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Five-dimensional PPN formalism and experimental test of Kaluza–Klein theory

Peng Xu, Yongge Ma *

Department of Physics, Beijing Normal University, Beijing 100875, China

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Abstract

The parametrized post-Newtonian formalism for 5-dimensional metric theories with a compact extra dimension is developed. The relation of the 5-dimensional and 4-dimensional formulations is then analyzed, in order to compare the higher dimensional theories of gravity with experiments. It turns out that the value of post-Newtonian parameter γ in the reduced 5-dimensional Kaluza–Klein theory is two times smaller than that in 4-dimensional general relativity. The departure is due to the existence of an extra dimension in the Kaluza–Klein theory. Thus the confrontation between the reduced 4-dimensional formalism and Solar system experiments raises a severe challenge to the classical Kaluza–Klein theory.

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As a candidate of fundamental theory, Kaluza–Klein (KK) theory unifies gravity with electromagnetic field (or Yang–Mills field) by certain higher dimensional general relativity (GR) [1,2]. Since the original 5-dimensional (5D) KK theory was proposed by Kaluza [3] and Klein [4], considerable works have been done along this line [5–7]. The fantastic idea that our spacetime has extra dimension, promotes various higher dimensional theories, including the well-known string theory [8]. Besides the potential function to unify the fundamental interactions, higher dimensional gravity theories are also shown to be effective in accounting for the dark constituent of the universe (see e.g. [9]). Given the fascinating virtues of extra dimensions, it becomes very desirable to confront higher dimensional theories of gravity with experiments. Works on this subject can be traced back to 1980s [10,11], while no agreement has been obtained in the literature. Different classes of solutions to higher dimensional GR are designed to represent Solar system (for soliton-like solution see [12–14], for Schwarzschild-like solu-

tion see [15,16]). However, whether the available experimental data permit higher dimensional theories gets quite different answers in different approaches. These ambiguities are caused by the freedom in choosing higher dimensional solutions which are supposed to represent the Solar system in 4 dimensions. On the other hand, in 4-dimensional (4D) case, a general framework, called canonical parameterized post-Newtonian (PPN) formalism, was established by Nordtvedt, Will et al. [17–19] in 1970s as a basic tool to connect gravitational theories with the Solar system experiments. In PPN formalism, the perturbative metric of a gravitational theory, which is generated by the matter distribution of the Solar system, is expanded by orders in terms of linear combinations of post-Newtonian potentials. The differences among various metric theories are represented by the coefficients (the PPN parameters) of these post-Newtonian potentials. Because of its high accuracy and well-defined procedure, PPN formalism has attained great achievements in testing 4D metric theories by Solar system experiments [20,21]. Thus, some crucial issues arise naturally. Is there a higher dimensional PPN formalism? If there is, what is the relation between the higher dimensional formalism and the 4D one? More crucially, can one test higher dimensional theories by the accurate Solar system experiments without the ambiguities motioned above?

* Corresponding author.

E-mail addresses: mooonbird@gmail.com (P. Xu), mayg@bnu.edu.cn (Y. Ma).

The purpose of this Letter is to address these issues first in terms of 5D gravity theories with a compact extra dimension. A 5D PPN formalism will be developed. Its relation with the 4D formalism will be set up. As one will see without any ambiguities, the concrete analysis reveals a severe contradiction between KK theory and the Solar system experiments.

The 5D gravitational theories which we consider are defined on some 5-manifold with topology $\mathbf{M}^4 \times \mathbf{S}^1$, where \mathbf{S}^1 is a compact extra dimension of radii R . Both gravity and matter fields are assumed to be distributed over the 5-manifold. Similar to 4D PPN formalism, the post-Newtonian coordinates system is chosen as certain asymptotic (in 4D sense) flat system $\{t, x^m\}$, $m = 1, 2, 3, 5$, where x^5 is the coordinate of extra space. Since the compactification radii R is sufficiently small, a killing vector field ξ^μ arises naturally along the extra dimension in the low energy regime [2]. It is convenient to take an adapted coordinate system such that its fifth coordinate basis vector $(\frac{\partial}{\partial x^5})^\mu$ coincides with ξ^μ . The 5-metric reads $\tilde{g}_{\mu\nu} = \tilde{\eta}_{\mu\nu} + \tilde{h}_{\mu\nu}$ with signature $(-, +, +, +, +)$, where $\tilde{h}_{\mu\nu}$ is the perturbative metric generated by the matter distribution, e.g., the Solar system. The gauge is chosen so that the spatial part of $\tilde{h}_{\mu\nu}$ is diagonal. As in canonical PPN formalism, we will expand $\tilde{h}_{\mu\nu}$ by orders in terms of linear combinations of our generalized post-Newtonian potentials which are functionals of matter variables. We assume that the matter composing the Solar system can be idealized as a perfect fluid. The matter variables which we considered for the 5D perfect fluid in Solar system include: 5D rest mass density $\tilde{\rho}$, 5D pressure \tilde{p} for the matter flow, the ratio $\tilde{\Pi}$ of 5D specific energy (including compressional energy, radiation, thermal energy, etc.) density to 5D rest mass density, and the coordinate velocity \tilde{v}^m of material particles or matter flow in post-Newtonian frame. The first three 5D matter variables give the corresponding effective 4D matter variables as

$$\begin{aligned} \int \sqrt{\tilde{g}_{55}} \tilde{\rho} dx^5 &= \rho, & \int \sqrt{\tilde{g}_{55}} \tilde{p} dx^5 &= p, \\ \int \sqrt{\tilde{g}_{55}} \tilde{\rho} \tilde{\Pi} dx^5 &= \rho \Pi. \end{aligned} \quad (1)$$

The general 5D post-Newtonian potentials which we used for KK-like theories are \tilde{U} , $\tilde{\Phi}_1$, $\tilde{\Phi}_2$, $\tilde{\Phi}_3$, $\tilde{\Phi}_4$, and \tilde{V}_m , which satisfy respectively the 5D Poisson equations with respect to the flat spatial background as:

$$\begin{aligned} \nabla^2 \tilde{U} &= -\frac{16}{3} \pi \tilde{G} \tilde{\rho}, & \nabla^2 \tilde{\Phi}_1 &= -\frac{16}{3} \pi \tilde{G} \tilde{\rho} v^2, \\ \nabla^2 \tilde{\Phi}_2 &= -\frac{16}{3} \pi \tilde{G} \tilde{\rho} \tilde{U}, & \nabla^2 \tilde{\Phi}_3 &= -\frac{16}{3} \pi \tilde{G} \tilde{\rho} \tilde{\Pi}, \\ \nabla^2 \tilde{\Phi}_4 &= -\frac{16}{3} \pi \tilde{G} \tilde{p}, & \nabla^2 \tilde{V}_m &= -\frac{16}{3} \pi \tilde{G} \tilde{\rho} \tilde{v}_m, \end{aligned}$$

where \tilde{G} denotes the 5D gravitational constant and we use the unit where the velocity of light $c = 1$. Note that one may add more potentials in this framework in order to consider more complicated 5D theories. Note also that the upper bound of the compactification radii R is constrained by the tests of gravitational inverse-square law to be about 10^{-4} m [22], which is sufficiently small compared with the characteristic length 10^{12} m

of Solar system. With this condition we can estimate the order relations of matter variables and potentials. Since $|\tilde{v}| \ll 1$, we denote its order of smallness as $\tilde{v} \sim \mathcal{O}(1)$. Note that in the adapted coordinate system the 5-metric components take the form [3,23]:

$$\tilde{g}_{\mu\nu} = \begin{pmatrix} g_{\alpha\beta} + \phi B_\alpha B_\beta & \phi B_\alpha \\ \phi B_\beta & \phi \end{pmatrix}, \quad (2)$$

where $\alpha, \beta = 0, 1, 2, 3$. Thus, the ‘‘effective’’ 4-spacetime can be understood as $(M^4, g_{\alpha\beta})$ with the local coordinate system $\{x^\alpha\}$ [23,24]. Denote the 5-velocity of a test particle as \tilde{U}^μ , then the 4-velocity of the particle in M^4 is defined as [23]

$$U^\alpha = \frac{\tilde{U}^\alpha}{\sqrt{-\tilde{U}^\alpha \tilde{U}_\alpha}}, \quad (3)$$

where $\tilde{U}^\alpha \tilde{U}_\alpha \equiv g_{\alpha\beta} \tilde{U}^\alpha \tilde{U}^\beta$. From Eq. (3) one can estimate the order relation between the coordinate velocities in five and four dimensions as $\tilde{v}^i = v^i + \mathcal{O}(3)$. From Virial’s theorem we have $\tilde{v}^2 \sim \tilde{U} \sim \mathcal{O}(2)$. Since the scale of the extra dimension is very small, one can approximate the solution of 5D Poisson equations by that of the corresponding 4D equations. Hence the Newtonian gravitational potentials in five and four dimensions are of the same order, i.e., $\tilde{U} \sim U$. The order relations between the matter variables in five and four dimensions can be estimated from Eq. (1) as $\tilde{\Pi} \sim \Pi$ and $\frac{\tilde{p}}{\tilde{\rho}} \sim \frac{p}{\rho}$. Therefore, in the light of the order relations in 4D PPN theory [20], we obtain $\tilde{U} \sim \tilde{\Pi} \sim U \sim \Pi \sim \mathcal{O}(2)$ and $\frac{\tilde{p}}{\tilde{\rho}} \sim \frac{p}{\rho} \sim \mathcal{O}(2)$. Moreover, the 5D continuous equation of perfect fluid ensures $\frac{|\partial/\partial t|}{|\partial/\partial x|} \sim \mathcal{O}(1)$. With all these instruments we can parametrize any 5D metric theories. Just as in canonical 4D PPN framework [20], to get nontrivial results, we should expand the components of a metric in terms of the linear combinations of our generalized post-Newtonian potentials to the following orders: $\tilde{g}_{00} \sim \mathcal{O}(4)$, $\tilde{g}_{0m} \sim \mathcal{O}(3)$, $\tilde{g}_{mn} \sim \mathcal{O}(2)$.

The concrete relations between the 5D post-Newtonian potentials and the 4D ones can be worked out by means of the Green function. Let $|\vec{x} - \vec{x}'|$ be the spatial distance between the source and field points in the post-Newtonian coordinate system measured by the 4D flat spatial metric, and $|\vec{x} - \vec{x}'|$ be its 3D projection. When $|\vec{x} - \vec{x}'| \gg R$, the Green function $\tilde{\mathbf{G}}(\vec{x}, \vec{x}')$ of the 5D Poisson equation can be approximated as [25]

$$\tilde{\mathbf{G}}(\vec{x}, \vec{x}') = \frac{G}{|\vec{x} - \vec{x}'|} + \frac{2G}{|\vec{x} - \vec{x}'|} e^{-\frac{|\vec{x} - \vec{x}'|}{R}},$$

where G is the 4D gravitational constant. Thus we have

$$\begin{aligned} \tilde{U}(\vec{x}) &= \int \tilde{\mathbf{G}}(\vec{x}, \vec{x}') \tilde{\rho}(\vec{x}') dx^{3'} \\ &= U(\vec{x}) - \gamma \Phi_2(\vec{x}) + \mathcal{O}(6), \end{aligned}$$

where we used in general $\tilde{g}_{55} = 1 + 2\gamma \tilde{U} + \mathcal{O}(4)$. By similar ways, we obtain the following relations:

$$\tilde{\Phi}_1 = \Phi_1 + \mathcal{O}(6) = \int \frac{G\rho(\vec{x}')v^2(\vec{x}')}{|\vec{x} - \vec{x}'|} d^3x' + \mathcal{O}(6), \quad (4)$$

$$\tilde{\Phi}_2 = \Phi_2 + \mathcal{O}(6) = \int \frac{G\rho(\vec{x}')U(\vec{x}')}{|\vec{x} - \vec{x}'|} d^3x' + \mathcal{O}(6), \quad (5)$$

$$\tilde{\Phi}_3 = \Phi_3 + \mathcal{O}(6) = \int \frac{G\rho(\vec{x}')\Pi(\vec{x}')}{|\vec{x} - \vec{x}'|} d^3x' + \mathcal{O}(6), \quad (6)$$

$$\tilde{\Phi}_4 = \Phi_4 + \mathcal{O}(6) = \int \frac{Gp(\vec{x}')}{|\vec{x} - \vec{x}'|} d^3x' + \mathcal{O}(6), \quad (7)$$

$$\tilde{V}_i = V_i + \mathcal{O}(5) = \int \frac{G\rho(\vec{x}')v_i(\vec{x}')}{|\vec{x} - \vec{x}'|} d^3x' + \mathcal{O}(5), \quad (8)$$

$$\tilde{V}_5 = \int \frac{G\rho(\vec{x}')\tilde{v}_5(\vec{x}')}{|\vec{x} - \vec{x}'|} d^3x' + \mathcal{O}(5). \quad (9)$$

The procedure of parametrizing 5D theories is similar to that of 4D ones [20]. Here we just outline the main steps and key points. The field equation of KK theory with matter fields reads

$$\tilde{R}_{\mu\nu} - \frac{1}{2}\tilde{g}_{\mu\nu}\tilde{R} = 8\pi\tilde{G}\tilde{T}_{\mu\nu}, \quad (10)$$

where $\tilde{T}_{\mu\nu} = (\tilde{\rho} + \tilde{\rho}\tilde{\Pi} + \tilde{p})\tilde{U}_\mu\tilde{U}_\nu + \tilde{p}\tilde{g}_{\mu\nu}$ is the energy-momentum tensor of the 5D perfect fluid. Eq. (10) is equivalent to

$$\tilde{R}_{\mu\nu} = 8\pi\tilde{G}\left(\tilde{T}_{\mu\nu} - \frac{1}{3}\tilde{g}_{\mu\nu}\tilde{T}\right), \quad (11)$$

where $\tilde{T} \equiv \tilde{g}^{\mu\nu}\tilde{T}_{\mu\nu}$. The Ricci tensor can be expanded in terms of the perturbative metric to the necessary order around the flat background as

$$\begin{aligned} \tilde{R}_{00} = & -\frac{1}{2}\nabla^2\tilde{h}_{00} - \frac{1}{2}\sum_m\partial_0\partial_0\tilde{h}_{mm} + \partial_m\partial_0\tilde{h}_{m0} \\ & + \frac{1}{2}\partial_m\tilde{h}_{00}\left(\partial_n\tilde{h}_{mn} - \frac{1}{2}\sum_n\partial_m\tilde{h}_{nn}\right) - \frac{1}{4}\nabla\tilde{h}_{00}\cdot\nabla\tilde{h}_{00} \\ & + \frac{1}{2}\tilde{h}_{mn}\partial_m\partial_n\tilde{h}_{00} + \frac{1}{2}(\partial_m\tilde{h}_{n0}\partial_m\tilde{h}_{n0} - \partial_m\tilde{h}_{n0}\partial_n\tilde{h}_{m0}), \end{aligned} \quad (12)$$

$$\begin{aligned} \tilde{R}_{mn} = & -\frac{1}{2}\left(\nabla^2\tilde{h}_{mn} - \partial_m\partial_n\tilde{h}_{00} + \sum_l\partial_m\partial_n\tilde{h}_{ll} \right. \\ & \left. - \partial_l\partial_n\tilde{h}_{ml} - \partial_l\partial_m\tilde{h}_{nl}\right), \end{aligned} \quad (13)$$

$$\tilde{R}_{0m} = -\frac{1}{2}\left(\nabla^2\tilde{h}_{0m} - \partial_n\partial_m\tilde{h}_{0n} + \sum_n\partial_0\partial_m\tilde{h}_{nn} - \partial_n\partial_0\tilde{h}_{mn}\right). \quad (14)$$

The components of the perturbative metric can be solved order by order.

- \tilde{h}_{00} to $\mathcal{O}(2)$: To the required order,

$$\tilde{R}_{00} \approx -\frac{1}{2}\nabla^2\tilde{h}_{00}, \quad \tilde{T}_{00} = -\tilde{T} = \tilde{\rho}, \quad \tilde{g}_{00} = -1.$$

Thus we have

$$\nabla^2\tilde{h}_{00} = -\frac{32}{3}\pi\tilde{G}\tilde{\rho}, \quad \tilde{h}_{00} = 2\tilde{U}.$$

Which justifies that \tilde{U} is the 5D Newtonian potential. Note that the $\mathcal{O}(2)$ term of \tilde{h}_{00} should be same for any 5D metric theories in order to have the same 5D Newtonian limitation.

- \tilde{h}_{mn} to $\mathcal{O}(2)$: Here we impose the gauge condition

$$\frac{1}{2}\partial_m\tilde{h}_\mu^\mu - \partial_\mu\tilde{h}_m^\mu = 0. \quad (15)$$

From Eq. (13) we have

$$\tilde{R}_{mn} = -\frac{1}{2}\nabla^2\tilde{h}_{mn},$$

and then

$$-\frac{1}{2}\nabla^2\tilde{h}_{mn} = \frac{8\pi}{3}\tilde{G}\tilde{\rho}\delta_{mn}.$$

Hence we gets

$$\tilde{h}_{mn} = \tilde{U}\delta_{mn} = U\delta_{mn}. \quad (16)$$

- \tilde{h}_{0m} to $\mathcal{O}(3)$: By imposing the gauge condition

$$\frac{1}{2}\partial_0\tilde{h}_\mu^\mu - \partial_\mu\tilde{h}_0^\mu = \frac{1}{2}\tilde{h}_{00,0}, \quad (17)$$

from Eq. (14) we get

$$\tilde{R}_{0m} = -\frac{1}{2}\nabla^2\tilde{h}_{0m},$$

and thus

$$-\frac{1}{2}\nabla^2\tilde{h}_{0m} + \tilde{U}_{,0m} = -8\pi\tilde{G}\tilde{\rho}\tilde{v}^m. \quad (18)$$

Hence we obtain

$$\tilde{h}_{0i} = -\frac{5}{2}V_i - \frac{1}{2}W_i, \quad h_{05} = 3\tilde{V}_5, \quad (19)$$

where $W_i \equiv \int \frac{G\rho(\vec{x}')[\tilde{v}'\cdot(\vec{x}-\vec{x}')](x_i-x'_i)}{|\vec{x}-\vec{x}'|^3} d^3x'$ is another 4D post-Newtonian potential [20].

- \tilde{h}_{00} to $\mathcal{O}(4)$: To evaluate this part we use all the lower-order solutions of $h_{\mu\nu}$. From Eqs. (12), (16) and (19) we get

$$\tilde{R}_{00} = -\frac{1}{2}\nabla^2\tilde{h}_{00} - \nabla^2\tilde{U}^2 + 3\nabla^2\tilde{\Phi}_2. \quad (20)$$

Thus from Eq. (11) we have

$$\begin{aligned} \nabla^2\tilde{h}_{00} = & 2\nabla^2\tilde{U} - 2\nabla^2\tilde{U}^2 + 3\nabla^2\tilde{\Phi}_1 + 2\nabla^2\tilde{\Phi}_2 \\ & + 2\nabla^2\tilde{\Phi}_3 + 4\nabla^2\tilde{\Phi}_4, \end{aligned}$$

and hence

$$\tilde{h}_{00} = 2\tilde{U} - 2\tilde{U}^2 + 3\tilde{\Phi}_1 + \tilde{\Phi}_2 + 2\tilde{\Phi}_3 + 4\tilde{\Phi}_4.$$

Now we are facing the problem how to relate the parametrized KK theory to the experiments. For most gravitational experiments in Solar system, we may consider only the free test particles without electric charge. From the viewpoint of the KK theory, this implies that the test particles do not mover along the extra dimension, i.e., $\tilde{U}^\mu\xi_\mu = 0$. In this case, it is easy to show that the 5D geodesic equations for both massive and massless test particles are reduced to the 4D geodesic equations in the effective 4D spacetime. Thus the reduced 4D theory behaves just like a metric theory for these particular test particles or photons. Along the reduction procedure previously discussed, we can reduce the parametrized 5-metric to the effective 4-metric

Table 1
The PPN parameters of GR and KK

Theory	PPN parameters				
	γ	β	ξ	$(\alpha_1, \alpha_2, \alpha_3)$	$(\zeta_1, \zeta_2, \zeta_3, \zeta_4)$
GR	1	0	0	(0, 0, 0)	(0, 0, 0, 0)
KK	$\frac{1}{2}$	0	0	(0, 0, 0)	(0, 0, 0, $\frac{1}{6}$)

$g_{\alpha\beta}$ as

$$g_{00} = -1 + 2U - 2U^2 + 3\Phi_1 + \Phi_2 + 2\Phi_3 + 4\Phi_4, \quad (21)$$

$$g_{0i} = -\frac{5}{2}V_i - \frac{1}{2}W_i, \quad (22)$$

$$g_{ij} = (1 + U)\delta_{ij}. \quad (23)$$

According to the general form of the post-Newtonian metric [20]

$$\begin{aligned} g_{00} = & -1 + 2U - 2\beta U^2 - 2\xi\Phi_w \\ & + (2\gamma + 2 + \alpha_3 + \zeta_1 - 2\xi)\Phi_1 \\ & + 2(3\gamma - 2\beta + 1 + \zeta_2 + \xi)\Phi_2 + 2(1 + \zeta_3)\Phi_3 \\ & + 2(3\gamma + 3\zeta_4 - 2\xi)\Phi_4 - (\zeta_1 - 2\xi)\mathcal{A} \\ & - (\alpha_1 - \alpha_2 - \alpha_3)w^2U - \alpha_2w^i w^j U_{ij} \\ & + (2\alpha_3 - \alpha_1)w^i V_i + \mathcal{O}(6), \end{aligned} \quad (24)$$

$$\begin{aligned} g_{0i} = & -\frac{1}{2}(4\gamma + 3 + \alpha_1 - \alpha_2 + \zeta_1 - 2\xi)V_i \\ & - \frac{1}{2}(1 + \alpha_2 - \zeta_1 + 2\xi)W_i - \frac{1}{2}(\alpha_1 - 2\alpha_2)w^i U \\ & - \alpha_2w^j U_{ij} + \mathcal{O}(5), \end{aligned} \quad (25)$$

$$g_{ij} = (1 + 2\gamma U)\delta_{ij} + \mathcal{O}(4), \quad (26)$$

the relevant post-Newtonian parameters for the reduced KK theory and 4D GR are compared in Table 1.

Therefore, given the same (reduced) 4D matter distribution such as a 4D perfect fluid, the detail comparison between the above two theories leads to significant conclusions. Firstly, the metric component g_{00} in the post-Newtonian coordinates system in KK theory is smaller than that in 4D GR. But the departure appear only in $\mathcal{O}(4)$ terms, and hence the reduced 5D KK theory has the right Newtonian limitation. Secondly, the metric component g_{0i} in KK theory are 7/5 times smaller than those in 4D GR in an $\mathcal{O}(3)$ term. This departure may in principle be detected by the current precise gravitational experiments in Solar system. At last, the metric components g_{ij} together with g_{00} and g_{0i} in KK theory determine the post-Newtonian parameter $\gamma = \frac{1}{2}$, which is obviously different from $\gamma = 1$ in 4D GR. It is obvious that the above departure is due to the existence of

an extra dimension in KK theory. The disaster of KK theory is that the value of the parameter γ has been accurately measured by Solar system experiments. In time delay experiment one obtains $\gamma - 1 = (2.1 \pm 2.3) \times 10^{-5}$ [21,26], and in light deflection experiment one gets $\gamma - 1 = (-1.7 \pm 4.5) \times 10^{-4}$ [21,27,28]. Hence there is a severe contradiction between 5D KK theory and the Solar system experiments.

Our PPN formalism and related discussion can be generalized straightforwardly to higher dimensional KK theories with compact extra dimensions. Therefore, although the original idea of Kaluza and Klein is rather beautiful, the classical KK theories cannot survive the experiments.

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