Scattering of a uniaxial anisotropic sphere located in dual Gaussian beam with arbitrary propagation directions

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Abstract

The scattering characteristics of a uniaxial anisotropic sphere illuminated by two Gaussian beams with arbitrary propagation and polarization directions are investigated. On the basis of spherical vector wave functions and coordinate rotation theory, the fields of any incident Gaussian beam with arbitrary propagation and polarization directions are expanded in terms of the spherical vector wave functions. Then the total expansion coefficients of the incident fields are derived by superposition of the vectors. Based on the Fourier transform and boundary conditions, the analytical expressions of the scattering coefficients of dual Gaussian beams with arbitrary propagation directions by a uniaxial anisotropic sphere are obtained. The effects of the incident angle, polarization angle of the dual Gaussian beams on the angle distributions of the radar cross sections (RCSs) are numerically analyzed in detail. This investigation may provide an effective test for further research on the scattering characteristic of anisotropic particles by a multiple beams and radiation forces, which are important in optical tweezers and related particle manipulation applications.

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1. Introduction

Since the Mie theory is proposed in 1908, the electromagnetic (EM) scattering of a homogenous isotropic sphere illuminated by a plane wave or a Gaussian beam has been researched and developed by many scholars [1, 2]. In recent years, the scattering characteristics of anisotropic media have attracted much attention because of their increasing applications in such areas as optical signal processing, radar cross section (RCS) controlling and microwave device fabrication. Interaction of a plane wave with a uniaxial anisotropic particle was extensively researched [3-5]. Though the analytical methods can obtain the exact solution and gain profound physical insight to the problem. Numerical approximate approaches based on FDTD, the Moment Method and integral equations have also been developed. Using Fourier transformation method, the team led by Wu investigated further the scattering characteristics of a uniaxial anisotropic sphere located in a Gaussian beam [6-8]. However, the incident wave in all these researches is limited to a plane wave or a single Gaussian beam. To our knowledge, the published works on the scattering of an isotropic sphere [9], especially the anisotropic sphere by dual Gaussian beam are exiguous. Since the trait of non-contact manipulation of nano-objects of optical trapping, many scholars studied the dual beam trap or standing-wave trap for a very small isotopic spherical particle due to its advantage comparing with the single beam trap [10]. But the scattering force is always neglected because of small particle size. So it is an urgent to exploit the scattering problem of an anisotropic sphere illuminated by a dual focused beam if we want to trap an anisotropic sphere by dual beam trap. In addition, the distribution of the scattering fields will become more complicated when the beam waist center and the propagation direction of the dual beam is arbitrary. In this paper, a rigorous analytical solution on the scattering of a uniaxial anisotropic sphere illuminated by a dual Gaussian beam with arbitrary propagation directions based on the Fourier transformation method are described. In the subsequent analysis, a time dependence of the form \( \exp(-i\omega t) \) is assumed for all the EM fields but is ignored throughout the treatment.

2. Theory

Consider a homogeneous uniaxial anisotropic sphere of radius \( a \) centrally located in a particle coordinate system \( Oxyz \). The primary optical axis is coincident with the \( z \)-axis. As Fig. 1 shows, the particle is obliquely illuminated by two Gaussian beams with arbitrary directions of propagation and polarization. One is polarized in \( x_1 \) axis and propagates in the \( z_1 \)-direction in Cartesian coordinate system \( OXYZ_1 \), with the beam center located at \( O_1 \). The other one is polarized in \( x_2 \) axis and propagates in the \( z_2 \)-direction in Cartesian coordinate system \( OXYZ_2 \), with the beam center located at \( O_2 \). The coordinates of beam center \( O_1 \) and \( O_2 \) in \( OXYZ \) are \((x_1, y_1, z_1)\) and \((x_2, y_2, z_2)\), respectively. We denote the propagation direction of the first beam as two angles in particle coordinate system \( OXYZ \); that is,
incident angle $\alpha_1$ with reference to the $z$-axis, and polarization angle $\beta_1$ between the $x$-axis and the propagation direction of the beam on the $xoy$-plane. These two angles are $\alpha_2$ and $\beta_2$ for the second beam. Temporary coordinate system $Ox_1y_1z_1$ is established parallel to $Ox_1y_1z_1$ in order to expand this incident field.

According to the rotation addition theorem and orthogonality of spherical vector wave functions (SVWFs), the fields of the first Gaussian beam can be expanded in terms of the SVWFs utilizing the Temporary coordinate system $Ox_1y_1z_1$ as \[ E^{(1)} = E_0 \sum_{n=1}^{\infty} \sum_{m=-n}^{n} [a_{nm}^{(1)} M_{nm}^{(1)}(r, k_0) + b_{nm}^{(1)} N_{nm}^{(1)}(r, k_0)] \]

\[ H^{(1)} = E_0 \frac{k_0}{\io \mu_0} \sum_{n=1}^{\infty} \sum_{m=-n}^{n} [a_{nm}^{(1)} N_{nm}^{(1)}(r, k_0) + b_{nm}^{(1)} M_{nm}^{(1)}(r, k_0)] \] (1)

where $k_0 = 2\pi/\lambda$ and $\lambda$ is a surrounding medium wavelength, $\mu_0$ denotes the permeability of the surrounding medium, and $E_0$ is the amplitude of the electric field at the beam center. The expansion coefficients of the incident fields are:

\[
\left( a_{nm}^{(1)}, b_{nm}^{(1)} \right) = \sum_{s=0}^{\infty} \rho(s, m, n) C_{nm} \left( i g_{n,TE}^s, g_{n,TM}^s \right) 
\]

\[
\rho(s, m, n) = (-1)^{s+m} e^{-is\alpha_1} \frac{(n+s)(n-m)!}{(n-s)!} u^{(s)}(-\beta_1) 
\]

\[
u^{(s)}(-\beta_1) = \left[ \frac{(n+m)(n-m)!}{(n+s)(n-s)!} \right]^{1/2} \sum_{\sigma=0}^{n-s} \left[ \frac{n+s}{n-s-\sigma} \right]^{n-s-\sigma} \left[ \frac{n-s}{\sigma} \right]^{n-s} \left( -1 \right)^{n-s-\alpha} \cos \left[ \frac{\beta_1}{2} \right]^{2s+w+\alpha} \sin \left[ \frac{\beta_1}{2} \right]^{2s-2w+\alpha} \] (4)

In Eq. (2), $g_{n,TE}^s$ and $g_{n,TM}^s$ are the beam shape coefficients and decided by the parameters of the first beam. Similarly, a temporary coordinate system $Ox_2y_2z_2$ is established parallel to $Ox_2y_2z_2$, the second beam can also be expanded in terms of the SVWFs as:

\[ E^{(2)} = E_0 \sum_{n=1}^{\infty} \sum_{m=-n}^{n} [a_{nm}^{(2)} M_{nm}^{(2)}(r, k_0) + b_{nm}^{(2)} N_{nm}^{(2)}(r, k_0)] \]

\[ H^{(2)} = E_0 \frac{k_0}{\io \mu_0} \sum_{n=1}^{\infty} \sum_{m=-n}^{n} [a_{nm}^{(2)} N_{nm}^{(2)}(r, k_0) + b_{nm}^{(2)} M_{nm}^{(2)}(r, k_0)] \] (5)

The expansion coefficients can be obtained through substituting $\alpha_1$ and $\beta_1$ into $\alpha_2$ and $\beta_2$ in Eqs. (2)-(4). Then total incident fields can be expanded superposing the two electromagnetic (EM) fields shown in Eqs. (1) and (5), and the total expansion coefficients are:

\[ a_{nm} = a_{nm}^{(1)} + a_{nm}^{(2)}, \quad b_{nm} = b_{nm}^{(1)} + b_{nm}^{(2)} \] (6)

The scattered fields can be expanded with the SVWFs as follows:

\[ E^s = E_0 \sum_{n=1}^{\infty} \sum_{m=-n}^{n} \left[ A_{nm}^{(s)} M_{nm}^{(3)}(r, k_0) + B_{nm}^{(s)} N_{nm}^{(3)}(r, k_0) \right] \]

\[ H^s = E_0 \frac{k_0}{\io \mu_0} \sum_{n=1}^{\infty} \sum_{m=-n}^{n} \left[ A_{nm}^{(s)} N_{nm}^{(3)}(r, k_0) + B_{nm}^{(s)} M_{nm}^{(3)}(r, k_0) \right] \] (7)

For a uniaxial anisotropic sphere whose permittivity and permeability tensors $\varepsilon$ and $\mu$ are characterized by

\[
\varepsilon = \begin{bmatrix}
\varepsilon_t & 0 & 0 \\
0 & \varepsilon_0 & 0 \\
0 & 0 & \varepsilon_z
\end{bmatrix}, \quad \mu = \begin{bmatrix}
\mu_t & 0 & 0 \\
0 & \mu_0 & 0 \\
0 & 0 & \mu_z
\end{bmatrix}
\]

(8)

The internal fields can be expanded in terms of the SVWFs using Fourier transform\([3, 6, 8]\). Utilizing the continuity of the tangential electric and magnetic field components at $r = a$, the scattering coefficients can be obtained \([3, 6]\):
The unknown expansion coefficient $G_{m,n}^{q}$ is decided by the expansion coefficients of incident fields\[6, 8\]. The expressions of coefficients $A_{m,n}^{e}$, $B_{m,n}^{e}$, $C_{m,n}^{e}$, $A_{m,n}^{h}$, $B_{m,n}^{h}$, and $C_{m,n}^{h}$ are detailed in [3].

With these solved coefficients, the field components of the total scattered can be obtained by corresponding substitutions. Then, according to the definition of the RCS for the total far-region scattered fields,

$$\sigma = \lim_{r \to \infty} \left( 4 \pi r^2 \left| \mathbf{E}' \right|^2 \right)$$  \hfill (11)

where $\mathbf{E}'$ denotes the initial incident electric field of one of the beam.

3. Numerical results and discussion

In this section, some numerical results in relation to a dual Gaussian beam scattering by a uniaxial anisotropic spheres for the TM mode are presented. The E-plane corresponds to the $xoz$-plane, and the H-plane corresponds to the $yoz$-plane.

![Diagram](image-url)

**Fig.2.** Results reduced to the case of single beam incident compared with those by literature (a) E-plane, (b) H-plane.

The angular distributions of the RCSs in the E-plane and H-plane are shown in Fig. 2. The second Gaussian beam is off-axis incidence and the coordinate in the particle coordinate system is $(10, 0, 0)^*a$. By virtue of the generalized Lorenz-Mie theory (GLMT), the interaction is very weak and negligible when the off-axis is so obvious. So the dual Gaussian beam incidence can be reduced to a single Gaussian beam incidence, and the angular distributions of the RCSs of a uniaxial anisotropic sphere illuminated by an on-axis or obliquely incident Gaussian beam which is from published paper [8] is also shown in Fig. 2 (denoted by “literature”). It can found that these two results are in a good agreement, which can verify the correctness of the theory and codes in this paper.
In Fig.3, the angular distributions of the RCSs in the E-plane and H-plane of a uniaxial anisotropic sphere by a dual Gaussian beams when the second beam is on-axis obliquely incidence but not off-axis. The other parameters are the same with those shown in Fig.2. It can be found that the value of the forward RCSs is maximal when both of the two beams propagates along the same direction, namely z-axis, $\alpha_1=\beta_1=0^\circ$. All of the RCSs are not two times of those values when the uniaxial anisotropic sphere is illuminated by only a single Gaussian beam shown in Fig. 2 due to that the two Gaussian beams have an interference effect before interacting with the particle. However the shape of the angle distribution curve has little change which is decided by the parameter of the uniaxial anisotropic particle. When the second Gaussian beam is obliquely incidence, there are two peak values in E-plane along the propagation direction of the dual beam, namely there are dual forward RCSs. It is obvious when the intersection angle between the propagation directions of the dual beam is very large so that the interference effect is weekly. Consequently, dual peak values in E-plane are not very obvious when $\alpha_1=30^\circ$ because of week interference effect. The angular distributions of the RCSs in E-plane $\alpha_1=\alpha_2=0^\circ$ is symmetrical and the symmetry is destroyed when $\alpha_1$ is not zero. Moreover, the angle distributions of the RCSs in H-plane are always symmetrical due to that the polarized direction of the dual Gaussian is vertical to H-plane. These properties indicate that the symmetry is decided by the incident angle and polarized angle.

4. Conclusion

The scattering characteristics of a uniaxial anisotropic sphere illuminated by two Gaussian beams with arbitrary propagation and polarization directions are investigated. The analytical solution is obtained, and the correctness is verified through numerical results. The RCSs are not two times of those values when the particle is illuminated by only a single Gaussian beam due to interference effect. The theory and numerical results is hopeful to provide an effective help for further research on the dual focused beam trapping of anisotropic spherical particle.

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