. D 52 (1995) 2254;
Z.C. Wu, Int. J. Mod. Phys. D 6 (1997) 199, gr-qc/9801020;
Z.C. Wu, Phys. Lett. B 445 (1999) 274, gr-qc/9810012.
[8] Z.C. Wu, Gen. Relativ. Gravit. 34 (2002) 1121, hep-th/0105021;
Z.C. Wu, Phys. Lett. B 585 (2004) 6, hep-th/0309178.