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Letter to the Editor

A Remark on Rational Approximation

A. R. REDDY

School of Mathematics, Hyderabad University, Hyderabad, India, and Department of Mathematics and Statistics, University of Pittsburgh, Pittsburgh, Pennsylvania 15260

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THEOREM. Let f(x) be a real function continuous on [0, 1] with f(1) = 0. Then for any real polynomials P(x), Q(x) which are $\neq 0$ on [0, 1], we have

$$\left\| f - \frac{P}{Q} \right\|_{L^{\infty}[0,1]} \ge \frac{f(0)}{\left[1 + (P(0) Q(1))/(Q(0) P(1)) \right]}.$$

Proof. Let

$$\left\| f - \frac{P}{Q} \right\|_{L^{\infty}[0,1]} = \varepsilon.$$

Then

$$\varepsilon \geqslant \frac{P(1)}{Q(1)} = \left(\frac{P(0)}{Q(0)} - f(0)\right) \frac{Q(0) P(1)}{P(0) Q(1)} + \frac{f(0) Q(0) P(1)}{P(0) Q(1)}$$

$$\geqslant -\frac{\varepsilon Q(0) P(1)}{P(0) Q(1)} + \frac{f(0) Q(0) P(1)}{P(0) Q(1)}$$

and, so,

$$\varepsilon \geqslant \frac{f(0)}{[1 + (P(0) Q(1))/(Q(0) P(1))]}$$

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