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Streamlines near a closed curve and chaotic streamlines in steady cavity flows

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Abstract

Numerical simulations are carried out for the three-dimensional steady flow in a lid-driven rectangular cavity. We study the incompressible flow in a cavity with spanwise aspect ratio 1.0 and aspect ratios 0.4, 0.6, 1.0 and 1.4. Streamlines are obtained from the steady velocity field and Poincaré sections are plotted from the streamlines for Reynolds numbers ranging from 100 and 400. There are two types of streamlines: localized streamlines near a closed curve and chaotic streamlines. In the Poincaré sections we can find various structures of ovals of invariant tori and resonant islands by localized streamlines in the regions near the end-wall and irregularly distributed points by chaotic streamlines. The structures in the Poincaré sections are similar to those in the phase portraits of one-dimensional non-autonomous Hamiltonian system. The Reynolds number ranges where the 3:1 and 2:1 resonances occur are presented for various aspect ratios.

1. Introduction

The cavity flows have been one of the most important subjects in fluid mechanics because studies of the internal flows in the closed system with the simplest geometry are important theoretically, numerically and practically. In the present study we carry out numerical simulations for the three-
dimensional steady flow in a lid-driven rectangular cavity. Figure 1 shows the Cartesian coordinate for a rectangular cavity with width $H$, depth $D$ and span $L$. The upper wall moves in positive-$x$ direction with constant speed $U$, and it generates the internal flow. There are two non-dimensional geometrical parameters of the cavity: one is the aspect ratio $\Gamma = D/H$ and the other is the spanwise aspect ratio $\Lambda = L/H$. The non-dimensional flow parameter is the Reynolds number $Re = U/H\nu$ where $\nu$ is the kinematic viscosity.

Ishii et al. [1,2] numerically studied three-dimensional viscous incompressible flows in a cubic cavity ($\Gamma = \Lambda = 1$). They obtained steady solutions for $Re$ less than 2000, and they found that the streamlines exhibit complicated structures even in the steady flow for $100 \leq Re \leq 400$. They showed that the Poincaré sections have various structures of invariant curves, resonant islands and chaotic distribution, which are all familiar in the non-autonomous Hamiltonian system. A striking similarity was found between the numerically obtained Poincaré sections and the phase portraits of the normal form Hamiltonians in the resonant cases.

Ishii and Adachi[3] studied the steady flows in a square cavity ($\Gamma = 1$) with a large span $\Lambda = 6.55$ for $Re$ ranging from 100 to 400 by numerical simulations. They found that in the Poincaré sections there are numerous dots by intersections of chaotic streamlines moving globally from the near-end-wall region to the central region for all Reynolds numbers. At $Re = 100$, 200 and 300, the same structures as those in the cubic cavity which result from the localized streamlines were found in the region near the end-wall. Ishii and Adachi[4] then studied the flow structure in a cavity with $\Lambda = 6.55$ for aspect ratios 0.5, 1.0 and 1.5. Dependence of streamline patterns on the aspect ratio was examined for $Re$ ranging from 100 to 500. They found that when the aspect ratio is 1.0, the Reynolds number at which the 3:1 resonance occurs is smaller than those for other aspect ratios.

In the present paper we report the numerical results for the incompressible steady flows in cavities with a spanwise aspect ratio $\Lambda = 1.0$ and aspect ratios $\Gamma = 0.4$, 0.6, 1.0 and 1.4 for $Re$ ranging from 100 to 400. The Poincaré sections at $Re = 100$, 200, 300 and 400 are presented. For resonant case we focus our attention on the 3:1 and 2:1 resonances, and we present the Poincaré sections which correspond to the phase portraits of the normal form Hamiltonian near the resonances. The Reynolds number ranges where these resonances occur are given.

The formulation of the problem and details of the numerical methods are given in sect. 2. It is shown that the particle motion in a steady solenoidal velocity field is equivalent to a non-autonomous Hamiltonian system of one-degree-of-freedom, and the normal forms of Hamiltonian in the resonance are given in sect. 3. Numerically obtained Poincaré sections are presented in sect. 4. The last section is for concluding remarks.

2. Formulation and numerical method

2.1. The governing equations

The viscous incompressible flow is governed by the following three-dimensional Navier-Stokes and
continuity equations. The governing equations in non-dimensional form are

\[
\frac{\partial \mathbf{u}}{\partial t} = -\mathbf{u} \cdot \nabla \mathbf{u} - \nabla p + \frac{1}{Re} \nabla^2 \mathbf{u}
\]

\( (1) \)

\( \nabla \cdot \mathbf{u} = 0 \)  

\( (2) \)

where \( \mathbf{u} = (u, v, w) \) is the velocity and \( p \) is the pressure, and all quantities are normalized with the cavity width \( H \), the speed of the moving lid \( U \) and the constant density \( \rho \). The no-slip boundary conditions are \( \mathbf{u} = (1, 0, 0) \) on the moving lid \( (y = \Gamma) \), and \( \mathbf{u} = 0 \) on other walls \( (y = 0, x = 0, 1, z = 0, \Lambda) \). With these boundary conditions and the continuity equation (2), the derivative of the normal velocity component \( u_n \) in normal direction to the wall should be zero: \( \frac{\partial u_n}{\partial n} = 0 \) at the wall.

The Marker-and-Cell (MAC) method is used to solve the governing equations. In the MAC method, we have to solve the Poisson equation for pressure in place of the continuity equation (2). Details of the numerical method for solving the Poisson equation are presented in our previous paper [3]. When the velocity increment by a time-step becomes small enough we stop the time integration of eq. (1), and we consider the obtained state as a steady state. The fulfilled condition is

\[
\| \mathbf{u}(x, y, z, t) - \mathbf{u}(x, y, z, t - \Delta t) \| / \Delta t < \varepsilon,
\]

where \( \| \mathbf{u}(x, y, z, t) - \mathbf{u}(x, y, z, t - \Delta t) \| \) is the magnitude of the velocity increment averaged over all grid points, \( \Delta t = 1 \times 10^{-7} \) and \( \varepsilon = 1 \times 10^{-5} \).

### 2.2. CCD scheme

In the present study the spectral-like Combined Compact Difference (CCD) scheme [5] is adopted to evaluate first and second spatial derivatives in the momentum equation and the Poisson equation. It is possible to achieve high accuracy and high resolution at the same time with the CCD scheme.

In the CCD scheme we solve a linear system of equations for the function and its first, second and third derivatives at three neighboring points with a uniform spacing \( h \). Let \( f, f', f'' \) and \( f''' \) be the values of the function, its first, second and third derivatives at the \( i \)-th grid point respectively for \( 1 \leq i \leq N \). The linear system of equations in matrix form is

\[
\begin{bmatrix}
B_1 & C_1 & 0 & 0 & \cdots & 0 & 0 \\
A & B & C & 0 & \cdots & 0 & 0 \\
0 & A & B & C & 0 & \cdots & 0 \\
0 & 0 & \cdots & 0 & A & B & C \\
0 & 0 & \cdots & 0 & A & B & C
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
x_3 \\
x_4 \\
x_5
\end{bmatrix}
= \begin{bmatrix} g_1 \\ g_2 \\ g_3 \\ g_4 \\ g_5 \end{bmatrix}
\]

with \( x_i = \begin{bmatrix} h f'_i \\ h^2 f''_i \\ h^3 f'''_i \end{bmatrix} \) \( (1 \leq i \leq N) \).

For the interior points the matrices \( A \), \( B \) and \( C \) and the vector \( g \) are

\[
A = \begin{bmatrix} a_1 & -b_1 & c_1 \\ -a_1 & b_2 & -c_2 \\ a_3 & -b_3 & c_3 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad C = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}, \quad g_i = \begin{bmatrix} d_i (f_{i+1} - f_i) \\ d_i (f_{i+1} - 2f_i + f_{i-1}) \\ d_i (f_{i+1} - f_{i-1}) \end{bmatrix} \quad (2 \leq i \leq N-1)
\]
The coefficients $a_j, b_j, c_j$ and $d_j$ are determined so that the scheme has high accuracy and high resolution at the same time. The highest accuracy of this scheme is 8-th, 8-th and 6-th order for the first, second and third derivatives respectively. In the highest accuracy case all twelve coefficients are fixed. When we do not require the highest accuracy, we have several free parameters. In the present study we have two free parameters. With two free parameters $d_2$ and $d_3$, other coefficients are given as

$$
\begin{align*}
    a_1 &= \frac{8d_1 + 195}{240}, b_1 = -\frac{16d_1 + 255}{1200}, c_i = \frac{4d_i + 45}{1800}, d_i = \frac{8d_i + 315}{240}, \\
    a_2 &= \frac{11d_2 - 15}{16}, b_2 = -\frac{3d_2 - 7}{16}, c_2 = \frac{d_2 - 3}{48}, a_1 = d_1, b_1 = -\frac{8d_1 + 15}{20}, c_1 = \frac{4d_1 + 15}{60}.
\end{align*}
$$

Values of two parameters are chosen so as to give high resolution: $d_2 = 9.12992$ and $d_3 = 6.01486$. The present scheme has 8-th, 6-th and 4-th order accuracy for the first, second and third derivatives respectively at the interior points. When values of the function at the boundaries are given but values of its first derivative are not given, the scheme has 5-th and 4-th order accuracy for the first and second derivatives respectively at the boundaries. When both values of the function and its first derivative at the boundaries are given, the scheme gives 5-th order accuracy to the second derivative at the boundaries. In order to solve efficiently the block tridiagonal matrix in eq. (3) we introduce the parallelization, details of which are given in our previous paper [3].

3. Poincaré sections and the phase portraits of Hamiltonian system

3.1. Streamlines and Poincaré sections

Streamlines coincide with the trajectories of fluid particles in steady flows. The streamlines equations in Cartesian coordinate $x = (x, y, z)$ are

$$
\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}
$$

where the velocity is $u = (u, v, w)$. Streamlines are numerically obtained by solving the equations $dx/dt = u, dy/dt = v, dz/dt = w$ with the fourth order Runge-Kutta time integration.

Whiteman [6] showed for generalized coordinates that the line equations of a three-dimensional solenoidal vector field are the Hamiltonian system. The solenoidal velocity vector field of the steady incompressible fluid is shown to be equivalent to a non-autonomous Hamiltonian system of one-degree-of-freedom as follows [2]. Let $(x^1, x^2, x^3)$ be generalized coordinates, $v^i$ the contravariant components of the velocity and $g$ the determinant of the metric tensor. If the generalized coordinate $q$ and momentum $p$ are defined as

$$
q = x^1, \quad p = \int_0^{x^2} \sqrt{g v^i (x^1, x^2, \tau) dx^2}, \quad \tau = x^3
$$

where the third coordinate $x^3$ plays the role of time, the streamline equations are derived from the Hamiltonian.
We can reduce the Hamiltonian’s equations of motion

\[
\frac{dq}{d\tau} = \frac{\partial H}{\partial p}, \quad \frac{dp}{d\tau} = -\frac{\partial H}{\partial q}
\]

to the line equations

\[
\frac{dx}{dx'} = \frac{v'}{v}, \quad \frac{dx'}{dx} = \frac{v'}{v}
\]

if we use the solenoidal condition

\[
\sum_{i} \frac{\partial}{\partial x'} \left( \sqrt{g_v} v' \right) = 0.
\]

In the following section we present the Poincaré sections of several streamlines which cover a surface of invariant torus. There is a closed orbit inside the torus. We take the closed orbit as the \( x^3 \) axis and the plane perpendicular to the \( x^3 \) axis as the \((x^1,x^2)\)-plane. Because the generalized momentum \( p \) is rewritten as \( \sqrt{g_v} v' \) in the neighbourhood of the origin, \( p \) can be approximated as \( C x^3 \) in the phase space \((q,p)\) with a scale transformation. The Poincaré section of the velocity field in the cavity is therefore equivalent to a phase space of the Hamiltonian system, and we can apply the resonance theory of the non-autonomous Hamiltonian system of one-degree-of-freedom to the system considered in the present paper.

### 3.2. Resonances in non-autonomous Hamiltonian system

When a streamline spirals on a torus, the motion of the fluid particle on the streamline has two frequencies. One is the frequency \( \omega_0 \) which corresponds to the periodicity of the motion along the \( x^3 \) axis, and the other is the frequency \( \omega_2 \) which corresponds to that of the rotating motion in the \((x^1,x^2)\)-plane. When the ratio \( \omega_0 / \omega_2 \) of two frequencies is an irrational number the motion of the fluid particle covers the surface of a torus. When the ratio is a rational number, periodic points or island structures are observed in the Poincaré sections. Disruption of the invariant torus is brought by nonlinear resonance in a
Hamiltonian system. Normal forms of Hamiltonian systems near a closed trajectory at a resonance are given by Arnold et al. [7].

We study the case where the resonance relation \( n\omega_1 - m\omega_2 = 0 \) is satisfied for two integers \( n \) and \( m \). First, by a canonical linear transformation of variables \((q, p) \mapsto (Q, P)\), the quadratic part of the Hamiltonian can be reduced to the form \( \pm \omega_2(P^2 + Q^2) \). Then we introduce the canonically conjugate (action-angle) variables:

\[
\rho = \frac{1}{2}(P^2 + Q^2),
\]

\[
\psi = \varphi - \frac{m}{n} \frac{2\pi \tau}{L}, \quad \left( \omega_1 = \frac{2\pi}{L} \frac{d\tau}{dt} \right)
\]

where \( L \) is the total length of the closed orbit, and \( \rho \) and \( \varphi \) are the polar coordinates defined by

\[
Q = \sqrt{2\rho \sin \varphi}, \quad P = \sqrt{2\rho \cos \varphi}
\]

and \( \psi \) is in the following the normal form Hamiltonian is given for the resonances with \( m = 1 \), that is, \( \omega_1 : \omega_2 = n : 1 \). In the present paper we study the 3:1 and 2:1 resonances. For the resonance \( \omega_1 : \omega_2 = 3 : 1 \) the normal form of the Hamiltonian system is

\[
H_3 = -\varepsilon \rho + B\rho^{3/2} \cos(3\psi) + \rho^2 F(\rho)
\]

where \( B \) is a constant, \( F(\rho) \) is a polynomial in \( \rho \), and \( \varepsilon = \omega_2 - \omega_1 / n \) is the detuning parameter. We add the higher-order term to the right-hand side of eq. (4) in order to obtain a better comparison between the phase portraits and the Poincaré sections from the numerical simulation.

With the coordinate \((Q, P)\) in the rotating frame, the normal form for the resonance \( \omega_1 : \omega_2 = 2 : 1 \) is

\[
H_2 = -\frac{1}{2} \delta Q_2^2 + \frac{1}{2} AP_2^2 + aQ_4^4
\]

where \( A \) and \( a \) are constants, and \( \delta \) is the detuning parameter.

4. Numerical results

The results of the numerical calculations for the steady flow in a cavity of spanwise aspect ratio \( \Lambda = 1.0 \) with aspect ratio \( \Gamma = 0.4, 0.6, 1.0 \) and 1.4 are presented in this section. In the simulation we use a uniformly spaced mesh: 128-nodes in \( x \)-direction, 52, 79, 128 and 180-nodes in \( y \)-direction for \( \Gamma = 0.4, 0.6, 1.0 \) and 1.4 respectively, and 128-nodes in \( z \)-direction. The velocity field in the cavity is calculated by using the interpolation of velocity vectors and their derivatives at node points which are obtained with the CCD method. With this velocity field we draw a particle trajectory which starts at a certain point in the cavity, and we plot the Poincaré sections where dots are the intersections of the fluid particle path with a certain plane.

4.1. Poincaré sections at \( Re = 100, 200, 300 \) and 400

The Poincaré sections at \( Re = 100, 200, 300 \) and 400 are presented in figs. 2 and 3 for \( \Gamma = 0.4 \) and 1.0 respectively. The plane of the Poincaré section is \( x = 0.75 \) and 0.50 for \( \Gamma = 0.4 \) and 1.0 respectively.
These planes are chosen because they are perpendicular to the streamlines. In the figures the left side is the end-wall ($z=0.0$) and the right side is the center symmetry plane ($z=0.5$). Dots on the plane are the intersections of streamlines which move across the plane in negative-$x$ direction. In the steady cavity flows there are two types of streamlines. One is a localized streamline which moves on the surface of a torus and it gives various structures in the Poincaré section. The other is a streamline of chaotic motion which moves in a spiral between the end-wall and the center symmetry plane and it brings irregularly distributed dots in the Poincaré section.

In fig. 2 for $\Gamma = 0.4$, at $Re = 100$ many points form closed curves. Each closed curve consists of points of intersections by a certain streamline. The closed curves in the Poincaré section imply that a streamline covers an invariant torus. There are nine periodic points around the center of the ovals. Dots in the outer region surrounding the tori show chaotic motion. There are six resonant islands around the ovals at $Re = 200$ and five resonant islands outside the ovals at $Re = 300$. At $Re = 400$ there are four resonant islands outside the deformed closed curves. The area of invariant tori becomes smaller as $Re$ increases.

Fig. 2. Poincaré sections ($x=0.75$, $0 \leq y \leq 0.25$, $0 \leq z \leq 0.5$) for $\Gamma=0.4$. (a) $Re=100$; (b) $Re=200$; (c) $Re=300$; (d) $Re=400$
In the case of $\Gamma = 0.6$ and 1.4, the area where we can find the structures becomes smaller when $Re$ increases, but the structure exists at $Re = 400$ as in the case of $\Gamma = 0.4$.

In fig. 3 for the cubic cavity ($\Gamma =1.0$), the Poincaré section has the central fixed point which corresponds to the closed orbit inside the torus, five periodic points, six resonant islands and invariant curves at $Re = 100$, and these are surrounded by chaotic region. At $Re = 200$ there are four resonant islands around the ovals whose shapes have changed slightly. As $Re$ increases the area of closed curves becomes smaller, and at $Re = 400$ closed curves disappear and all points distribute irregularly in the cubic cavity. These Poincaré sections are compared with those by Ishii et al. [1], and we find that we have almost the same Poincaré sections as theirs though we use different numerical methods and grid system from those adopted by them.

![Fig. 3. Poincaré sections($x=0.50, 0 \leq y \leq 0.8, 0 \leq z \leq 0.5$) for $\Gamma=1.0$. (a) $Re=100$; (b) $Re=200$; (c) $Re=300$; (d) $Re=400$](image)

### 4.2. 3:1 Resonances

The phase portraits of $H_j$ are presented in fig. 4 for the detuning parameter $\varepsilon = -0.4, 0.0, +1.0$. The coefficient and the polynomial in eq. (4) are $B=1$ and $F(\rho) = 1$. With $\varepsilon$ changing from negative to positive number, triangles in the central region rotate by $\pi/3$ because near the origin the last term can be negligible compared to the first two terms of the right hand side of eq. (4).
In figs. 5-8 we present the Poincaré sections for $\Gamma=0.4, 0.6, 1.0$ and 1.4 respectively. The plane of the Poincaré section is $x=0.75, 0.62, 0.50$ and 0.55 for $\Gamma=0.4, 0.6, 1.0$ and 1.4 respectively. In each figure there are three Poincaré sections for three Reynolds numbers. The Poincaré section (a) corresponds to the phase portrait before the 3:1 resonance occurs, (b) to that near the resonance and (c) to that after the resonance. In the numerical simulation it is extremely difficult to find the exact Reynolds number at which the resonance occurs. But in the Poincaré sections (b) in figs. 5-8, dots near the fixed point of the closed orbit do not form closed curves, which is similar to the phase portrait in fig. 4(b).

Fig. 4. Phase portraits near the 3:1 resonance. (a) $\varepsilon = 0.4$; (b) $\varepsilon = 0.0$; (c) $\varepsilon = +1.0$ (From figure 6 of ref. 2)

Fig. 5. Poincaré sections ($x=0.75, 0.05 \leq y \leq 0.15, 0.05 \leq z \leq 0.25$) for $\Gamma=0.4$ near the 3:1 resonance. (a) $Re=400$; (b) $Re=410$; (c) $Re=420$
We can recognize the 3:1 resonance in four aspect ratios. Table 1 gives the Reynolds number range where the 3:1 resonance occurs.
Table 1. Reynolds number range of 3:1 resonance

<table>
<thead>
<tr>
<th>$\Gamma$</th>
<th>$Re$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.4</td>
<td>400-420</td>
</tr>
<tr>
<td>0.6</td>
<td>235-250</td>
</tr>
<tr>
<td>1.0</td>
<td>219-225</td>
</tr>
<tr>
<td>1.4</td>
<td>240-250</td>
</tr>
</tbody>
</table>

4.3. 2:1 Resonances

The phase portraits of $H_z$ are presented in fig. 9 for the detuning parameter $\delta = -0.1, +0.2$. The coefficients in eq. (5) are $A=1$ and $a=1/4$.

Fig. 9. Phase portraits near the 2:1 resonance. (a) $\delta = -0.1$; (b) $\delta = +0.2$ (From figure 8 of ref. 2)

In figs. 10-13 we present the Poincaré sections for $\Gamma =0.4, 0.6, 1.0$ and 1.4 respectively. In each figure there are two Poincaré sections for two Reynolds numbers. The Poincaré section (a) corresponds to the phase portrait before the 2:1 resonance occurs and (b) to that after the resonance. In the Poincaré sections (b) in figs. 10-13, dots near the point of intersection of the $\infty$ shape do not form ovals.

Fig. 10. Poincaré sections($x=0.75, 0.095 \leq y \leq 0.115, 0.09 \leq z \leq 0.13$) for $\Gamma =0.4$ near the 2:1 resonance. (a) $Re=500$; (b) $Re=515$
Fig. 11. Poincaré sections (x=0.62, 0.12 ≤ y ≤ 0.18 , 0.09 ≤ z ≤ 0.19 ) for Γ=0.6 near the 2:1 resonance. (a) Re=385; (b) Re=400

Fig. 12. Poincaré sections (x=0.50, 0.26 ≤ y ≤ 0.38 , 0.14 ≤ z ≤ 0.26 ) for Γ=1.0 near the 2:1 resonance. (a) Re=320; (b) Re=325

Fig. 13. Poincaré sections (x=0.55, 0.59 ≤ y ≤ 0.7 , 0.14 ≤ z ≤ 0.26 ) for Γ=1.4 near the 2:1 resonance. (a) Re=335; (b) Re=350
The 2:1 resonance can be recognized in all aspect ratios. The Reynolds number range for the 2:1 resonance is given in table 2. Tables 1 and 2 shows that for both 3:1 and 2:1 resonances, the Reynolds number range is the smallest in the cubic cavity ($\Gamma=1$) and the largest in that of $\Gamma=0.4$. There is a marked change between the cases of $\Gamma=0.4$ and $\Gamma=0.6$.

Table 2. Reynolds number range of 2:1 resonance

<table>
<thead>
<tr>
<th>$\Gamma$</th>
<th>$Re$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.4</td>
<td>500-515</td>
</tr>
<tr>
<td>0.6</td>
<td>385-400</td>
</tr>
<tr>
<td>1.0</td>
<td>320-325</td>
</tr>
<tr>
<td>1.4</td>
<td>335-350</td>
</tr>
</tbody>
</table>

5. Concluding remarks

We studied the streamline structures of the three-dimensional steady viscous incompressible flow in rectangular cavities with span aspect ratio 1.0 and aspect ratios 0.4, 0.6, 1.0 and 1.4 for $Re$ ranging from 100 to 400. The solenoidal velocity vector field of the steady incompressible fluid is shown to be equivalent to a non-autonomous Hamiltonian system of one-degree-of-freedom. The Poincaré sections show the various structures of invariant curves, resonant islands and chaotic distribution. For the cubic cavity the Poincaré sections of the present study and those by Ishii et al. [1] have almost the same structures though different numerical methods and grid systems are adopted in both studies. This shows that when computational errors are small enough, the torus structures are preserved in the numerically obtained Poincaré sections, which is interpreted as the stability of KAM torus. And we can apply the theory of Hamiltonian system to the system of streamlines obtained by numerical methods. We found that the phase portraits of the normal form Hamiltonian and the Poincaré sections of streamlines show striking similarities for the 3:1 and 2:1 resonances. The present study and the previous studies show that existence of the invariant tori and disruption by the resonances are generally observed in steady cavity flows in cavities with different aspect ratios and spanwise aspect ratios.

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