On total labelings of graphs with prescribed weights

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Abstract

Let $G = (V, E)$ be a finite, simple and undirected graph. The edge-magic total or vertex-magic total labeling of $G$ is a bijection $f$ from $V(G) \cup E(G)$ onto the set of consecutive integers $\{1, 2, \ldots, |V(G)| + |E(G)|\}$, such that all the edge weights or vertex weights are equal to a constant, respectively. When all the edge weights or vertex weights are different then the labeling is called edge-antimagic or vertex-antimagic total, respectively.

In this paper we provide some classes of graphs that are simultaneously super edge-magic total and super vertex-antimagic total, that is, graphs admitting labeling that has both properties at the same time. We show several results for fans, sun graphs, caterpillars and prisms.

Keywords: Super edge-magic total labeling; Super vertex-antimagic total labeling; Total labeling

1. Introduction

We consider $G = (V, E)$ finite undirected graphs without loops and multiple edges with vertex set $V(G)$ and edge set $E(G)$, where $n = |V(G)|, m = |E(G)|$. A labeling of a graph $G$ is any mapping that maps a certain set of graph elements to a certain set of positive integers. If the domain is the vertex (or edge) set, respectively, the labeling is called vertex (or edge) labeling, respectively. If the domain is both vertices and edges then the labeling is called a total labeling. The edge weight of an edge under the total labeling is the sum of the edge label and the labels of its end vertices. The vertex weight of a vertex under the total labeling is defined as sum of the vertex label itself and the labels of its incident edges.

A labeling is called edge-magic total (vertex-magic total) if the edge weights (vertex weights) are all the same. If the edge weights (vertex weights) are pairwise distinct then the total labeling is called edge-antimagic total (vertex-antimagic total). A graph that admits edge-magic total (edge-antimagic total) labeling or vertex-magic total (vertex-antimagic total) labeling is called an edge-magic total (edge-antimagic total) graph or vertex-magic total (vertex-antimagic total) graph, respectively.

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The subject of edge-magic total graph has its origin in the work of Kotzig and Rosa [1]. Vertex-magic total graphs are introduced by MacDougall, Miller, Slamin and Wallis in [2], see also [3,4]. The notation of edge-antimagic total labeling was introduced by Simanjuntak, Bertault and Miller in [5] as a natural extension of magic valuation defined by Kotzig and Rosa in [1]. The vertex-antimagic total labeling of graphs was introduced in [6], see also [7]. If moreover the vertices are labeled with the smallest possible labels then, as is customary, the labeling is referred to as super. The concept of super-magic graphs was introduced by Stewart [8]. For further information on these types of labelings, one can see [9,7,10,11].

In [12] Bača, Miller, Phanalasy, Ryan, Semaničová-Feňovčíková and Sillasen proved that there exist some classes of stars, paths and cycle graphs which are simultaneously edge-magic total and vertex-antimagic total or simultaneously vertex-magic total and edge-antimagic total.

In this paper we will prove that some classes of fans, suns, caterpillars and prism graphs are simultaneously super edge-magic total and super vertex-antimagic total. For every of these graph we describe a total labeling that has both properties at the same time.

2. Fans, sun graphs, caterpillars and prisms

A fan $F_n$, $n \geq 2$, is a graph obtained by joining all vertices of path $P_n$ on $n$ vertices to another vertex, called center. The fan graph $F_n$ contains $n+1$ vertices and $2n-1$ edges.

**Theorem 1.** The fan $F_n$ is simultaneously super edge-magic total and super vertex-antimagic total if and only if $3 \leq n \leq 6$.

**Proof.** In [13], see also [7], is proved that the fan $F_n$ has a super edge-magic total labeling if and only if $2 \leq n \leq 6$. The fan $F_2$ is isomorphic to the cycle $C_3$. In [12] is showed that the cycle $C_3$ is not simultaneously super edge-magic total and super vertex-antimagic total.

For $3 \leq n \leq 6$ are the required labelings depicted in Figs. 1 through 4.

Fig. 1 depicts a simultaneously super edge-magic total and super vertex-antimagic total labeling for $F_3$ with edge weights equal 12 and vertex weights 16, 17, 22, 25.

A super edge-magic total labeling of the fan $F_4$ with edge weights 15 is illustrated in Fig. 2. This total labeling is also super vertex-antimagic with vertex weights 18, 24, 27, 33, 39.

Fig. 3 shows a simultaneously super edge-magic total and super vertex-antimagic total labeling of the fan $F_5$ with edge weights 18 and with vertex weights 21, 29, 31, 38, 43, 57.

A total labeling of the fan $F_6$ with constant edge weights 21 is given in Fig. 4. This total labeling is also super vertex-antimagic with vertex weights 25, 34, 35, 39, 47, 52, 82. ■
In the following lemma we prove that number of pendant edges incident with a given vertex in a simultaneously super edge-magic total and super vertex-antimagic total graph is at most 1.

**Lemma 1.** Let $G$ be a simultaneously super edge-magic total and super vertex-antimagic total graph. Then every vertex of $G$ can be incident with at most one pendant edge.

**Proof.** Let $G$ be a simultaneously super edge-magic total and super vertex-antimagic total graph and let $f$ be the corresponding labeling of $G$. Let us assume that there is a vertex $v \in V(G)$ that is adjacent to more than one vertex of degree 1, say $u_1, u_2$ are two of them. As $f$ is a super edge-magic total labeling then the weights of edges $vu_1$ and $vu_2$ are the same, i.e.,

$$wt f(vu_1) = wt f(vu_2)$$

$$f(v) + f(vu_1) + f(u_1) = f(v) + f(vu_2) + f(u_2)$$

and thus

$$f(vu_1) + f(u_1) = f(vu_2) + f(u_2)$$

$$wt f(u_1) = wt f(u_2)$$

what is a contradiction to the fact that $f$ is also vertex-antimagic. ■

Now we will investigate graphs obtained by attaching $n$ pendant vertices to every vertex of a given graph $G$. These graphs can be alternatively obtained as a corona product of a graph $G$ and a totally disconnected graph on $m$ vertices, denoted by $G \odot K_m$. A cycle of order $n$ with $m$ pendant edges attached at each vertex, i.e., $C_n \odot mK_1$, is called an $m$-crown with cycle of order $n$. A 1-crown, or only a crown or a sun graph, is a cycle with exactly one pendant edge attached at each vertex of the cycle. We will use notation $Sun(n)$ for this graph. We denote the elements of the sun graph $Sun(n)$ such that its vertex set is

$$V(Sun(n)) = \{x_i, y_i : 1 \leq i \leq n\}$$

and its edge set is

$$E(Sun(n)) = \{x_i, x_{i+1}, x_i y_i : 1 \leq i \leq n\},$$

where the indices are taken modulo $n$. Thus the sun $Sun(n)$ has $2n$ vertices and $2n$ edges, see Fig. 5.

According to Lemma 1 we immediately have the following result.
Observation 1. If the \( m \)-crown with cycle of order \( n \), i.e., \( C_n \odot mK_1 \), is a simultaneously super edge-magic total and super vertex-antimagic total graph then \( m = 1 \).

Next we will deal with the existence of total labeling of the sun graph \( \text{Sun}(n) \) which has simultaneously super edge-magic properties and super vertex-antimagic properties.

**Theorem 2.** For every odd positive integer \( n \), \( n \geq 3 \), the sun graph \( \text{Sun}(n) \) is simultaneously super edge-magic total and super vertex-antimagic total.

**Proof.** Define a total labeling \( f : V(\text{Sun}(n)) \cup E(\text{Sun}(n)) \rightarrow \{1, 2, \ldots, 4n\} \) in the following way:

\[
\begin{align*}
  f(x_i) & = \begin{cases} 
    \frac{i+1}{2}, & \text{for } i \equiv 1 \pmod{2}, \ 1 \leq i \leq n \\
    \frac{n+i+1}{2}, & \text{for } i \equiv 0 \pmod{2}, \ 2 \leq i \leq n-1 
  \end{cases} \\
  f(y_i) & = 2n - i, \quad \text{for } 1 \leq i \leq n-1 \\
  f(y_n) & = 2n, \\
  f(xix_{i+1}) & = 4n - i, \quad \text{for } 1 \leq i \leq n-1 \\
  f(xnx_1) & = 4n, \\
  f(xiy_i) & = \begin{cases} 
    \frac{5n+i+2}{2}, & \text{for } i \equiv 1 \pmod{2}, \ 1 \leq i \leq n-2 \\
    \frac{4n+i+2}{2}, & \text{for } i \equiv 0 \pmod{2}, \ 2 \leq i \leq n-1 
  \end{cases} \\
  f(xny_n) & = 2n + 1.
\end{align*}
\]

It is not difficult to see that the labeling \( f \) is a total labeling and moreover the vertex labels are the smallest possible labels. For edge weights we have

\[
\begin{align*}
  wt_f(xix_{i+1}) & = f(x_i) + f(x_{i}x_{i+1}) + f(x_{i+1}) \\
  & = \begin{cases} 
    \frac{n+i+1}{2} + (4n - i) + \frac{i+2}{2} = \frac{9n+3}{2}, & \text{for } i \equiv 0 \pmod{2}, \ 2 \leq i \leq n-1 \\
    \left(\frac{i+1}{2}\right) + (4n-i) + \frac{n+i+2}{2} = \frac{9n+3}{2}, & \text{for } i \equiv 1 \pmod{2}, \ 1 \leq i \leq n-2 
  \end{cases} \\
  wt_f(xnx_1) & = f(x_n) + f(x_nx_1) + f(x_1) = \frac{n+1}{2} + 4n + 1 = \frac{9n+3}{2}, \\
  wt_f(xiy_i) & = f(x_i) + f(x_iy_i) + f(y_i) \\
  & = \begin{cases} 
    \frac{n+i+1}{2} + \frac{4n+i+2}{2} + (2n-i) = \frac{9n+3}{2}, & \text{for } i \equiv 0 \pmod{2}, \ 2 \leq i \leq n-1 \\
    \frac{i+1}{2} + \frac{5n+i+2}{2} + (2n-i) = \frac{9n+3}{2}, & \text{for } i \equiv 1 \pmod{2}, \ 1 \leq i \leq n-2 
  \end{cases} \\
  wt_f(xny_n) & = f(x_n) + f(x_ny_n) + f(y_n) = \frac{n+1}{2} + (2n+1) + 2n = \frac{9n+3}{2}.
\end{align*}
\]

Thus the total labeling \( f \) is a super edge-magic labeling of the sun graph \( \text{Sun}(n) \) for every odd \( n \), \( n \geq 3 \). Let us consider the vertex weights under the total labeling \( f \).
\[ wt_f(x_i) = f(x_i-1x_i) + f(x_i) + f(x_iy_i) + f(x_iy_{i+1}) \]
\[ = \begin{cases} 
(4n - (i - 1)) + \frac{n+i+1}{2} + \frac{4n+i+2}{2} + (4n - i) = \frac{21n+5}{2} - i, & \text{for } i \equiv 0 \pmod{2}, \ 2 \leq i \leq n - 1 \\
(4n - (i - 1)) + \frac{i+1}{2} + \frac{5n+i+2}{2} + (4n - i) = \frac{21n+5}{2} - i, & \text{for } i \equiv 1 \pmod{2}, \ 1 \leq i \leq n - 2 
\end{cases} \]
\[ wt_f(x_n) = f(x_{n-1}x_n) + f(x_n) + f(x_ny_n) + f(x_nx_1) = (4n - (n - 1)) + \frac{n+1}{2} + (2n + 1) + 4n = \frac{19n+5}{2}, \]
\[ wt_f(y_i) = f(y_i) + f(x_iy_i) \]
\[ = \begin{cases} 
(2n - i) + \frac{4n+i+2}{2} = 4n + 1 - \frac{i}{2}, & \text{for } i \equiv 0 \pmod{2}, \ 2 \leq i \leq n - 1 \\
(2n - i) + \frac{5n+i+2}{2} = \frac{9n+2-i}{2}, & \text{for } i \equiv 1 \pmod{2}, \ 1 \leq i \leq n - 2 
\end{cases} \]
\[ wt_f(y_n) = f(y_n) + f(x_ny_n) = 2n + (2n + 1) = 4n + 1. \]

As for \( 1 \leq i \leq n \)
\[ wt_f(x_i) = \frac{21n+5}{2} - i, \] (1)
we have that the weights of vertices \( x_i \) form an arithmetic progression with difference \( d = 1 \). Also the weights of vertices \( y_i \) constitute the set of consecutive integers
\[ \{ wt(y_i) : 1 \leq i \leq n \} = \left\{ \frac{7n+3}{2}, \frac{7n+5}{2}, \ldots, \frac{9n+1}{2} \right\}. \]

We can also easily see that \( wt_f(x_i) \neq wt_f(y_i) \) for all \( 1 \leq i \leq n \). This shows that \( f \) is vertex-antimagic as well. It means that for every odd \( n, n \geq 3 \), the sun graph \( Sun(n) \) is simultaneously super edge-magic total and super vertex-antimagic total.

If we remove one edge on the cycle of the sun graph \( Sun(n) \) then the resulting graph is a tree and it is a special class of caterpillar. We denote this tree as \( Cat(n) \).

**Theorem 3.** For every odd positive integer \( n, n \geq 3 \), the tree \( Cat(n) \) is simultaneously super edge-magic total and super vertex-antimagic total.

**Proof.** Let us consider the tree \( Cat(n) \) as the sun graph \( Sun(n) \) with the edge \( x_1x_n \) deleted. By the labeling \( f \) defined in the proof of **Theorem 2**, we define a labeling \( g \) of \( Cat(n) \) as follows:
\[ g(x) = f(x) \]
for every \( x \in V(Sun(n)) \cup E(Sun(n)) - \{x_1x_n\} \).

Since the label of removed edge \( x_1x_n \) in \( Sun(n) \) under the total labeling \( f \) is the largest one then the labeling \( g \) is a total labeling of the tree \( Cat(n) \) with labels from 1 up to \( 4n - 1 \). It is easy to see that the total labeling \( g \) preserves super edge-magic properties.

For proving that the total labeling \( g \) preserves also super vertex-antimagic properties it is sufficient to show that the vertex weights of vertices \( x_1 \) and \( x_n \) are distinct from other vertices of the tree \( Cat(n) \). In fact, using (1), for \( 2 \leq i \leq n - 1 \) we get
\[ wt_g(x_1) = wt_f(x_1) - f(x_nx_1) = \left( \frac{21n+5}{2} - 1 \right) - 4n = \frac{13n+3}{2} < wt_f(x_i) = \frac{21n+5}{2} - i \]
and for \( 1 \leq i \leq n \)
\[ wt_g(x_1) > wt_f(y_i). \]
Also for \( 2 \leq i \leq n - 1 \)
\[ wt_g(x_n) = wt_f(x_n) - f(x_nx_1) = \left( \frac{21n+5}{2} - n \right) - 4n = \frac{11n+5}{2} < wt_f(x_i) = \frac{21n+5}{2} - i \]
and for $1 \leq i \leq n$

$$w_{tg}(x_i) > w_{tf}(y_i).$$

This proves that the tree $Cat(n)$ is simultaneously super edge-magic total and super vertex-antimagic total for every odd $n$, $n \geq 3$.

The prism $D_n$, $n \geq 3$, is a cubic graph which can be represented as a Cartesian product $P_2 \square C_n$ of a path on two vertices with a cycle on $n$ vertices. Let

$$V(D_n) = \{x_i, y_i : 1 \leq i \leq n\}$$

be the vertex set and

$$E(D_n) = \{x_i x_{i+1}, y_i y_{i+1}, x_i y_i : 1 \leq i \leq n\},$$

where the indices are taken modulo $n$, be the edge set of the prism $D_n$, see Fig. 6. The prism $D_n$ has $2n$ vertices and $3n$ edges.

Next theorem shows that there exists a total labeling of the prism $D_n$ which has super edge-magic and also super vertex-antimagic properties.

**Theorem 4.** For every odd positive integer $n$, $n \geq 3$ and $n \not\equiv 0 \pmod{5}$ the prism $D_n$ is simultaneously super edge-magic total and super vertex-antimagic total.

**Proof.** We define a total labeling $h : V(D_n) \cup E(D_n) \to \{1, 2, \ldots, 5n\}$ as follows:

$$h(x_i) = \begin{cases} 
\frac{i+1}{2}, & \text{for } i \equiv 1 \pmod{2}, \ 1 \leq i \leq n \\
\frac{n+i+1}{2}, & \text{for } i \equiv 0 \pmod{2}, \ 2 \leq i \leq n-1
\end{cases}$$

$$h(y_i) = \begin{cases} 
\frac{3n+i}{2}, & \text{for } i \equiv 1 \pmod{2}, \ 1 \leq i \leq n \\
\frac{2n+i}{2}, & \text{for } i \equiv 0 \pmod{2}, \ 2 \leq i \leq n-1
\end{cases}$$

$$h(x_i x_{i+1}) = 5n - i, \quad \text{for } 1 \leq i \leq n-1$$

$$h(x_n x_1) = 5n,$$

$$h(y_i y_{i+1}) = 3n + 1 - i, \quad \text{for } 1 \leq i \leq n-1$$

$$h(y_n y_1) = 2n + 1,$$

$$h(x_i y_i) = 4n + 1 - i, \quad \text{for } 1 \leq i \leq n.$$

It is not difficult to check that $h$ is a bijection and the vertices of prism $D_n$ under the total labeling $h$ receive labels from the set $\{1, 2, \ldots, 2n\}$. So, the total labeling $h$ is super. For edge weights under the labeling $h$ we have
Thus the labeling $h$ is a super edge-magic total labeling.

Let us consider the vertex weights under the labeling $h$:

\[
wt_h(x_n) = h(x_{n-1}x_n) + h(x_n) + h(x_{n+1})x_n + h(x_{n+1})
\]

\[
= (5n - (n - 1)) + \frac{n+1}{2} + (4n - n + 1) + 5n = \frac{25n+5}{2},
\]

\[
wt_h(x_i) = h(x_{i-1}x_i) + h(x_i) + h(x_{i+1})x_i + h(x_{i+1})
\]

\[
= \begin{cases} 
(5n - (i - 1)) + \frac{n+1}{2} + (4n - i + 1) + (5n - i) = \frac{29n+5-5i}{2}, & \text{for } i \equiv 0 \pmod{2}, \ 2 \leq i \leq n - 1 \\
(5n - (i - 1)) + \frac{i+1}{2} + (4n - i + 1) + (5n - i) = \frac{28n+5-5i}{2}, & \text{for } i \equiv 1 \pmod{2}, \ 1 \leq i \leq n - 2 
\end{cases}
\]

\[
wt_h(y_1) = h(y_{1}y_1) + h(y_1) + h(x_{1}y_1) + h(y_1y_2) = (2n + 1) + \frac{3n+1}{2} + 4n + 3n = \frac{21n+3}{2},
\]

\[
wt_h(y_i) = h(y_{i-1}y_i) + h(y_i) + h(x_{i}y_i) + h(y_{i}y_{i+1})
\]

\[
= \begin{cases} 
(3n + 1 - (i - 1)) + \frac{2n+i}{2} + (4n + 1 - i) + (3n + 1 - i) = \frac{22n+8-5i}{2}, & \text{for } i \equiv 0 \pmod{2}, \ 2 \leq i \leq n - 1 \\
(3n + 1 - (i - 1)) + \frac{3n+i}{2} + (4n + 1 - i) + (3n + 1 - i) = \frac{23n+8-5i}{2}, & \text{for } i \equiv 1 \pmod{2}, \ 3 \leq i \leq n.
\end{cases}
\]

Since $n \not\equiv 0 \pmod{5}$ then $wt_h(x_i) = \frac{29n+5-5i}{2}$ for $i$ even, $2 \leq i \leq n - 1$, is distinct to $wt_h(x_i) = \frac{28n+5-5i}{2}$ for $i$ odd, $1 \leq i \leq n - 2$. Moreover, $wt_h(x_n) = \frac{25n+5}{2} \neq wt_h(x_i)$ for $1 \leq i \leq n - 1$.

By the same reason $wt_h(y_i) = \frac{22n+8-5i}{2}$ for $i$ even, $2 \leq i \leq n - 1$, is distinct to $wt_h(y_i) = \frac{23n+8-5i}{2}$ for $i$ odd, $3 \leq i \leq n$. It is easy to see that $wt_h(y_1) = \frac{21n+3}{2} \neq wt_h(y_i)$ for $2 \leq i \leq n$.

Hence $h$ is a super vertex-antimagic total labeling.

Therefore the prism $D_n$ is simultaneously super edge-magic total and super vertex-antimagic total for every odd $n$, $n \geq 3$, $n \not\equiv 0 \pmod{5}$. This concludes the proof. ■

3. Conclusion

In the paper we dealt with the problem of finding total labelings of some classes of graphs that are simultaneously super vertex-magic and super vertex-antimagic. We showed the existence of such labelings for certain classes of graphs, namely fans, sun graphs, one class of caterpillars and prisms. In [12] authors ask not only for finding classes of graphs that are simultaneously vertex-magic and edge-antimagic but also graphs that are simultaneously vertex-antimagic and edge-magic.

For the fan $F_n$ we proved that it is simultaneously super edge-magic total and super vertex-antimagic total if and only if $3 \leq n \leq 6$. In [14] it is proved that the fan $F_n$ is vertex-magic total if and only if $2 \leq n \leq 10$. Note that for
$n = 3, 5$ and $n = 6$ the described labelings induce also distinct edge weights. In Fig. 7 we present a simultaneously vertex-magic total and edge-antimagic total labeling of the fan $F_4$.

Moreover, in [15], it is showed that the fan $F_n$ is super vertex-magic total if and only if $n = 2$. However, in this case the considered edge weights are also the same, which means that no fan is simultaneously super vertex-magic total and super edge-antimagic total. So we pose the following open problem.

**Open Problem 1.** For the fan $F_n$, $7 \leq n \leq 10$, determine if there is a simultaneously vertex-magic total and edge-antimagic total labeling.

For sun graph we proved that for every $n$ odd, $n \geq 3$, the sun graph $Sun(n)$ is simultaneously super edge-magic total and super vertex-antimagic total. Another interesting question is to settle the existence question for even orders. We found a simultaneously super edge-magic total and super vertex-antimagic total labeling for sun graph $S_4$, see Fig. 8. Although we were unable to find such labelings for bigger even numbers, we still believe that a computer aided search would help to find them.

**Open Problem 2.** For the sun graph $Sun(n)$, $n$ even, $n \geq 6$, determine if there exists a total labeling which is simultaneously super edge-magic and super vertex-antimagic.

It is a simple observation that the minimum degree of a super vertex-magic total graph must be at least 2. Thus, immediately from this we have that no $m$-crown with cycle of order $n$, i.e., $C_n \circ mK_1$, and thus no $Sun(n)$, is simultaneously super vertex-magic total and super edge-antimagic total. On the other hand, it is evident that no simultaneously vertex-magic total and edge-antimagic total graph can contain $K_{1,2}$ as an induced subgraph. This means that if an $m$-crown with the cycle of order $n$, i.e., $C_n \circ mK_1$, is a simultaneously vertex-magic total and edge-antimagic total graph, then $m = 1$. Thus we state the following for further investigation.

**Open Problem 3.** For the sun graph $Sun(n)$, $n \geq 3$, determine if there exists a total labeling which is simultaneously vertex-magic total and edge-antimagic total.

For prism we showed that for every odd $n$, $n \geq 3$ and $n \not\equiv 0 \pmod{5}$, the prism $D_n$ is simultaneously super edge-magic total and super vertex-antimagic total. For prism $D_n$, when $n \equiv 0 \pmod{5}$ and $n$ is odd or $n$ is even, we did not find any total labeling with required properties. Therefore we propose the following open problem.

**Open Problem 4.** For the prism $D_n$, $n \equiv 5 \pmod{10}$ or $n$ even, find a total labeling which is simultaneously super edge-magic and super vertex-antimagic.
For further investigation we also state the following open problem.

**Open Problem 5.** For the prism \(D_n\), \(n \geq 3\), determine if there exists a total labeling which is simultaneously super vertex-magic and super edge-antimagic.

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