Heterogeneous traffic flow modelling using macroscopic continuum model

Ranju Mohana\textsuperscript{a,1}, Gitakrishnan Ramadurai\textsuperscript{b}

\textsuperscript{a}Ph. D. Research Scholar, Indian Institute of Technology, Madras, Chennai-600036, India
\textsuperscript{b}Assistant Professor, Indian Institute of Technology, Madras, Chennai-600036, India

Abstract

Modelling heterogeneous traffic flow for Asian countries is one of the emerging research areas in the past few years. The two main challenges in modelling are: capturing the effect of varying size of vehicles, and the lack in lane discipline, both of which together lead to the ‘capacity filling’ behavior of vehicles. The same section length of the road can be occupied by different types of vehicles at the same time, and the conventional measure of traffic concentration, density (vehicles per lane per unit length), is not a good measure for heterogeneous traffic modelling. This paper addresses the above mentioned two challenges by extending the Aw-Rascle macroscopic model based on continuum theory using area occupancy for traffic concentration instead of density. The aim of the model is to have a parsimonious model of heterogeneous traffic for network wide applications that can capture unique phenomena in heterogeneous traffic flow such as capacity filling. The paper calibrates and validates the model using data from an arterial road in Chennai city.

1. Objective

Modelling heterogeneous traffic flow for Asian countries is one of the emerging research areas in the past few years. The two main challenges in modelling are: capturing the effect of varying size of vehicles, and the lack in lane discipline, both of which together lead to the ‘capacity filling’ behaviour (a phenomenon of congested traffic where vehicles try to percolate through the available gaps in the road section ahead) of vehicles. Existing conventional continuum type traffic flow models are well suited for developed countries with homogeneous traffic and perfect lane discipline. In fact, even in the so called homogeneous traffic, the vehicles’ sizes may vary. Frequent lateral/lane changing movements in heterogeneous traffic forces modelling approaches to consider the entire road width as a whole instead of multiple lanes. The capacity filling behavior makes the road capacity in
Asian countries much higher when compared to the other developed countries since the same section length of the road can be occupied by different types of vehicles at the same time. Thus, the conventional measure of traffic concentration, the density which is measured as vehicles per unit length, is not a good choice for heterogeneous traffic. Recently, Mallikarjuna (2006) and Arasan and Dhivya (2008) introduced the concept of area occupancy for measuring heterogeneous traffic concentration. This paper addresses the above mentioned two challenges by extending the Aw-Rascle macroscopic model based on continuum theory using area occupancy for traffic concentration instead of density. The aim of the model is to have a parsimonious model of heterogeneous traffic for network wide applications that can capture unique phenomena in heterogeneous traffic flow such as capacity filling.

### Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N$</td>
<td>Number of vehicle types</td>
</tr>
<tr>
<td>$i$</td>
<td>Vehicle type</td>
</tr>
<tr>
<td>$k$, $k_i$</td>
<td>Density per lane per unit length, density of $i$ per unit length</td>
</tr>
<tr>
<td>$u$, $u_i$</td>
<td>Speed, speed of $i$</td>
</tr>
<tr>
<td>$q$, $q_i$</td>
<td>Flow, flow of $i$</td>
</tr>
<tr>
<td>$u_{eq}$, $u_{eq_i}$</td>
<td>Equilibrium speed, equilibrium speed of $i$</td>
</tr>
<tr>
<td>$(x,t)$</td>
<td>(space, time)</td>
</tr>
<tr>
<td>$\tau$, $\tau_i$</td>
<td>Relaxation time, relaxation time of $i$</td>
</tr>
<tr>
<td>$c_{jam}$</td>
<td>Jam density in homogeneous traffic</td>
</tr>
<tr>
<td>$u_f$, $u_{fi}$</td>
<td>Free flow speed, free flow speed of $i$</td>
</tr>
<tr>
<td>$AO$</td>
<td>Area occupancy</td>
</tr>
<tr>
<td>$AO_{max_i}$</td>
<td>Maximum area occupancy for $i$</td>
</tr>
<tr>
<td>$L$, $W$, $T$</td>
<td>Length, width, and observation period for the section for which $AO$ is to be determined</td>
</tr>
<tr>
<td>$t_m$, $a_m$</td>
<td>Area of, and time occupied by vehicle $m$ on the section</td>
</tr>
<tr>
<td>$\nabla$</td>
<td>Gradient of $f$</td>
</tr>
<tr>
<td>$f_i(.)$</td>
<td>Function of $f$</td>
</tr>
<tr>
<td>$f(.)$</td>
<td>Function of $f$</td>
</tr>
</tbody>
</table>

### 1.1. Introduction

Macroscopic continuum models of traffic flow started with the first order LWR model by Lighthill and Whitham (1955) and Richards (1956) where the traffic is assumed to be always in steady state equilibrium condition. The model consists of the flow conservation equation, an equilibrium speed density relationship and the fundamental equation of traffic flow. Though the model proved its ability to capture the traffic shockwave formation explicitly, because of the presence of instantaneous speed density relationship, it failed to explain some well-known traffic phenomena such as hysteresis, stop and go waves, and platoon dispersion. Realizing that the model’s performance can be improved by including the inertial effect in the dynamics of velocity and driver’s
anticipation to the traffic ahead, Lighthill and Whitham (1955) proposed higher order extension of this model. Higher order models remained under-explored until the introduction of Payne (1971) and Payne-Whitham (1979) macroscopic models which are derived from the microscopic car following logic. Along with the flow conservation and fundamental traffic flow equations, these models express the dynamics of velocity using the relaxation and anticipation terms borrowed from Newtonian physics. The relaxation term shows how the vehicle adjusts to its equilibrium velocity in some relaxation time and hence contributes to the inertial effect of speed. The anticipation term includes a pressure term and considers driver’s reaction to the spatially changing traffic condition ahead. Daganzo (1995) criticized this model stating the dissimilarities between infinitely small particle flow in fluid and the finite size vehicle flow in the traffic. He also mentioned about the ‘negative speed’ experienced by vehicles in the tails of the congestion region. The higher order models developed after Payne-Whitham (1979) model tried to resolve the limitation of negative speeds modifying the anticipation term and includes the model by Zhang (1998, 2002), Aw and Rascle (2000) and Jiang, Wu, and Zhu (2002). The literature for higher order traffic flow models is still vast and only few of them are highlighted above. A comprehensive review of macroscopic traffic flow models is given by Mohan and Ramadurai (2013).

Higher order traffic flow models, when apply for network level application, perform better than first-order LWR model (Mohan & Ramadurai, 2012). This paper concentrates on the macroscopic model by Aw and Rascle (2000), well known as AR model. The model uses the convective derivative instead of the spatial derivative, of traffic pressure in the velocity dynamics of Payne-Whitham (1979) model. Thus unlike Payne type models, if a driver sees a high density ahead, he will not decelerate if that high density is moving faster than him. This paper extends AR model for heterogeneous traffic, observes the qualitative properties of the extended model, and checks the validity using real traffic data.

The paper is organized as follows. The next section briefly explain AR model along with the equation system and the third section describe the concept of area occupancy. In the fourth section, the extended AR model is presented using area occupancy and its qualitative properties are analyzed. Calibration of the model is given in the fifth section and the sixth section validates and compares the model with multi-class LWR model. The last section draws conclusion on the paper.

2. AR model of traffic flow

The flaws (Daganzo, 1995) in the well-known Payne-Whitham (1979) higher order model of traffic flow lead Aw and Rascle (2000) to the formulation of a new model modifying the velocity dynamics equation. The velocity dynamics equation in Payne-Whitham (1979) model is given as:

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} = \frac{u_r - u}{\tau} \frac{1}{k} \frac{\partial p}{\partial x}$$  \hspace{1cm} (1)

where $p$ is the traffic pressure expressed as a function of density ahead to vehicles. When solving the above equation along with the flow conservation, the characteristic speeds from the model are given by $u_\pm (dp/dk)$. The speed $u_+ (dp/dk)$ indicates that some part of the information always travels faster than the velocity $u$ of cars and hence causes negative speeds of vehicles at some part of the congested region. It can be avoided by using a convective derivative of the pressure instead of the spatial derivative. Thus, the equation system of AR model is given by

$$\frac{\partial k}{\partial t} + \frac{\partial (ku)}{\partial x} = 0$$  \hspace{1cm} (2)

$$\frac{\partial (u + p(k))}{\partial t} + u \frac{\partial (u + p(k))}{\partial x} = \frac{u_r (k) - u}{\tau}$$  \hspace{1cm} (3)
The traffic pressure $p(k)$ can be expressed as an increasing function of density and is given by $p(k) = C_0 k^\gamma$ where $\gamma$ is a dimensionless parameter ($\gamma > 0$) and $C_0$ is a constant that equals 1. For choosing the pressure function, the two qualitatively important conditions to be satisfied are: the strict convexity of the function $kp(k)$ and $p(k) \sim C_0 k^\gamma$, near $k = 0$. The two characteristic speeds from equations (2) and (3) are $\lambda_1 = u$ and $\lambda_2 = u \gamma p(k)$ indicating that the information will travel with speed at most equal to that of the vehicles.

For traffic lacking lane following discipline, the same section length may be occupied by different vehicle types at the same time depending on the space availability. Hence the traffic pressure in terms of density is valid only for homogeneous traffic condition where clear lane and queue discipline exists. For heterogeneous traffic conditions in Asian countries where the concept of lane based movement is not strictly followed, traffic concentration can be better expressed in terms of $AO$.

3. Area occupancy (Mallikarjuna & Rao, 2006; Arasan & Dhivya, 2008) for heterogeneous traffic

Traffic density is defined as the number of vehicles occupying unit length of roadway at any instant of time. It is usually measured as vehicles per lane per km length of the road neglecting individual vehicle speed and dimensions and hence more suitable for homogeneous, lane disciplined traffic. In order to account for vehicles’ varying speeds and dimensions even under homogeneous traffic condition, a new measure of concentration, occupancy is introduced which is a dimensionless variable, measured directly by the amount of time a particular point of the roadway is occupied by all the vehicles. Even though occupancy is mentioned as a point measurement, based on practical consideration, it is defined as the percentage of time a detection zone is occupied by vehicles. Hence this measure varies with change in detection zone length even for the same site with identical traffic. Thus it is required to consider also the section length in the measurement. Again, in order to account for the lack in lane discipline, the entire width of the section (width of the road as a whole) is to be considered for measuring occupancy. Thus, a standard measurement for the concentration of heterogeneous traffic, lacking in lane discipline, can be expressed as:

$$AO = \frac{\sum m t_m a_m}{TA}$$

(4)

If the observation period $T$ is too small such that even vehicles with the highest speed cannot pass the section in this time period, equation (4) converges to the area occupancy measured over space and can be expressed as

$$AO = \frac{\sum m k_i L_i}{A} = \frac{\sum m k_i a_i}{WL} = \frac{1}{W}$$

(5)

where $L$ and $W$ are the length and width of the road section for which $AO$ is to be measured.

4. Model’s extensions to heterogeneous traffic

For heterogeneous traffic, the traffic pressure experienced by different types of vehicles will be different and can be expressed as $p_i(AO) = C_0^2 (AO)^\gamma_i$ where $\gamma_i$ is a dimensionless parameter for vehicle type $i$ ($\gamma_i > 1$), $C$ is a constant, and $AO$ is defined by equation (5). Now, AR model can be extended to heterogeneous traffic modelling with modified pressure term using area occupancy as follows:

$$\frac{\partial k_i}{\partial t} + \frac{\partial (k_i v_i)}{\partial x} = 0$$

(6)

$$\frac{\partial (v_i + p_i(AO))}{\partial t} + v_i \frac{\partial (v_i + p_i(AO))}{\partial x} = \frac{v_i}{\tau_i} (AO) - v_i$$

(7)

In equations (5) and (6), class-wise densities are retained to account for the number of vehicles of each type, but the ‘traffic concentration’ in the road section is replaced by $AO$ which is a dimensionless variable. Also, the
The fundamental equation of traffic flow, \( q = kv \) for homogeneous traffic is valid separately for each vehicle type \( i \) \((q_i = k(v_i))\). The system of equations (6) and (7) can be expressed in conservative form as:

\[
\frac{\partial U}{\partial t} + \frac{\partial F(U)}{\partial x} = S(U)
\]

where

\[
U = \begin{bmatrix}
U_1 \\
U_2 \\
\vdots \\
U_N
\end{bmatrix}, \quad
F(U) = \begin{bmatrix}
F(U_1) \\
F(U_2) \\
\vdots \\
F(U_N)
\end{bmatrix}, \quad
S(U) = \begin{bmatrix}
S(U_1) \\
S(U_2) \\
\vdots \\
S(U_N)
\end{bmatrix}
\]

\[
U_i = \begin{bmatrix}
k_i \\
m_i
\end{bmatrix}, \quad
F(U_i) = \begin{bmatrix}
m_i - p_i(AO)k_i \\
\frac{m_i^2}{k_i} - m_i p_i(AO)
\end{bmatrix}, \quad
S(U_i) = \begin{bmatrix}
0 \\
\frac{m_{ei} - m_i}{\tau_i}
\end{bmatrix}
\]

The conservative variables \( m_i \) and \( m_{ei} \) are given by: \( m_i = k_i(v_i + p_i(AO)) \) and \( m_{ei} = k_i(v_{ei} + p_i(AO)) \)

### 4.1 Qualitative properties of the model

- **The pressure function \( p(AO) \):** One qualitatively important condition for the pressure function in the original AR model is the strict convexity of the function \( kp(k) \). In the extended model, \( k_i \frac{\partial^2 p_i(AO)}{\partial k_i^2} + 2 \frac{\partial p_i(AO)}{\partial k_i} > 0 \) \( \forall i \) (or) \( AO p_i(AO) \) is strictly convex.

- **The system of equations should be hyperbolic:** The equation system (8) is (strictly) hyperbolic if the Jacobian matrix \( \frac{\partial F(U)}{\partial U} \) is diagonalizable and has (distinct) real Eigen values. For heterogeneous traffic with \( N \) vehicle classes this Jacobian is a square matrix of size \( 2N \times 2N \) and will have \( 2N \) Eigen values. For both models, as an example for case \( N=2 \), the Eigen values of the Jacobian matrix are given by:

\[
\lambda_1 = u_1; \quad \lambda_2 = u_2; \quad \lambda_3 = \frac{A + \sqrt{B}}{2k_1k_2}; \quad \lambda_4 = \frac{A - \sqrt{B}}{2k_1k_2}
\]

where \( A = f_1(u_1, u_2, k_1, k_2, \frac{dp_1}{dk_1}, \frac{dp_2}{dk_2}) \); \( B = f_2(u_1, u_2, k_1, k_2, \frac{dp_1}{dk_1}, \frac{dp_2}{dk_2}) \). All Eigen values are real and distinct and hence, the equation system (8) is strictly hyperbolic.

- **Speed of the characteristics:** For heterogeneous traffic to ensure anisotropic property, it is not necessary to have all the Eigen values of a particular vehicle type less than the corresponding vehicle’s speed, since different vehicle types travels with different speeds. However they should be less than the maximum speed among all vehicles. Several combinations of density, speed, and pressure values from their possible ranges were tried and found that all Eigen values are less than the maximum speed among vehicles.
Properties of Eigen values: Let $\lambda_s$ and $r_s$ ($s=1, 2, ..., 2N$) respectively be the Eigen values and Eigen vectors obtained from the Jacobian of the flux matrix. Then, for genuinely non-linear Eigen values, $\nabla \lambda_s(U)r_s(U) \neq 0$ and for linearly degenerate Eigen values, $\nabla \lambda_s(U)r_s(U) = 0$ for all $U$. It can be shown (LeVeque, 1992) that the Eigen values equivalent to vehicle speeds are linearly degenerate and others are genuinely non-linear. Linearly degenerate Eigen values correspond to contact discontinuities (one vehicle follows other) and genuinely non-linear values corresponds to shock (deceleration) or rarefaction (acceleration) waves.

4.2 Speed–Area Occupancy relationship for heterogeneous traffic flow

Del Castillo (1995) mentioned the significant impact of the parameter $c_{jam}$ in the speed-density relationship and formulated a double exponential model. In this paper, for the equilibrium speed-AO relationship, this double exponential relationship is assumed and modified in terms of AO as given in equation (9).

$$u_{es}(AO) = u_f \left(1 - \exp\left(1 - \exp\left(c_i \left(\frac{AO_{\text{max},i}}{AO} - 1\right)\right)\right)\right)$$

(9)

For the above expression, the fraction $k_j/k$ in the original speed-density relationship is replaced by $AO_{\text{max},i}/AO$. Since $AO_{\text{max},i}/AO > k_j/k$ always, the ratio $c_{jam}/u_f$ (ratio of kinematic wave speed under jam density to free flow speed) in the original speed density relationship is replaced by a dimensionless parameter $c_i$ for each vehicle type $i$. Calibration of speed-AO relationship is given in the next section.

5. Model Calibration

Model parameters are calibrated for real traffic data collected from a 0.9 km road section, with a bottleneck because of reduction in road width from 13.7 m to 8.85 m for a length of 0.2 km ending at a merge junction, in Chennai city. Video data collected from the field is extracted using MCME (Manual Counting Made Easy), a newly developed code in Visual C++ (Ramadurai, 2013). Traffic flow data are simulated for the section using McCormack predictor-corrector method (Trieber and Kesting, 2013). The space and time grid for simulation is set as 0.05 km and 2 sec respectively. The considered road section is thus divided into 18 sub-sections in which bottleneck starts from 15th subsection. The input data given to the model are: lengths and widths of the road section, inflow into and outflow from section, and the outflow from a subsection (12th section). Since the overall aim of the model is to be used at network wide applications, model is calibrated minimizing the error ($E$) formulated in terms of cumulative outflow, given by:

$$E = \sum_t \sum_i \left(\text{Outflow}_{\text{field}}^{\text{simulated}} - \text{Outflow}_{\text{simulated}}^{\text{simulated}}\right)^2 + \sum_t \sum_i \left(\text{Outflow}_{\text{field}}^{\text{field-section12}} - \text{Outflow}_{\text{simulated}}^{\text{simulated-section12}}\right)^2$$

(10)

where

$\text{Outflow}_{\text{field}}^{\text{simulated}}$ - Simulated cumulative outflow of vehicle type $i$ from the road section at time $t$

$\text{Outflow}_{\text{field}}^{\text{field}}$ - Actual cumulative outflow of vehicle type $i$ from the road section at time $t$

$\text{Outflow}_{\text{field}}^{\text{field-section12}}$ - Simulated cumulative outflow of vehicle type $i$ from section 12 at time $t$

$\text{Outflow}_{\text{field}}^{\text{field-section12}}$ - Actual cumulative outflow of vehicle type $i$ from section 12 at time $t$

The second order models are sensitive to the model parameters and small changes of parameter set will lead to completely different model performance. Thus popular and convenient approach for higher order model calibration is the random search techniques (Ngoduy & Maher, 2012). This paper uses SRES (Stochastic Ranking Evolution Strategy) algorithm in NLOPT (Non-Linear OPTimization) library (Ab-Initio, MIT, 2008) for model calibration. The vehicles are grouped into four different classes as Motorized Two Wheeler (MTW), Motorized three wheeler (AUTO), Car (CAR) and Other Vehicles (OV). Parameters to be calibrated for each vehicle type
are: maximum area occupancy \((AO_{\text{max}})\), free flow speed \((u_f)\), a dimensionless constant \(c\), and the relaxation time, \(\tau\).

Because of complex vehicle interactions in heterogeneous traffic and lack in lane discipline, parameters’ sensitivity is again high, and also holds the drawback of multiple optimum values. Again, the parameter \(AO_{\text{max}}\) is found to have multiple optimum which is mainly due to: a) varying vehicle sizes b) spatial arrangement of vehicles. The spatial arrangement of vehicles/ vehicle types is stochastic and the proposed model does not address this in this paper. To address vehicle types of varying size, \(AO_{\text{max}}\) is calibrated separately for vehicle types. This is intuitive in the sense that at one particular \(AO\), since the traffic pressure experienced by each vehicle type is different, the \(AO_{\text{max}}\) at which different vehicle types reach a jammed condition will also be different. For calibration of \(AO_{\text{max}}\), the upper bound is fixed as follows. Assuming minimum lateral and longitudinal clearance requirements for various vehicle types, the maximum number of vehicles that can be accommodated in the considered mid-block is calculated for each vehicle class separately, if they alone are present. Thus, \(AO_{\text{max}}\) for each vehicle type is calculated to be \(\sim0.90\). However, in heterogeneous traffic mix this value can be still lower because of spatial arrangement of vehicles and thus the range for \(AO_{\text{max}}\) is fixed as 0.6-0.9.

To compare the proposed model with state-of-the-art, an existing multi-class extension of the LWR model (Wong & Wong, 2002) is also calibrated for the same speed-\(AO\) relationship. Calibrated values of parameters are shown in Table 1.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Proposed model</th>
<th>LWR model</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>MTW</td>
<td>AUTO</td>
</tr>
<tr>
<td>For (AO_{\text{max}})</td>
<td>0.85</td>
<td>0.84</td>
</tr>
<tr>
<td>Equilibrium (u_f)</td>
<td>40.1</td>
<td>40.0</td>
</tr>
<tr>
<td>speed-(AO) (c)</td>
<td>0.64</td>
<td>0.54</td>
</tr>
<tr>
<td>Other parameters</td>
<td>(\tau)</td>
<td>0.0025</td>
</tr>
<tr>
<td></td>
<td>(C)</td>
<td>75</td>
</tr>
<tr>
<td></td>
<td>(\gamma)</td>
<td>2.23</td>
</tr>
</tbody>
</table>

The calibrated values of \(AO_{\text{max}}\) can be considered only as theoretical values since spatial arrangement of vehicles is not considered in the models. For both the proposed and LWR models, free flow speed \(u_f\) of CAR is found to be higher among the modes followed by OV, MTW, and AUTO respectively which is intuitive. Value of \(c\) has been found to have great impact on the model result. For less \(c\) values, speed reduction for all vehicles is found to happen at low \(AO\) values, and as \(c\) increases, the rate of reduction in speed also decreases. \(c\) values were high in the LWR model compared to the proposed model, leading to the freely flowing vehicles for a wide range of \(AO\) that lead to less \(AO\) and free flow speed of vehicles for the simulated period. \(AO_{\text{max}}\) of TW is high compared to the other modes and highest to lowest ranking among remaining vehicle types is AUTO, CAR and OV.

The equilibrium speed-\(AO\) graph calibrated is shown in Fig. 1. Vehicles speed reduction is found to happen even at low \(AO\) particularly for CAR and OV. Even at low \(AO\), if few vehicles cluster in some space, hindering the movement of other vehicles, speed reduction can happen. Thus, it is related to the spatial arrangement of vehicles that is not addressed in this paper and will be considered in future work. However, the model effectively captures the order in which different vehicle types reduce their speed. At a certain \(AO\), speed reduction happens first for OV, followed by car, auto, and M TW respectively. Relaxation time \(\tau\) for a mode is the time required for the acceleration or deceleration to its equilibrium velocity. In few existing heterogeneous traffic flow models
where vehicles are categorized as slow and heavy vehicles (Jiang & Wu, 2004) or as car and bus (Tang, Huang, Zhao & Shang, 2009) relaxation time depends on the weight of the vehicle type. In the proposed model, AUTO has lower relaxation time, CAR and OV have the highest, followed by M TW. Ideally, for heterogeneous traffic, But in real heterogeneous traffic, this value need to have vary dynamically depending on traffic regime and speed of the vehicles.

6. Model validation and comparison with multi-class LWR model

Validation of the proposed model and its comparison with the multi-class LWR model using the speed-AO relationship is described in this section. The model is validated for 27 minutes and first 8 minutes are avoided as warm-up period. The validation results are shown in Table 2. Mean Absolute Percentage Error (MAPE) in outflow from two different sections are lesser in most cases for the proposed model. These differences could have more significance when the models are applied at network level. The error is higher for AUTO followed by OV, CAR, and M TW respectively.

<table>
<thead>
<tr>
<th>Vehicle type</th>
<th>Mean Absolute Percentage Error (%) in the cumulative outflow</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Proposed model</td>
</tr>
<tr>
<td></td>
<td>Section x =18</td>
</tr>
<tr>
<td>M TW</td>
<td>5.0</td>
</tr>
<tr>
<td>AUTO</td>
<td>5.0</td>
</tr>
<tr>
<td>CAR</td>
<td>6.0</td>
</tr>
<tr>
<td>OV</td>
<td>17.0</td>
</tr>
</tbody>
</table>

The variation of AO over sections and over time is shown in Fig. 2 (the initial 8 minutes data is the warm-up period and is not shown in the plot). The considered traffic is in one direction (in Fig. 2, to the left). In the proposed model the sections near to intersection and bottleneck have significantly higher AO, whereas in LWR model AO is less in all the sections. LWR model appears to under-predict congestion.

The speed variation for the section 12 is shown in Fig. 3. Multi-class LWR model predicts free flow speed at the section for the entire simulation period, whereas, the proposed model predicts speed reduction due to increase in AO. In the proposed model, around 780 minutes, vehicles speed reduces and free flow traffic changes to stop-and-go traffic. During congestion (16th minute), M TW travel with higher speed followed by AUTO, CAR and OV.
7. Conclusion

The aim of the paper is to have a parsimonious model of heterogeneous traffic flow for network wide applications. The paper shows a formulation of heterogeneous traffic flow model for Asian countries where lane discipline is not strictly followed. The proposed formulation is an extension of an existing macroscopic second order model, the AR model. The extended model uses ‘area occupancy’, which is recently proposed as a good measure of traffic concentration instead of the measure ‘density’. The velocity dynamics in the model depends on the equilibrium speed in terms of area occupancy, the relaxation time and the relative speed of vehicles. The proposed model can be expressed as a hyperbolic conservative system of equations with Eigen values that correspond to contact discontinuity and acceleration /deceleration waves. However, velocity dynamics while
writing in the conservative form, assumes that the momentum of each vehicle type is conserved separately and is possibly a drawback of the formulation.

The model is calibrated and validated with real data for a road section with a bottleneck ending at a merge intersection. When compared with an existing multi-class extension of LWR model, the MAPE in the cumulative outflows are found to be same in both the models, however, congestion propagation is poorly captured in the LWR model. Lack of consideration of spatial arrangement of vehicles is another drawback of this model, still calibrated equilibrium speed-AO graph better captures the real traffic trend on how different vehicle types reduces speed, when moving from uncongested to congested regime.

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