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Physics Letters B 747 (2015) 27–35

Contents lists available at ScienceDirect

Physics Letters B

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A new spin on neutrino quantum kinetics

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ARTICLE INFO

Article history:
Received 16 March 2015
Received in revised form 25 April 2015
Accepted 29 April 2015
Available online 14 May 2015
Editor: W. Haxton

ABSTRACT

Recent studies have demonstrated that in anisotropic environments a coherent spin-flip term arises in the Quantum Kinetic Equations (QKEs) which govern the evolution of neutrino flavor and spin in hot and dense media. This term can mediate neutrino–antineutrino transformation for Majorana neutrinos and active-sterile transformation for Dirac neutrinos. We discuss the physical origin of the coherent spin-flip term and provide explicit expressions for the QKEs in a two-flavor model with spherical geometry. In this context, we demonstrate that coherent neutrino spin transformation depends on the absolute neutrino mass and Majorana phases.

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1. Introduction

The evolution of an ensemble of neutrinos in hot and dense media is described by an appropriate set of quantum kinetic equations (QKEs), accounting for kinetic, flavor, and the often neglected spin degrees of freedom [1–10]. QKEs are the essential tool to obtain a complete description of neutrino transport in the early universe, core collapse supernovae, and compact object mergers, valid before, during, and after the neutrino decoupling epoch (region). A self-consistent treatment of neutrino transport is highly relevant because in such environments neutrinos carry a significant fraction of the energy and entropy, and through their flavor- and energy-dependent weak interactions play a key role in setting the neutron-to-proton ratio [11], a critical input for the nucleosynthesis process.

Recent studies [8,10] have demonstrated that the QKEs acquire a coherent spin-flip in regions where the spatial (anti)neutrino fluxes are anisotropic or where there exist anisotropic matter currents. Such anisotropy can exist in a core-collapse supernovae or compact object merger environments. This spin-flip term can mediate neutrino–antineutrino transformation for Majorana neutrinos and active-sterile transformation for Dirac neutrinos. Moreover, it was shown in Ref. [7] that a general treatment of neutrino ensembles should include correlations that pair neutrinos and antineutrinos of opposite momenta. The coupling to these new densities to the standard density matrices has been worked out explicitly in Ref. [10]. In this work we neglect these terms as their effect primarily generates coherence of opposite-momentum neutrinos only for very long-wavelength modes, with $\lambda_{\text{de Broglie}} \sim \lambda_{\text{scale-height}}$, where $\lambda_{\text{scale-height}}$ is the length scale characterizing a given astrophysical environment. Significant feedback effects from the long-wavelength modes could alter the analysis presented below, and this deserves a separate study.

In this letter we further elaborate on the terms of the QKEs describing coherent neutrino evolution (i.e. neglecting inelastic collisions). The novel aspects of this work are:

- We discuss the physical origin of the coherent spin-flip term in the framework of an MSW-like effective Hamiltonian, in analogy to the spin-(flavor) oscillations induced by neutrino magnetic moments in a magnetic field.
- We provide explicit expressions for the coherent QKEs in a two-flavor model with spherical geometry, amenable for a computational implementation. This is the first step towards a realistic exploration of the impact of helicity oscillations in astrophysics environments.

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http://dx.doi.org/10.1016/j.physletb.2015.04.066
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- We point out the dependence of the QKEs (through the neutrino–antineutrino conversion term) on the neutrino absolute mass scale and Majorana phases. We also compare and contrast neutrino-less double beta decay and neutrino spin transformation in astrophysical environments as probes of these parameters.

2. Spin-mixing term

Refs. [8,10] have pointed out that in anisotropic environments the QKEs entail a new term that drives coherent conversion between different helicity states (of any flavor). An important feature of the new term is that it induces qualitatively different effects for Dirac and Majorana neutrinos. In the Dirac case, the mixing term converts active left-handed neutrinos to sterile right-handed states. On the other hand, in the Majorana case the mixing term enables conversion of neutrinos into antineutrinos. Given the potentially high impact of the spin-flip term, here we discuss its physical origin in a framework that does not involve the intricacies of non-equilibrium quantum field theory. Indeed, as argued below, the basic physics of this term can be understood in the case of one-flavor Dirac neutrinos even at the first-quantized level.

Physically, spin oscillations are induced by the axial-vector potential generated by forward scattering of neutrinos on the background matter and background neutrinos themselves. To illustrate this point, let us first consider the evolution of neutrinos in *external* chiral four-vector potentials $\Sigma^{\mu}_{L,R}$ (we will give their explicit expressions later on). Since our discussion parallels the analysis of spin-flip transition induced by a neutrino magnetic moment in an external magnetic field [12,13], we also include in the interaction Lagrangian the familiar magnetic-moment term. Suppressing flavor indices (μ_ν and $\Sigma_{L,R}$ are matrices in flavor space) the interaction Lagrangian is given by

$$\mathcal{L}_{\rm int} = -\bar\nu_L \not{\Sigma}_R \nu_L - \bar\nu_R \not{\Sigma}_L \nu_R + \left(\frac{\mu_\nu}{2}\,\bar\nu_R \sigma_{\mu\nu} F^{\mu\nu} \nu_L + \text{h.c.}\right) . \tag{1}$$

The Majorana case is obtained by replacing $\nu_R \to \nu_L^c$, $\Sigma_L \to -\Sigma_R^T$, and setting to zero the diagonal elements μ_ν^{ii} (for Majorana neutrinos $\mu_\nu^{ji} = -\mu_\nu^{ij}$). Given this interaction, our goal is to obtain an effective Hamiltonian in spin(-flavor) space, with off-diagonal components giving the helicity mixing [13]. Since the essential physics of spin oscillations is already present in the one-flavor case, to keep the discussion as simple as possible we consider the case of one-flavor Dirac neutrinos, with real magnetic moment. In this case the interaction Lagrangian is

$$\mathcal{L}_{\rm int} = \frac{\mu_\nu}{2}\,\bar\nu \sigma_{\mu\nu} F^{\mu\nu} \nu - \frac{1}{2}\bar\nu \not{\Sigma}_V \nu - \frac{1}{2}\bar\nu \not{\Sigma}_A \gamma_5 \nu , \tag{2}$$

where we have defined the vector and axial-vector potentials as $\Sigma^{\mu}_{V,A} \equiv \Sigma^{\mu}_{L} \pm \Sigma^{\mu}_{R} = (\Sigma^0_{V,A}, \vec\Sigma_{V,A})$.

In a first-quantized approach [12], the Dirac Hamiltonian corresponding to the interaction (2) is

$$H = H_0 + \Delta H , \qquad H_0 = \hat p \cdot \vec\alpha + \beta m , \qquad \Delta H = \mu_\nu\, \beta\, \vec\Sigma \cdot \vec B + \left(\Sigma^0_V - \vec\Sigma_V \cdot \vec\alpha\right) + \left(\gamma_5 \Sigma^0_A - \vec\Sigma_A \cdot \vec\Sigma\right) , \tag{3}$$

with $\beta = \gamma^0$, $\vec\alpha = \gamma^0 \vec\gamma$, and $\vec\Sigma = \text{diag}(\vec\sigma, \vec\sigma)$. Defining the helicity operator $h \equiv \hat p \cdot \Sigma$, already at this level one sees that while $[H_0, h] = 0$, in general $[\Delta H, h] \neq 0$, unless $\vec\Sigma_A$ and $\vec B$ are parallel to the momentum $\vec p$. So the energy eigenstates are in general mixtures of helicity eigenstates, and we reach the conclusion that magnetic fields *and/or* axial-vector potentials transverse to the direction of motion induce helicity oscillations.

To quantify the helicity mixing effect, it is more convenient to work within the second-quantized quantum field theory approach [13]. One can define the 2×2 effective Hamiltonian in helicity space $\mathcal{H}_{hh'}$ by computing transition amplitudes between massive neutrino states labeled by momentum $\vec p$ and helicity $h \in \{L, R\}$ namely

$$\langle \vec p', h' \,|\, \vec p, h\rangle \equiv -i(2\pi)^4\, 2E_{\vec p}\, \delta^{(4)}(p - p')\, \mathcal{H}_{h'h}(p). \tag{4}$$

To first order in the interaction (2) and to all orders in $m/|\vec p|$ (with the notation $p \equiv |\vec p|$, $E = \sqrt{m^2 + p^2}$), following the steps outlined in Appendix A we find

$$\mathcal{H}_{LL}(p) = \frac{E+p}{4E}\Big\{ -4r(p)\,\mu_\nu\, \hat p \cdot \vec B - (1 - r(p)^2)\Sigma^0_A + (1 + r(p)^2)\hat p \cdot \vec\Sigma_A + (1 + r(p)^2)\Sigma^0_V - (1 - r(p)^2)\hat p \cdot \vec\Sigma_V \Big\} \tag{5}$$

$$\mathcal{H}_{RR}(p) = \frac{E+p}{4E}\Big\{ +4r(p)\,\mu_\nu\, \hat p \cdot \vec B + (1 - r(p)^2)\Sigma^0_A - (1 + r(p)^2)\hat p \cdot \vec\Sigma_A + (1 + r(p)^2)\Sigma^0_V - (1 - r(p)^2)\hat p \cdot \vec\Sigma_V \Big\} \tag{6}$$

$$\mathcal{H}_{LR}(p) = \frac{E+p}{2E}\Big\{ (1 + r(p)^2)\,\mu_\nu\, \hat x_+ \cdot \vec B \; - \; r(p)\, \hat x_+ \cdot \vec\Sigma_A \Big\} \tag{7}$$

$$\mathcal{H}_{RL}(p) = \frac{E+p}{2E}\Big\{ (1 + r(p)^2)\,\mu_\nu\, \hat x^*_+ \cdot \vec B \; - \; r(p)\, \hat x^*_+ \cdot \vec\Sigma_A \Big\} , \tag{8}$$

where

$$r(p) = \frac{m}{E+p} \qquad 1 + r(p)^2 = \frac{2E}{E+p} \qquad 1 - r(p)^2 = \frac{2p}{E+p} , \tag{9}$$

and $\hat x_+ \equiv e^{i\phi_p}(\hat x_1 + i\hat x_2)$ with $\hat x_{1,2}$ defined so that $(\hat x_1, \hat x_2, \hat p)$ form a right-handed triad. The choice of $\hat x_{1,2}$ orthogonal to $\hat p$ is arbitrary up to a rotation along the $\hat p$ axis. We use here the "standard gauge" specified by choosing the same azimuthal angle for $\hat x_1$ and $\hat p$ ($\phi_{x_1} = \phi_p$),[1] with unit vectors Cartesian coordinates expressed in terms of the polar and azimuthal angles (θ_p, ϕ_p) by

[1] In Ref. [8] a different "gauge" was used, in which a rotation by $-\phi_p$ was made in the $\hat x_1$–$\hat x_2$ plane. With this choice the phase factors $e^{\pm i\phi_p}$ disappear from all formal expressions, but the algebra to obtain dot products of $\hat x_{1,2}(p)$ with other vectors is more cumbersome.

$$\hat{p} = (\sin\theta_p \cos\phi_p\ ,\ \sin\theta_p \sin\phi_p\ ,\ \cos\theta_p) \tag{10}$$

$$\hat{x}_1 = (\cos\theta_p \cos\phi_p\ ,\ \cos\theta_p \sin\phi_p\ ,\ -\sin\theta_p) \tag{11}$$

$$\hat{x}_2 = (-\sin\phi_p\ ,\ \cos\phi_p\ ,\ 0)\ . \tag{12}$$

From the results in (5)–(8) one sees explicitly that helicity mixing occurs only due to components of $\vec{B}$ and $\vec{\Sigma}_A$ transverse to the momentum. Note that factors of $r(p)$ involving one power of mass appear whenever needed to provide the appropriate helicity flip: in absence of mass, axial-vector couplings are helicity conserving while magnetic dipole couplings are helicity-flipping. Note that $r(p)$ provides a suppression factor for axial-induced spin flip amplitude at $|\vec{p}| \gg m$, while it is $O(1)$ at $|\vec{p}| \leq m$. Besides displaying helicity-flip transitions, the results in (5)–(8) also encode the known medium birefringence effect [14]: from parity-violating interactions ($\Sigma_A \neq 0$) left-handed and right-handed states of momentum $\vec{p}$ acquire different energy shifts, with energy splitting proportional to $\Sigma_A^0 - \vec{\Sigma}_A \cdot \hat{p}$. Finally, taking the limit $m/|\vec{p}| \ll 1$, these results reproduce the findings of Refs. [8,10], where the more general multi-flavor case was considered.

While so far we have treated the potentials $\Sigma^\mu_{V,A}$ as external fields, in a complete calculation these are induced by forward scattering on a background of matter and (anti)neutrinos. The only difference with respect to the standard MSW [15–17] analyses is that here we keep non-zero space-like components of the matter- and neutrino-induced potentials ($\vec{\Sigma}_{V,A} \neq 0$), as done for example in Ref. [18]. The explicit expressions (given below) are not crucial to understand the physical origin of the helicity mixing effect, the key point being the spin-dependent axial coupling. So in summary, neutrino interactions in a non-isotropic medium induce a coupling of the neutrino axial current to an axial-vector potential $\Sigma^\mu_A \equiv \Sigma^\mu_L - \Sigma^\mu_R = (\Sigma^0_A, \vec{\Sigma}_A)$ (see Eq. (2)). The time-like component Σ^0_A induces the well known birefringence effect. The space-like potential $\vec{\Sigma}_A$ has a twofold effect: (i) its component $\vec{\Sigma}_A \cdot \hat{p}$ parallel to the neutrino propagation gives an additional contribution to the energy splitting of L and R states; (ii) its component transverse to $\vec{p}$ induces mixing of the L and R states. In general these effects are flavor dependent, as Σ^μ_A carries flavor indices.

3. QKE's for coherent neutrino evolution

Having established the existence of the helicity-mixing term in the effective Hamiltonian through simple quantum-mechanical considerations, we next summarize how this new term appears in the QKEs [8,10]. In Ref. [8] the QKEs describing the evolution of Majorana neutrinos were derived using field-theoretic methods. These QKEs generalize earlier work [1–6] in two respects: (i) They include spin degrees of freedom; (ii) They include effects up to second order in small ratios of scales characterizing the neutrino environments we are interested in. Specifically, we treat neutrino masses, mass-splitting, and matter potentials induced by forward scattering, as well as external gradients as much smaller than the typical neutrino energy scale E, set by the temperature or chemical potential: namely $m_\nu/E \sim \Delta m_\nu/E \sim \Sigma_{\rm forward}/E \sim \partial_X/E \sim O(\epsilon)$.[2] The inelastic scattering can also be characterized by a potential $\Sigma_{\rm inelastic} \sim \Sigma_{\rm forward} \times G_F E^2$ which we therefore power-count as $\Sigma_{\rm inelastic}/E \sim O(\epsilon^2)$. This power-counting is tantamount to the statement that physical quantities vary slowly on the scale of the neutrino de Broglie wavelength.

3.1. Neutrino density matrices

QKEs are the evolution equations for suitably defined dynamical quantities that characterize a neutrino ensemble, which we will refer to (with slight abuse of language) as neutrino density matrices. In the most general terms a neutrino ensemble is described by the set of all $2n$-field Green's functions, encoding n-particle correlations. These obey coupled integro-differential equations, equivalent to the BBGKY equations [19]. As discussed in Refs. [1,8], for weakly interacting neutrinos ($\Sigma/E \sim O(\epsilon, \epsilon^2)$) the set of coupled equations can be truncated by using perturbation theory to express all higher order Green's functions in terms of the two-point functions. In this case the neutrino ensemble is characterized by the full set of one-particle correlations.[3] One-particle states of massive neutrinos and antineutrinos are specified by the three-momentum $\vec{p}$, the helicity $h \in \{L, R\}$, and the family label i (for eigenstates of mass m_i), with corresponding annihilation operators $a_{i,\vec{p},h}$ and $b_{j,\vec{p},h}$ satisfying the canonical anti-commutation relations $\{a_{i,\vec{p},h}, a^\dagger_{j,\vec{p}',h'}\} = (2\pi)^3\, 2\,\omega_i(\vec{p})\, \delta_{hh'}\, \delta_{ij}\, \delta^{(3)}(\vec{p} - \vec{p}')$, etc., where $\omega_i(\vec{p}) = \sqrt{\vec{p}^2 + m_i^2}$. Then, the ensemble is specified by the matrices $f^{ij}_{hh'}(\vec{p})$ and $\bar{f}^{ij}_{hh'}(\vec{p})$ defined by

$$\langle a^\dagger_{j,\vec{p}',h'}\, a_{i,\vec{p},h}\rangle = (2\pi)^3\, 2 n_{ij}(\vec{p})\, \delta^{(3)}(\vec{p} - \vec{p}')\ f^{ij}_{hh'}(\vec{p})\ , \tag{13}$$

$$\langle b^\dagger_{i,\vec{p}',h'}\, b_{j,\vec{p},h}\rangle = (2\pi)^3\, 2 n_{ij}(\vec{p})\, \delta^{(3)}(\vec{p} - \vec{p}')\ \bar{f}^{ij}_{hh'}(\vec{p})\ , \tag{14}$$

where $\langle \ldots \rangle$ denotes the ensemble average and the normalization factor can be chosen as $n_{ij} = 2\omega_i\omega_j/(\omega_i + \omega_j)$.[4] For inhomogeneous backgrounds, the density matrices depend also on the space–time label, denoted by x in what follows.

Despite the intimidating index structure, the physical meaning of the generalized density matrices $f^{ij}_{hh'}(\vec{p})$ and $\bar{f}^{ij}_{hh'}(\vec{p})$ is dictated by simple quantum mechanical considerations: the diagonal entries $f^{ii}_{hh}(\vec{p})$ represent the occupation numbers of neutrinos of mass m_i, momentum $\vec{p}$, and helicity h; the off diagonal elements $f^{ij}_{hh}(\vec{p})$ represent quantum coherence of states of the same helicity and different mass (familiar in the context of neutrino oscillations); $f^{ii}_{hh'}(\vec{p})$ represent coherence of states of different helicity and the same mass, and finally $f^{ij}_{hh'}(\vec{p})$ represent coherence between states of different helicity and mass.

[2] In the early universe, the small lepton number implies $\Sigma_{\rm forward} \sim G_F n_e \ll m_\nu \sim \Delta m_\nu$. This is not the case in supernovae.

[3] As discussed in the introduction, we neglect here correlations that pair particles and antiparticles of opposite momenta [7,10].

[4] The interchange $i \leftrightarrow j$ in the definition of antiparticle distribution matrices is chosen so that under unitary transformations $\nu' = U\nu$, f and $\bar{f}$ transform in the same way, i.e. $f' = U f U^\dagger$.

Fig. 1. Feynman graphs contributing to $\Sigma(x)$. External lines represent neutrinos. Internal lines represent ν, e, n, p propagators.

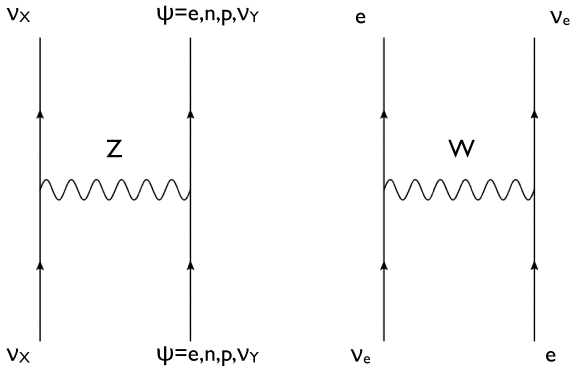

Fig. 2. Tree-level Feynman graphs whose forward-scattering contributions generate the 4-vector potential $\Sigma^\mu(x)$.

In summary, the basic dynamical objects describing ensembles of neutrinos and anti-neutrinos are the $2n_f \times 2n_f$ matrices,

$$F(\vec{p},x) = \begin{pmatrix} f_{LL} & f_{LR} \\ f_{RL} & f_{RR} \end{pmatrix}; \qquad \bar{F}(\vec{p},x) = \begin{pmatrix} \bar{f}_{RR} & \bar{f}_{RL} \\ \bar{f}_{LR} & \bar{f}_{LL} \end{pmatrix}, \tag{15}$$

where we have suppressed the generation indices (each block $f_{hh'}$ is a square $n_f \times n_f$ matrix). For Dirac neutrinos, one needs both F and $\bar{F}$, with f_{LL} and $\bar{f}_{RR}$ describing active states. For Majorana neutrinos, one can choose the phases so that $a_i(\vec{p},h) = b_i(\vec{p},h)$ and therefore $f_{hh'} = \bar{f}^T_{hh'}$ (transposition acts on flavor indices). Therefore the dynamics is specified by $f \equiv f_{LL}$, $\bar{f} \equiv \bar{f}_{RR} = f^T_{RR}$, and $\phi \equiv f_{LR}$, and one needs evolution equations only for the matrix $\mathcal{F}$ [8]:

$$F \to \mathcal{F} = \begin{pmatrix} f & \phi \\ \phi^\dagger & \bar{f}^T \end{pmatrix}. \tag{16}$$

Strictly speaking, the above discussion in terms of creation and annihilation operators makes sense only within the mass eigenstate basis [20]. One can still define "flavor basis" density matrices $f_{\alpha\beta}$ in terms of the mass-basis f_{ij} as $f_{\alpha\beta} = U_{\alpha i} f_{ij} U^*_{\beta j}$, where U is the unitary transformation $\nu_\alpha = U_{\alpha i}\nu_i$ that puts the inverse neutrino propagator in diagonal form. While the QKEs can be written in any basis, we give our results below in the "flavor" basis.

3.2. Anatomy of the QKEs

A detailed derivation of the QKEs using field-theoretic methods is given in Ref. [8]. Keeping terms up to $O(\epsilon^2)$ in the power counting discussed earlier on, the QKEs take the compact $2n_f \times 2n_f$ form:

$$\begin{aligned} D_{\vec{p},x}\, F(\vec{p},x) &= -i\big[\, H(\vec{p},x)\,,\, F(\vec{p},x)\,\big] + C(\vec{p},x)\,; \\ \bar{D}_{\vec{p},x}\, \bar{F}(\vec{p},x) &= -i\big[\, \bar{H}(\vec{p},x)\,,\, \bar{F}(\vec{p},x)\,\big] + \bar{C}(\vec{p},x)\,. \end{aligned} \tag{17}$$

The differential operator on the left-hand side generalizes the usual "Vlasov" term of transport equations. The first term on the right-hand side controls coherent evolution due to mass and forward scattering, generalizing the standard MSW [15–17]. Finally, the second term on the right-hand side encodes inelastic collisions and generalizes the standard Boltzmann collision term used in supernova neutrino analyses [21–29]. Here we focus on the "Vlasov"-type differential operators (D, $\bar{D}$) and the Hamiltonian-like operators (H, $\bar{H}$), describing coherent neutrino evolution. The analysis of inelastic collisions (C, $\bar{C}$) was outlined in Ref. [8], where only a small subset of the contributions to C and $\bar{C}$ was explicitly calculated. Full details on the collision terms will be presented elsewhere [30].

In order to provide the explicit form of the various operators appearing in (17), it is extremely useful to introduce the following notation. Given an ultra-relativistic neutrino of momentum $\vec{p}$, one can naturally introduce a basis formed by two light-like four-vectors $n^\mu(p) = (1,\hat{p})$ and $\bar{n}^\mu(p) = (1,-\hat{p})$ (satisfying $n\cdot n = \bar{n}\cdot\bar{n} = 0$, $n\cdot\bar{n} = 2$) and two transverse four vectors $x^\mu_{1,2}(p) = (0,\hat{x}_{1,2})$ so that $n\cdot x_i = \bar{n}\cdot x_i = 0$ and $x_i \cdot x_j = -\delta_{ij}$. As discussed below Eq. (9), $\hat{p}$ and the space-like components $\hat{x}_{1,2}$ of $x^\mu_{1,2}$ form a right-handed triad.

The key ingredients controlling coherent neutrino evolution are the neutrino mass matrix m and the 4-potential induced by forward scattering on matter and other neutrinos. In the non-equilibrium field-theory approach, forward scattering is encoded in the one-loop self-energy diagrams of Fig. 1. In the more familiar amplitude-based approach this physics is described by the diagrams in Fig. 2. The chiral 4-potentials can be arranged in the $2n_f \times 2n_f$ structure

$$\Sigma^\mu(x) = \begin{pmatrix} \Sigma^\mu_R(x) & 0 \\ 0 & \Sigma^\mu_L(x) \end{pmatrix}. \tag{18}$$

Σ_R and Σ_L are the potentials for left-handed and right-handed neutrinos, respectively. For Dirac neutrinos $\Sigma_R \neq 0$ while $\Sigma_L \propto G_F m^2 \sim O(\epsilon^3)$ (massless right-handed neutrinos do not interact). On the other hand, in the Majorana case one has $\Sigma_L = -\Sigma_R^T$, with transposition acting on flavor indices. The potential induced by a background of electrons and positrons is given for any geometry by the following expressions:

$$\left[\Sigma_R^\mu \Big|_e\right]_{IJ} = 2\sqrt{2} G_F \left[\left(\delta_{eI}\delta_{eJ} + \delta_{IJ}\left(\sin^2\theta_W - \frac{1}{2}\right)\right) J^\mu_{(e_L)} + \delta_{IJ}\, \sin^2\theta_W\, J^\mu_{(e_R)}\right] \tag{19}$$

$$J^\mu_{(e_L)}(x) = \int \frac{d^3q}{(2\pi)^3}\, v^\mu_{(e)}(q)\left(f_{e_L}(\vec{q},x) - \bar{f}_{e_R}(\vec{q},x)\right), \tag{20}$$

$$J^\mu_{(e_R)}(x) = \int \frac{d^3q}{(2\pi)^3}\, v^\mu_{(e)}(q)\left(f_{e_R}(\vec{q},x) - \bar{f}_{e_L}(\vec{q},x)\right), \tag{21}$$

where $v^\mu_{(e)} = (1, \vec{q}/\sqrt{m_e^2+q^2})$, and we use the notation $f_{e_L}(\vec{q},x)$ ($\bar{f}_{e_L}(\vec{q},x)$) for the distribution function of L-handed electrons (positrons), etc. The nucleon-induced potentials have similar expressions, with appropriate replacements of the L- and R-handed couplings to the Z and the distribution functions $f_{e_L} \to f_{N_L}$, etc. For unpolarized electron and nucleon backgrounds of course one has $f_{e_L} = f_{e_R} = (1/2) f_e$, etc., and the nucleon contribution to the potential is

$$\left[\Sigma_R^\mu \Big|_N\right]_{IJ} = \sqrt{2} G_F\, C_V^{(N)}\, J^\mu_{(N)}\, \delta_{IJ}\,, \qquad C_V^{(n)} = -\frac{1}{2}\,, \quad C_V^{(p)} = \frac{1}{2} - 2\sin^2\theta_W\,. \tag{22}$$

On the other hand, the neutrino-induced potentials are given by

$$\left[\Sigma_R^\mu \Big|_\nu\right]_{IJ} = \sqrt{2} G_F \left(\left[J^\mu_{(\nu)}\right]_{IJ} + \delta_{IJ}\, \mathrm{Tr} J^\mu_{(\nu)}\right) \tag{23}$$

$$J^\mu_{(\nu)}(x) = \int \frac{d^3q}{(2\pi)^3} n^\mu(q)\left(f_{LL}(\vec{q},x) - \bar{f}_{RR}(\vec{q},x)\right), \tag{24}$$

with $n^\mu(q) = (1,\hat{q})$. For a test-neutrino of three-momentum $\vec{p}$, these potentials can be further projected along the basis vectors: with light-like component $\Sigma^\kappa \equiv n(p)\cdot\Sigma$ along the neutrino trajectory (in the massless limit); and space-like component $\Sigma^i \equiv x^i(p)\cdot\Sigma$, transverse to the neutrino trajectory. In particular, for the neutrino-induced contribution we find $\Sigma^\kappa(x) \propto \int d^3q\ (1-\cos\theta_{pq})\cdot(f_{LL}(\vec{q},x) - \bar{f}_{RR}(\vec{q},x))$, consistently with the familiar results in the literature ([31] and references therein).

In terms of the mass matrix m and the potentials $\Sigma^\mu_{L,R}$, the Hamiltonian-like operators controlling the coherent evolution are given by

$$H = \begin{pmatrix} H_R & H_{LR} \\ H_{LR}^\dagger & H_L \end{pmatrix} \qquad \bar{H} = \begin{pmatrix} \bar{H}_R & H_{LR} \\ H_{LR}^\dagger & \bar{H}_L \end{pmatrix}, \tag{25}$$

with

$$H_R = \Sigma_R^\kappa + \frac{1}{2|\vec{p}|}\left(m^\dagger m - \epsilon^{ij}\partial^i \Sigma_R^j + 4\Sigma_R^+\Sigma_R^-\right) \tag{26}$$

$$H_L = \Sigma_L^\kappa + \frac{1}{2|\vec{p}|}\left(m m^\dagger + \epsilon^{ij}\partial^i \Sigma_L^j + 4\Sigma_L^-\Sigma_L^+\right) \tag{27}$$

$$H_{LR} = -\frac{1}{|\vec{p}|}\left(\Sigma_R^+ m^\dagger - m^\dagger \Sigma_L^+\right), \tag{28}$$

where $\Sigma^\pm_{L,R} \equiv (1/2)\, e^{\pm i\phi}\,(x_1 \pm i x_2)_\mu\, \Sigma^\mu_{L,R}$. The antineutrino operators $\bar{H}_{L,R}$ can be obtained from $H_{L,R}$ by flipping the sign of the entire term multiplying $1/(2|\vec{p}|)$. The first two terms in $H_{L,R}$ are included in all analyses of neutrino oscillations in medium. $\Sigma^\kappa_{L,R}$ include the usual forward scattering of matter and neutrinos, and are functions of F, $\bar{F}$ thereby introducing non-linear effects in the coherent evolution. The $m^\dagger m/|\vec{p}|$ term encodes vacuum oscillations. The additional terms in $H_{L,R}$ and the spin-flip term H_{LR} complete the set of $O(\epsilon^2)$ terms, and can be as important as $m^2/|\vec{p}|$ in supernova environments. The spin-flip term H_{LR} is given in compact matrix form in (28), and its physical origin has been discussed in Section 2. Note that the spin-flip term H_{LR} depends linearly on the mass matrix m, while the vacuum Hamiltonian depends on $m^\dagger m$. Therefore, as we will show explicitly later, the spin-flip term is sensitive to the absolute mass scale of the neutrino spectrum and (for Majorana neutrinos) to the Majorana phases.

Finally, using the compact notation $\partial^\kappa \equiv n(p)\cdot\partial = \partial_t + \hat{p}\cdot\partial_{\vec{x}}$, $\partial^i \equiv x^i(p)\cdot\partial = \hat{x}_i\cdot\partial_{\vec{x}}$, the generalized Vlasov operators are (recall that the $2n_f \times 2n_f$ potential Σ^μ is defined in (18))

$$D_{\vec{p},x} F(\vec{p},x) = \partial^\kappa F + \frac{1}{2|\vec{p}|}\left\{\Sigma^i, \partial^i F\right\} - \frac{1}{2}\left\{\frac{\partial\Sigma^\kappa}{\partial\vec{x}}, \frac{\partial F}{\partial\vec{p}}\right\} \tag{29}$$

$$\bar{D}_{\vec{p},x} \bar{F}(\vec{p},x) = \partial^\kappa \bar{F} - \frac{1}{2|\vec{p}|}\left\{\Sigma^i, \partial^i \bar{F}\right\} + \frac{1}{2}\left\{\frac{\partial\Sigma^\kappa}{\partial\vec{x}}, \frac{\partial \bar{F}}{\partial\vec{p}}\right\}. \tag{30}$$

The physical meaning of D and $\bar{D}$ becomes more transparent by noting that they can be re-written as

$$\partial_t + \frac{1}{2}\{\partial_{\vec{p}}\omega_\pm, \partial_{\vec{x}}\} - \frac{1}{2}\{\partial_{\vec{x}}\omega_\pm, \partial_{\vec{p}}\}, \tag{31}$$

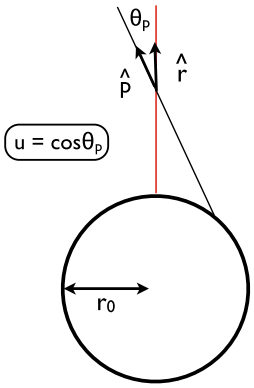

Fig. 3. Bulb model geometry.

with $\omega_+ = |\vec{p}| + \Sigma^\kappa$ in D and $\omega_- = |\vec{p}| - \Sigma^\kappa$ in $\bar{D}$. Recalling that $\omega_\pm(\vec{p}) = |\vec{p}| \pm \Sigma^\kappa$ are the $O(\epsilon)$ neutrino $(+)$ and anti-neutrino $(-)$ Hamiltonian operators, one sees that D and $\bar{D}$ generalize the total time-derivative operator $d_t = \partial_t + \dot{\vec{x}}\ \partial_{\vec{x}} + \dot{\vec{p}}\ \partial_{\vec{p}}$, with $\dot{\vec{p}} = -\partial_{\vec{x}}\,\omega$ and $\dot{\vec{x}} = \partial_{\vec{p}}\,\omega$, thus encoding the familiar drift and force terms.

4. The bulb model

The simplest realistic setup to explore the impact of helicity oscillations in supernovae is provided by the so-called bulb model. In this model one assumes spherical symmetry and further assumes that neutrinos are emitted isotropically, with a given spectrum and luminosity, from a sharply defined neutrino-sphere of radius r_0 (see Fig. 3). In this section we provide explicit expressions for the coherent QKEs describing two-flavor Majorana neutrinos in the bulb model. These expressions are amenable to computational implementation and allow us to explicitly point out the dependence of QKEs on the absolute neutrino mass scale and the Majorana phase characterizing the two-flavor problem. Throughout the discussion, it is useful to keep in mind that the Σ^μ_R potential in this geometry only has time-like and radial space-like components (the transverse components cancel).

To make contact with the existing literature on the subject [1,10,32,33], we introduce the following notation for the 2×2 blocks of the Majorana density matrix

$$f = \begin{pmatrix} \rho_{ee} & \rho_{ex} \\ \rho^*_{\bar{e}\bar{x}} & \rho_{xx} \end{pmatrix} \qquad \bar{f} = \begin{pmatrix} \rho_{\bar{e}\bar{e}} & \rho_{\bar{e}\bar{x}} \\ \rho^*_{\bar{e}\bar{x}} & \rho_{\bar{x}\bar{x}} \end{pmatrix} \qquad \phi = \begin{pmatrix} \rho_{e\bar{e}} & \rho_{e\bar{x}} \\ \rho_{x\bar{e}} & \rho_{x\bar{x}} \end{pmatrix}, \tag{32}$$

so that the flavor $\times$ spin density matrix for two flavors $(\nu_e,\ \nu_x)$ is given by

$$\mathcal{F} = \begin{pmatrix} f & \phi \\ \phi^\dagger & \bar{f}^T \end{pmatrix} = \begin{pmatrix} \rho_{ee} & \rho_{ex} & \rho_{e\bar{e}} & \rho_{e\bar{x}} \\ \rho^\star_{ex} & \rho_{xx} & \rho_{x\bar{e}} & \rho_{x\bar{x}} \\ \rho^\star_{e\bar{e}} & \rho^\star_{x\bar{e}} & \rho_{\bar{e}\bar{e}} & \rho^\star_{\bar{e}\bar{x}} \\ \rho^\star_{e\bar{x}} & \rho^\star_{x\bar{x}} & \rho_{\bar{e}\bar{x}} & \rho_{\bar{x}\bar{x}} \end{pmatrix}. \tag{33}$$

The coherent QKEs for the density matrix are

$$D_{\vec{p},x}\,\mathcal{F}(\vec{p},x) = -i\,[\mathcal{H},\mathcal{F}]\ , \tag{34}$$

and the assumption of spherical symmetry implies that the density matrix depends only on $p \equiv |\vec{p}| \simeq E$, $u \equiv \cos\theta_p$, and r, so that $\mathcal{F}(\vec{p},x) \to \mathcal{F}(E,u,r)$.

Both the Vlasov differential operator $D_{\vec{p},x}$ and the Hamiltonian $\mathcal{H}$ depend on the 4-potential Σ^μ (18), that in spherical symmetry has a time-like and a space-like radial component

$$\Sigma^\mu = \left(\Sigma^0_{\rm matter} + \Sigma^0_\nu,\ \left(\Sigma^r_{\rm matter} + \Sigma^r_\nu\right)\hat{r}\right). \tag{35}$$

Assuming the absence of muons, and neglecting corrections of order G_F^2, the matter contribution is given by

$$\Sigma^0_{\rm matter} = \frac{G_F n_B}{\sqrt{2}} \begin{pmatrix} 3Y_e - 1 & 0 & 0 & 0 \\ 0 & Y_e - 1 & 0 & 0 \\ 0 & 0 & -(3Y_e - 1) & 0 \\ 0 & 0 & 0 & -(Y_e - 1) \end{pmatrix} \qquad \Sigma^r_{\rm matter} = V_{\rm out}\ \Sigma^0_{\rm matter}\ , \tag{36}$$

where n_B is the baryon number density, $Y_e = (n_e - n_{\bar{e}})/n_B$ is the electron lepton number fraction, and $V_{\rm out}$ is the (radial) matter outflow speed as a fraction of the speed of light. Similarly, the neutrino-induced potential is

$$\Sigma^0_\nu = \sqrt{2} G_F \begin{pmatrix} 2J^0_{ee} + J^0_{xx} & J^0_{ex} & 0 & 0 \\ J^{0\star}_{ex} & 2J^0_{xx} + J^0_{ee} & 0 & 0 \\ 0 & 0 & -2J^0_{ee} - J^0_{xx} & -J^{0\star}_{ex} \\ 0 & 0 & -J^0_{ex} & -2J^0_{xx} - J^0_{ee} \end{pmatrix} \qquad \Sigma^r_\nu = \Sigma^0_\nu\Big|_{J^0_{IJ} \to J^r_{IJ}}\ , \tag{37}$$

with time-like and radial components of the neutrino current J^0_{IJ} and J^r_{IJ} given by[5]

$$J^0_{IJ} = \int \frac{E'^2 dE'}{(2\pi)^2} \int_{u_{\min}}^{1} du' \left[f_{IJ}\left(E', u'\right) - \bar{f}_{JI}\left(E', u'\right) \right] \tag{38}$$

$$J^r_{IJ} = \int \frac{E'^2 dE'}{(2\pi)^2} \int_{u_{\min}}^{1} du'\, u' \left[f_{IJ}\left(E', u'\right) - \bar{f}_{JI}\left(E', u'\right) \right] . \tag{39}$$

In terms of the potentials explicitly given above, the Vlasov operator is

$$D_{\vec{p},x}\mathcal{F} = u\,\frac{\partial \mathcal{F}}{\partial r} + \frac{1-u^2}{r}\frac{\partial \mathcal{F}}{\partial u} - \frac{1-u^2}{2E}\left\{\Sigma^r,\ \frac{\partial \mathcal{F}}{\partial r}\right\} - \frac{1}{2}\left\{\frac{\partial \Sigma^0}{\partial r} - u\frac{\partial \Sigma^r}{\partial r},\ u\frac{\partial \mathcal{F}}{\partial E} + \frac{1-u^2}{E}\,\frac{\partial \mathcal{F}}{\partial u}\right\} . \tag{40}$$

Within current "multi-angle" simulations of the bulb model the terms proportional to Σ^r and Σ^0 are usually dropped, retaining only the first two terms of the above Vlasov operator. In a consistent analysis to second order in gradients and interactions all terms in the above expression should be kept.

The Hamiltonian contains vacuum, matter and neutrino contributions. In the presence of spacelike currents, the matter and neutrino contributions give a spin flip term. We break up the terms in the Hamiltonian as follows:

$$\mathcal{H} = \mathcal{H}_{\rm vac} + \mathcal{H}_{\rm matter} + \mathcal{H}_{\nu} + \mathcal{H}_{\rm sf} . \tag{41}$$

The traceless part of the vacuum Hamiltonian is ($\Delta m^2 \equiv m_2^2 - m_1^2 > 0$)

$$\mathcal{H}_{\rm vac} = \frac{\Delta m^2}{4E}\begin{pmatrix} -c_{2\theta} & s_{2\theta} & 0 & 0 \\ s_{2\theta} & c_{2\theta} & 0 & 0 \\ 0 & 0 & -c_{2\theta} & s_{2\theta} \\ 0 & 0 & s_{2\theta} & c_{2\theta} \end{pmatrix} , \tag{42}$$

where $s_{2\theta} = \sin 2\theta$, $c_{2\theta} = \cos 2\theta$ and θ is the two-flavor mixing angle. The matter and neutrino Hamiltonians are

$$H_{\rm matter} = \Sigma^0_{\rm matter} - u\,\Sigma^r_{\rm matter} \tag{43}$$

$$H_{\nu} = \Sigma^0_{\nu} - u\,\Sigma^r_{\nu} . \tag{44}$$

The spin-flip Hamiltonian includes a matter term (in the presence of bulk motion of matter) and a neutrino term. In 2×2 block form, the spin-flip Hamiltonian is

$$\mathcal{H}_{\rm sf} = \begin{pmatrix} 0 & e^{i\phi_p}\left[\left(H^{\rm matter}_{\rm sf} + H^{\nu}_{\rm sf}\right)\frac{m^\star}{E} + \frac{m^\star}{E}\left(H^{\rm matter}_{\rm sf} + H^{\nu}_{\rm sf}\right)^T\right] \\ e^{-i\phi_p}\left[\left(H^{\rm matter}_{\rm sf} + H^{\nu}_{\rm sf}\right)\frac{m^\star}{E} + \frac{m^\star}{E}\left(H^{\rm matter}_{\rm sf} + H^{\nu}_{\rm sf}\right)^T\right]^\dagger & 0 \end{pmatrix} , \tag{45}$$

where the phase factor can be set to $e^{i\phi_p} = 1$ in spherical symmetry and the 2×2 matrices $H^{\rm matter}_{\rm sf}$ and $H^{\nu}_{\rm sf}$ are

$$H^{\rm matter}_{\rm sf} = -\frac{G_F n_B}{2\sqrt{2}} V_{\rm out}\sqrt{1-u^2}\begin{pmatrix} 3Y_e - 1 & 0 \\ 0 & Y_e - 1 \end{pmatrix} \tag{46}$$

$$H^{\nu}_{\rm sf} = -\sqrt{2}G_F\sqrt{1-u^2}\begin{pmatrix} 2J^r_{ee} + J^r_{xx} & J^r_{ex} \\ J^{r\star}_{ex} & 2J^r_{xx} + J^r_{ee} \end{pmatrix} . \tag{47}$$

As observed in the previous sections, the spin-mixing Hamiltonian depends linearly on the neutrino mass matrix. In the two-flavor case, the Majorana mass matrix can be written as

$$m = U^* m_d U^\dagger ; \qquad \text{with} \qquad m_d = \begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix} , \qquad U = \begin{pmatrix} c_\theta & s_\theta \\ -s_\theta & c_\theta \end{pmatrix} \times \begin{pmatrix} 1 & 0 \\ 0 & e^{i\alpha/2} \end{pmatrix} . \tag{48}$$

In terms of the observable parameters $\Delta m^2 \equiv m_2^2 - m_1^2 > 0$ and $m_0 \equiv (1/2)(m_1 + m_2)$ (so that $m_{1,2} = m_0 \mp \Delta m^2/(4m_0)$) we find

$$m = m_0\begin{pmatrix} c_\theta^2 + e^{-i\alpha}s_\theta^2 & (e^{-i\alpha} - 1)s_\theta c_\theta \\ (e^{-i\alpha} - 1)s_\theta c_\theta & s_\theta^2 + e^{-i\alpha}c_\theta^2 \end{pmatrix} + \frac{\Delta m^2}{4m_0}\begin{pmatrix} -(c_\theta^2 - e^{-i\alpha}s_\theta^2) & (e^{-i\alpha} + 1)s_\theta c_\theta \\ (e^{-i\alpha} + 1)s_\theta c_\theta & c_\theta^2 e^{-i\alpha} - s_\theta^2 \end{pmatrix} . \tag{49}$$

Eq. (49) shows explicitly the dependence of the mass matrix on the absolute mass scale of the neutrino mass spectrum, m_0, the mixing angle θ, and the Majorana phase α. The phase α can significantly alter the spin-flavor mixing structure compared to the Dirac case ($\alpha = 0$),

[5] In general the radial component of the current receives an additional contribution [10]

$$\delta J^r_{IJ} = \int_0^{2\pi}\frac{d\phi'}{2\pi}\int \frac{E'dE'}{2(2\pi)^2}\int_{u_{\min}}^{1} du'\sqrt{1-u'^2}\left[m^\dagger\left(\phi^\dagger - \phi^\star\right)e^{i\phi'} + e^{-i\phi'}\left(\phi - \phi^T\right)m\right]_{IJ} .$$

This term, while suppressed by m/E, might be important near neutrino–antineutrino resonance. In spherical symmetry, however, $\delta J^r_{IJ} = 0$ as the dynamical functions $\phi(E', u', r)$ do not depend on the azimuthal angle ϕ'.

for any value of $m_0/\sqrt{\Delta m^2}$. For example, in the degenerate limit $m_0 \gg \sqrt{\Delta m^2}$ (in which we expect $\mathcal{H}_{\rm sf}$ to have the largest impact) the first term in (49) dominates, and a non-zero Majorana phase α can induce $O(1)$ off-diagonal terms compared to the vanishing ones in the Dirac case ($\alpha = 0$).

In absence of a full-fledged calculation no conclusion can be drawn on the impact of Majorana phases on supernova neutrinos. However, we find it very interesting that at least in principle astrophysical processes are sensitive to these parameters. In fact, until now neutrino-less double beta decay experiments offer the only way to probe a subset of these quantities [34], namely the element m_{ee} of the mass matrix given in (49). The detection of large scale neutrino–antineutrino transformation in a supernova neutrino burst could provide information complementary to that obtained from neutrino-less double beta decay searches.

5. Conclusions

In this letter we have discussed he physical origin of the coherent spin-flip term in the neutrino QKEs in the framework of an MSW-like effective Hamiltonian. The key point is that in anisotropic environments, neutrino forward scattering on matter and other neutrinos induces not only a time-like but also a space-like axial-vector potential: the latter couples to neutrino spin and generates helicity mixing. We have also provided explicit expressions for the coherent QKEs in a two-flavor model with spherical geometry: this exercise is a necessary step towards a computational analysis of the QKEs in astrophysical environments, and gives us the opportunity to illustrate an under-appreciated point: through the spin-flip term, neutrino evolution is sensitive to the absolute scale of the neutrino mass spectrum and to the Majorana phases.

While the spin-mixing effect is in general small, $O(G_F \times m_\nu/E)$, it may become dominant if a resonance occurs. In fact, an exploration within a simplified setup [35] indicates that non-linearities can keep the system near resonance thus leading to large scale neutrino–antineutrino conversion. In a different context, it has been shown [32,33] that magnetic-induced spin-flavor oscillations can significantly impact supernova neutrinos. In fact, assuming typical magnetic fields in a supernova envelope ($B \sim 10^{10-12} G$) and Majorana transition magnetic moments a factor of 100 larger than the SM values, so that $\mu_\nu^{ij} B(r) \sim 10^{-18}$ eV $(50\ \text{km}/r)^2$, Refs. [32,33] find that magnetic spin-flip transitions lead to significant effects on collective neutrino oscillations in supernovae. A naive estimate based on $\Sigma \sim \sqrt{2} G_F (n_\nu - n_{\bar\nu})$, with net neutrino density $n_\nu - n_{\bar\nu}$, suggests that $H_{LR} \geq 10^{-18}$ eV at $r \leq 100$ km (for $m_\nu = 0.01$ eV and $|\vec p| = 10$ MeV), in the same ballpark as the magnetic term. While these estimates are rough since they ignore the flavor structure and the effects of geometry, combined with the results of Refs. [32,33], they nevertheless suggest potential implications for supernova neutrinos.

The conditions for significant neutrino–antineutrino conversion require large ν luminosities and the presence of level-crossings (resonances) [35]. These conditions are likely to be realized during the supernova neutronization burst, where there is a large dominance of ν_e over the $\bar\nu_e$ fluxes and the overall neutrino luminosities are large. Additionally, by influencing the competition between the charged current neutrino capture processes $\nu_e n \leftrightarrow p e^-$ and $\bar\nu_e p \leftrightarrow n e^+$ [11], ν_e–$\bar\nu_e$ transformation can directly affect the neutron-to-proton ratio, which is a key determinant of nucleosynthesis in core-collapse supernova ejecta and compact object ejecta.

To assess the impact of our findings on neutrino evolution in supernovae, additional studies are called for. First, one needs to have a full-fledged (i.e. multi-angle) numerical implementation of the coherent QKEs in a spherically symmetric model [36]. Moreover, one eventually needs to work out analytically [30] and implement numerically inelastic collision terms in the QKEs, including the dependence on the full density matrix in flavor and spin space.

Acknowledgements

This work was supported in part by NSF grant PHY-1307372 at UCSD, by the LDRD Program at LANL, by the University of California Office of the President (grant 12-LR-237070), and by the UC HIPACC Collaboration. We would also like to acknowledge support from the DOE Topical Collaboration on "Neutrino and Nucleosynthesis". We thank J. Carlson, J.F. Cherry, and S. Reddy for useful discussions.

Appendix A

In what follows we outline the steps needed to obtain the amplitudes (5)–(8) starting from the interaction Lagrangian (2), for the case of one massive Dirac neutrino. In order to compute the amplitude (4) in perturbation theory, we first express the Lagrangian density $\mathcal{L}_{\rm int}$ in terms of free fields, written as linear combinations of creation and annihilation operators for the helicity states introduced in Section 3:

$$\nu(x) = \int \frac{d^3p}{(2\pi)^2 2E} \sum_{h=\pm} \left[a_{\vec p,h}\, u(p,h)\, e^{-ip\cdot x} + b^\dagger_{\vec p,h}\, v(p,h)\, e^{ip\cdot x} \right] . \tag{50}$$

Here $p = |\vec p|$, $E = \sqrt{p^2 + m^2}$, and the helicity spinors $u(p,\pm)$ are given by

$$u(p,+) = \sqrt{E+p} \begin{pmatrix} r(p)\,\xi_+(\hat p) \\ \xi_+(\hat p) \end{pmatrix} \qquad u(p,-) = \sqrt{E+p} \begin{pmatrix} \xi_-(\hat p) \\ r(p)\,\xi_-(\hat p) \end{pmatrix} \qquad r(p) = \frac{m}{E+p} , \tag{51}$$

with (denoting by θ_p, ϕ_p the polar and azimuthal angles of $\hat p$)

$$\xi_+(\hat p) = \begin{pmatrix} \cos\frac{\theta_p}{2} \\ e^{i\phi_p} \sin\frac{\theta_p}{2} \end{pmatrix} \qquad \xi_-(\hat p) = \begin{pmatrix} -e^{-i\phi_p} \sin\frac{\theta_p}{2} \\ \cos\frac{\theta_p}{2} \end{pmatrix} \qquad (\vec\sigma\cdot\hat p)\,\xi_\pm(\hat p) = \pm\,\xi_\pm(\hat p) . \tag{52}$$

Using the above results and observing that the interaction Lagrangian density has the bilinear structure $\mathcal{L}_{\rm int}(x) = \bar\nu(x)\,\Gamma\,\nu(x)$ (Γ can be identified from Eq. (2)), we write the interaction Hamiltonian $H_{\rm int} = -\int d^3x \mathcal{L}_{\rm int}(x)$ as

$$H_{\rm int} = -\sum_{h,h'} \int \frac{d^3p}{(2\pi)^3} \frac{1}{4E^2}\; a^\dagger_{\vec p,h'} a_{\vec p,h}\; T_{h'h}(p) + \dots , \qquad T_{h'h}(p) \equiv \bar u(p,h')\,\Gamma\,u(p,h) , \tag{53}$$

where the dots indicate the corresponding anti-neutrino operators. Using this interaction Hamiltonian we compute the amplitudes (4) in terms of $T_{h'h}(p)$. The final results (5)–(8) follow after explicit calculation of $T_{h'h}(p)$, requiring some straightforward Dirac algebra and use of the relations

$$\xi^{\dagger}_{\pm}(\hat{p})\,\vec{\sigma}\,\xi_{\pm}(\hat{p}) = \pm\hat{p} \qquad \xi^{\dagger}_{\pm}(\hat{p})\,\vec{\sigma}\,\xi_{\mp}(\hat{p}) = e^{\mp i\phi_p}\left(\hat{x}_1(\hat{p}) \mp i\hat{x}_2(\hat{p})\right), \tag{54}$$

with $\hat{p}$, $\hat{x}_{1,2}(\hat{p})$ defined in (10)–(12).

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