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An ideal MEMS parametric resonator using a tapered comb-drive

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Abstract

We present an ideal MEMS parametric resonator. The resonator is ideal in the sense that it is driven by modulation of a linear stiffness. The resonator is based on an electrostatic anti-spring which only affects stiffness, and in contrast to previous transducers, does not induce any other linear or nonlinear force. The significance of this work is that it allows, for the first time, to experimentally explore classic parametric resonators such as described by the Meissner equation. We present experimental results showing parametric excitation of a Meissner resonator, and characterization of the first unstable region of its stability map.

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1. Introduction

Micro resonators are used in many applications such as chemical sensors, RF-MEMS filters, oscillators, micro-gyros and many more. Electrostatic resonators were first proposed in the pioneering work of Nathanson [1]. Since then, MEMS resonators have been developed and improved to achieve the desired dynamic response required for filtering, clocking and sensing technologies. In the last two decades, significant work was devoted to utilize the advantages of nonlinear dynamics in micro-systems, and to the design and application of parametric resonators [2].

Parametric resonators are dynamic systems in which at least one physical parameter varies in time, resulting in special properties of their dynamic response. One interesting property is that the applied driving frequency may

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differ from the response frequency. This allows to attenuate feed-through noise in sensing applications, and to use parametric resonators in filtering [3] (e.g. frequency dividers). Another attractive property is sharp transition between stable and unstable responses. These sharp transitions enable sensitive frequency-shift detection for sensing applications [4]. Moreover, the (theoretically predicted) exponentially unbounded response is also appealing for large displacement applications [5] and for parametric amplification [6]. Hence, parametric actuation offers many advantages for MEMS transducers.

However, existing MEMS parametric resonators induce nonlinear modulation of stiffness, which mixes together several nonlinear effects. The parametric resonator presented in this work allows pure linear stiffness modulation, without introducing any unwarranted linear or nonlinear effects. Such a system enables the design and fabrication of a parametric system that behaves exactly as a classic Meissner resonator.

In section 2 the ideal parametric resonator is introduced. This parametric resonator is based on a tapered comb-drive [7], where its unique design allows time-modulation of the system stiffness, without introducing any nonlinear force to the system. Experimental validation of the response of the ideal parametric resonator performing as a Meissner resonator is presented in Section 3. The last section addresses the nonlinear capping phenomena observed in the experiments.

2. An ideal parametric resonator with linear stiffness modulation

Consider a symmetric comb-drive with a linear tapered finger lengths rotor [7], as presented in Figure 1. This design enables to change the linear stiffness of the system by applying dc voltage to the stator fingers. The equation of motion of this system is given by,

\[ m\ddot{x} + c\dot{x} + \left(k - n\frac{\varepsilon_0 w}{gOL}V^2\right)x = 0 \]  

(1)

Here \( m \) is the rotor mass, \( c \) is the linear damping coefficient, \( k \) is the linear stiffness of the suspension, \( n \) is the number of tapered fingers of the rotor on each side, \( \varepsilon_0 \) is the permittivity of free-space, \( w \) is the device thickness, \( g \) is the gap between rotor and stator fingers, and \( OL \) is the initial overlap where motion is in the range \(-OL < x < OL\).

![Fig. 1. Schematic view of the symmetric comb-drive with linearly tapered rotor fingers [7].](image)

It is clear from Eq. (1) that the stiffness of the system can be tuned down by application of a dc bias, and that this stiffness is otherwise constant, i.e. it is unaffected by motion. This actuator induces a diverging electrostatic force which is a perfectly linear function of displacement. This force is exactly the opposite of the restoring force in a linear spring, and hence we refer to it as a linear anti-spring. Moreover, applying time-modulated voltage will cause time-modulation of the system stiffness, resulting in parametric excitation. However, the modulated stiffness is proportional to the voltage-squared. This means that if the applied voltage has a square waveform \( V(t) = V_{dc} + V_{ac}\text{sgn}(\cos(\omega t)) \), then the average voltage (and hence the average stiffness) will also be affected by \( V_{ac} \), and not only by \( V_{dc} \), as would be preferable.

Therefore, we choose to apply a voltage of the form...
where \( V_{eq}^2 = V_{dc}^2 + V_{ac}^2 \) is an arbitrary constant.

Applying this voltage scheme to Eq. (1) yields,

\[
mx + cx + \left[ k - \frac{1}{2} \mu \frac{E_0}{gOL} V_{eq}^2 - \mu \frac{E_0}{gOL} V_{ac} \sqrt{V_{eq}^2 - V_{ac}^2 \text{sgn}(\cos(\omega t))} \right] x = 0
\]

The first two terms in the square brackets constitute the constant stiffness of the system and the third term constitutes the time-modulated stiffness. The form of this equation resembles that of the Meissner equation, and includes a linear damping term.

The next section presents experimental results of the ideal parametric resonator, when it is driven as a Meissner resonator.

3. Experimental results

Test devices were fabricated using SOIMUMPs technology in a (100) single crystalline silicon, and the beams were oriented in the (110) direction (run 43, [8]). A typical device is presented in Fig. 2. The test devices were designed such that the shuttle can be excited parametrically by an electrostatic anti-spring (tapered comb-drive). The rotor is suspended on four folded-beam springs and its nominal natural frequency is about 700 Hz. The finger width and minimal trench are both designed to be 5 \( \mu \text{m} \), and the device layer thickness is 25 \( \mu \text{m} \). The overlap length of the anti-spring fingers was tapered by 0.4 \( \mu \text{m/tooth} \). The device was designed to allow maximal travel range of \( \pm 10 \mu \text{m} \).

The test device was operated as a Meissner parametric resonator by applying modulated voltage (2) to the anti-spring (Fig. 2), which in turn modulated the stiffness of the device. The experiments were performed with an ambient vacuum of 6 mTorr. Modulation frequency was slowly swept in the vicinity of double the natural frequency of the system. This frequency is associated with the first instability window (inset in Fig. 3b). At the detection of an unbounded motion increase, the system was brought to rest. Then, the parametric actuation was repeated in that very same frequency, to ensure that the unbounded response indeed results from parametric excitation.

Figure 3a presents a typical measured response of the device. It is evident from the double frequency actuation with respect to the response frequency, that the response is indeed a typical parametric response. The dynamic response is eventually bounded at 4 \( \mu \text{m} \) stroke by a nonlinear effect which is not associated with parametric resonance and will be discussed in section 4. To characterize the first instability window of the Meissner resonator, different voltage modulation amplitudes were applied. For each modulation amplitude, the frequency was swept.
back and forth, to identify the window in which the response becomes unstable. Figure 3b presents the measured points at which the response switched from stable to unstable. The very good correlation between the measured points and the predicted stability map, confirms that the parametric resonator presented in this work can perform as an ideal parametric resonator.

Fig. 3. (a) The response due to a modulated square wave signal of 6[V]. The modulated signal has twice the frequency of the response signal, as expected. (b) A non-dimensional stability map of the first instability window of a Meissner resonator. The inset presents a larger region of the map. Theoretical boundaries are indicated by the solid line, and the measured results are indicated by ‘x’.

4. Discussion and conclusion

In this work we have presented an ideal parametric resonator based on a symmetric comb-drive anti-spring [7]. This ideal parametric resonator enables to modulate stiffness without inducing any additional unwarranted nonlinear effects. Specifically, we demonstrated that this resonator may be operated as an ideal Meissner resonator. Though the option was not demonstrated in the present study, a Mathieu resonator may also be implemented by applying a different voltage modulation scheme.

However, the response measured in Fig. 3a demonstrates that the motion was bounded by some unforeseen effect - the maximal motion was far less than the maximal designed travel range of ±10μm. We attribute this capping phenomenon to mechanical nonlinear stiffening, which is inherent to folded-beam suspensions [9]. Nevertheless, this stiffening is insignificant for small displacements, and does not affect the investigated instability threshold.

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References