Shear strength of an unsaturated weakly expansive soil

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Abstract: To study the weakly expansive clay obtained from a slope along Wuhan—Shiyan expressway in Hubei Province, soil-water property tests and some unsaturated triaxial tests with suction control were conducted, and the soil-water retention curve (SWRC) and unsaturated shear strength of this soil were obtained. Results show that the air-entry suction and the residual degree of saturation of the tested soil are 106 kPa and 8%, respectively. The boundary effect zone and the transition zone can be identified on the desorption curve, but the residual zone is not so obvious. The unsaturated shear strength increases as suction increases within the range of controlled suction in the test, and friction angle, $\phi_b$, in the triaxial shear test is 17.6°. Based on the results, constitutive models for predicting the unsaturated shear strength using the SWRC were evaluated, and comparisons between prediction and measurement were made. It is concluded that for engineering purpose, the constitutive model should be carefully selected based on soil properties when predicting the unsaturated shear strength using the SWRC.

Key words: unsaturated soil; soil-water retention curve (SWRC); weakly expansive soil; suction; shear strength

1 Introduction

Expansive soils encountered in engineering practice widely spread all over the world. They are often in unsaturated state. Furthermore, with a high content of hydrophilic minerals such as montmorillonite, illite, etc., their unsaturated shear strength is strongly influenced by the physico-chemical interaction between water and clay minerals. This is why the shear strength is always an important issue for this kind of expansive soil.

A large quantity of investigations into unsaturated expansive soils strength has been made by many scholars both experimentally and theoretically. Change in shear strength with suction for an expansive soil from Ningxia Hui Autonomous Region, China, was explored by Xu et al. [1] using an improved triaxial apparatus. The shear strength characteristics of expansive soils in Nanyang Basin in Henan Province, China, were compared with those from some areas in northwest Hubei Province by Gong et al. [2]. Based on triaxial tests on remoulded unsaturated expansive soil at different dry densities and suctions, the relationships among hardening, softening, volumetric change behaviors and pore size distributions were investigated by Miao et al. [3] on Guangxi expansive soil, and a hyperbolic relationship between the unsaturated shear strength and the suction was established. The relationship between the unsaturated shear strength and the water content was further discussed by Yang et al. [4]. Zhan and Ng [5] found that suction could lead to the hardening of unsaturated soils without modifying the internal friction angle, and the unsaturated shear strength increased nonlinearly with suction.

As shear strength tests on unsaturated soils are costly and time-consuming, simple and indirect methods have been developed to obtain the shear strength of unsaturated soils for engineering purpose. Many researchers suggested that the shear strength of unsaturated soils could be predicted using different equations [6]. Based on the soil-water retention curve (SWRC), some prediction models were proposed by Lamborn et al. [7–11]. Some empirical models for prediction of unsaturated shear strength were
suggested by Escario et al. [12–14]. Based on the 
analysis of the characteristics of unsaturated shear 
strength, the relationship between the shear strength 
parameters and the initial water content of compacted 
unsaturated expansive soils was developed by Kong 
and Tan [15].

In the present work, the expansive soil taken from 
Hubei Province was studied. The SWRC and the 
unsaturated shear strength were determined from 
soil-water retention tests and unsaturated triaxial tests. 
Based on the results, different constitutive models 
were evaluated by comparisons between prediction 
and measurement results.

2 Unsaturated shear strength prediction 
models

2.1 Background

Bishop [16] proposed a simple equation to 
determine of the shear strength of unsaturated soil 
based on the Mohr-Coulomb criterion:

$$\tau_t = c' + [(\sigma - u_a) + \chi(u_s - u_w)]\tan\phi'$$ (1)

where $\tau_t$ is the shear strength, $c'$ is the effective 
cohesion in saturated state, $u_a$ is the pore air pressure, 
$u_s$ is the pore water pressure, $u_s - u_w$ is the matric 
suction, $\phi'$ is the effective angle of internal friction, 
$\chi$ is a parameter depending on the degree of saturation, 
equal to 1 and 0 when the soil is fully saturated and 
totally dry, respectively.

The Bishop’s equation has the advantage to be 
simple. However, $\chi$ depends on many factors and it is 
difficult to be determined. Moreover, the suction is 
difficult to be measured. Therefore, some other similar 
equations [17] were suggested successively by Croney 
et al.

Currently, the double-variable shear strength 
formula proposed by Fredlund et al. [18] seems to be 
the mostly used one in geotechnical engineering 
involving unsaturated shear strength:

$$\tau_t = \tau_{sat} + \tau_{us} = [c' + (\sigma - u_a)\tan\phi'] + (u_s - u_w)\tan\phi^b$$ (2)

where $\tau_{sat}$ is the saturated shear strength, $\tau_{us}$ is the 
 shear strength related to matric suction, and $\phi^b$ is the 
friction angle related to matric suction $u_s - u_w$. Based on 
an analysis of $\phi^b$ for various unsaturated soils, 
Fredlund and Rahardio [19] found that $\phi^b$ is 
generally less than or equal to the value of internal 
friction angle $\phi'$. More precisely, Gan et al. [20] have 
demonstrated that when the suction is lower than a 
certain value (such as the air-entry suction) $\phi^b = \phi'$; 
when the suction is higher than a certain value, $\phi^b$ 
decreases nonlinearly with suction. That is to say, for 
most soils, $\phi^b$ is not a constant.

2.2 Unsaturated shear strength prediction models 
involving the water retention characteristics

The SWRC describes the relationship between 
water content or degree of saturation and suction, and 
it is widely used to describe the hydraulic behavior of 
unsaturated soils. Based on the SWRC and the 
saturated shear strength parameters, many empirical 
models have been developed for prediction of 
unsaturated shear strength.

Based on an investigation into the microscopic 
mechanical properties of unsaturated soil, the 
following equation for prediction of unsaturated shear 
strength was proposed by Lamborn [7]:

$$\tau_w = (u_s - u_w)\theta\tan\phi'$$ (3)

$$\theta = \frac{S_e}{1 + e}$$ (4)

where $\theta$ is the volumetric water content, $S_e$ is the 
degree of saturation, and $e$ is the void ratio.

Vanapalli et al. [8] also proposed a model for 
calculation of unsaturated shear strength using the 
SWRC:

$$\tau_w = (u_s - u_w)\tan\phi'\left(\frac{\theta - \theta_1}{\theta_e - \theta_1}\right)$$ (5)

where $\theta_1$ is the saturated volumetric water content, 
and $\theta_e$ is the residual volumetric water content. It 
should be noticed that in practice $\theta_e$ is difficult to be 
determined because the residual zone for clayey soils 
is not so obvious.

Oberg and Sallfors [9] proposed the following 
model for both sand and muddy clay:

$$\tau_w = (u_s - u_w)S_e\tan\phi'$$ (6)

This model is similar to the Bishop’s one, and 
assumes that $\chi$ in the Bishop’s equation is equal to $S_e$.

Based on the experimental results, Khalili and 
Khambaz [10] proposed $\chi = [(u_s - u_w) / (u_s - u_w)_{se}]^{0.55}$ 
($u_s - u_w$ is the air-entry suction), and substituted it 
into the Bishop’s equation and finally obtained the 
following expression:

$$\tau_w = (u_s - u_w)\left[\frac{u_s - u_w}{(u_s - u_w)_{se}}\right]^{0.55}\tan\phi'$$ (7)

According to Eq.(7), it can be used for all kinds of 
soils.
Rassam and Cook [11] proposed the following model:

\[ \tau_u = (u_s - u_a) \tan \varphi' - \psi(u_s - u_a) - (u_s - u_a) h \]  

\[ \psi = \left( (u_s - u_a) \right) \tan \varphi - \tau_u \left( (u_s - u_a) \right) h \]  

\[ \beta = \frac{\tan \varphi(u_s - u_a) - (u_s - u_a) h}{(u_s - u_a) \tan \varphi - \tau_u} \]  

where \( \psi \) and \( \beta \) are fitting parameters, \( u_s - u_a \) is the residual suction, and \( \tau_u \) is the contribution of matric suction to the shear strength at residual suction. \( \tau_u \) should be experimentally determined. This model is suitable for coarse and medium sands, for which \( \tau_u \) is relatively easy to be determined.

Kayadelen et al. [6] proposed the following equation for predicting the unsaturated shear strength:

\[ \tau_u = \tan \varphi(u_s - u_a) + P_a \ln \left( \frac{u_s - u_a + P_a}{P_a} \right) \]  

where \( P_a \) is the atmospheric pressure.

3 Experimental tests

3.1 Basic properties

The soil specimen was obtained from a slope along Wuhan—Shiyuan expressway at Laohekou, Hubei Province, with a bedding depth of 1–1.5 m. According to the geological investigation report, the soils involved are alluvial and diluvial cohesive soils formed in Quaternary pleistocene, colored in dark-yellow, rigid-plastic, containing some Fe, Mn oxide nodules and a little incanus kaolin bands. It has a low natural water content and a relatively high bearing capacity. The basic physical properties of soil samples are summarized in Table 1.

<table>
<thead>
<tr>
<th>Depth (m)</th>
<th>Free expansive ratio, ( \alpha_e ) (%)</th>
<th>Specific gravity, ( G_s )</th>
<th>Natural water content, ( w ) (%)</th>
<th>Density, ( \rho ) (g/cm³)</th>
<th>Initial void ratio, ( e_0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1–1.5</td>
<td>58</td>
<td>2.74</td>
<td>20.6</td>
<td>2.01</td>
<td>0.64</td>
</tr>
</tbody>
</table>

Table 1 Basic physical properties of soil samples.

Degree of saturation, \( S_h \) (%): 89

Liquid limit, \( w_l \) (%): 42.08

Plastic limit, \( w_p \) (%): 23.68

Plasticity index, \( I_p \): 18.40

Clay grain (<0.005 mm) content (%): 16.6

3.2 Test methods

3.2.1 Determination of the SWRC

The vapor equilibrium method and the osmotic method were employed for high suctions (4.2–309 MPa) and low suctions (below 2 MPa) control, respectively [21]. The saturated salt solutions used for the vapor equilibrium method and the corresponding suctions at a temperature of 20 °C are listed in Table 2.

<table>
<thead>
<tr>
<th>Saturated salt solution</th>
<th>Suction (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>K₂SO₄</td>
<td>4.2</td>
</tr>
<tr>
<td>ZnSO₄·7H₂O</td>
<td>12.6</td>
</tr>
<tr>
<td>NaCl</td>
<td>38</td>
</tr>
<tr>
<td>Mg(NO₃)₂·6H₂O</td>
<td>82</td>
</tr>
<tr>
<td>LiCl</td>
<td>309</td>
</tr>
</tbody>
</table>

Table 2 Saturated salt solutions and corresponding suctions (20 °C).

For the osmotic method, PEG 20000 solution and \((1.2–1.4) \times 10^4\) semi-permeable membrane were used. The brix of the PEG 20000 solution and the corresponding suctions are presented in Table 3.

<table>
<thead>
<tr>
<th>Brix (%)</th>
<th>Suction (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.634</td>
<td>0.01</td>
</tr>
<tr>
<td>10.694</td>
<td>0.2</td>
</tr>
<tr>
<td>15.816</td>
<td>0.5</td>
</tr>
<tr>
<td>20.850</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 3 Brix of PEG 20000 solution and corresponding suctions.

3.2.2 Triaxial shear tests

The unsaturated triaxial shear test apparatus used in Ref. [22] was employed for shear tests in this study. The dimensions of the specimen are 76 mm in height and 38 mm in diameter.

Nine specimens divided into 3 sets were tested, and each specimen underwent different stress paths. The test conditions and the corresponding results are listed in Table 4. Note that if a peak deviatoric stress was reached before 15% deformation, the deviatoric stress considered for shear strength determination was taken as the peak value, otherwise the deviatoric stress at 15% deformation was taken for this purpose.

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Suction (kPa)</th>
<th>Net confining pressure (kPa)</th>
<th>Deviatoric strength (kPa)</th>
<th>Cohesion (kPa)</th>
<th>Friction angle (°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2-1</td>
<td>0</td>
<td>100</td>
<td>222</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2-2</td>
<td>0</td>
<td>200</td>
<td>385</td>
<td>14.8</td>
<td>28.2</td>
</tr>
<tr>
<td>2-3</td>
<td>0</td>
<td>400</td>
<td>753</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2-4</td>
<td>100</td>
<td>100</td>
<td>300</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2-5</td>
<td>100</td>
<td>200</td>
<td>513</td>
<td>76.9</td>
<td>24.6</td>
</tr>
<tr>
<td>2-6</td>
<td>100</td>
<td>400</td>
<td>801</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2-7</td>
<td>200</td>
<td>100</td>
<td>400</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2-8</td>
<td>200</td>
<td>200</td>
<td>578</td>
<td>141.5</td>
<td>22.8</td>
</tr>
<tr>
<td>2-9</td>
<td>200</td>
<td>400</td>
<td>910</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 4 Testing conditions and corresponding results.

3.3 Results and discussions

3.3.1 The SWRC
The suction and the corresponding degree of saturation measured in the tests are listed in Table 5, and the SWRC is presented in Fig.1.

Table 5: Testing results of suction and degree of saturation.

<table>
<thead>
<tr>
<th>Suction (kPa)</th>
<th>Degree of saturation (%)</th>
<th>Suction (kPa)</th>
<th>Degree of saturation (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>100</td>
<td>12 600</td>
<td>47.51</td>
</tr>
<tr>
<td>10</td>
<td>99.49</td>
<td>38 000</td>
<td>34.25</td>
</tr>
<tr>
<td>200</td>
<td>91.38</td>
<td>82 000</td>
<td>21.29</td>
</tr>
<tr>
<td>500</td>
<td>85.71</td>
<td>309 000</td>
<td>5.98</td>
</tr>
<tr>
<td>1 000</td>
<td>73.47</td>
<td>1 000 000</td>
<td>0</td>
</tr>
<tr>
<td>4 200</td>
<td>59.66</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure 1 shows that, for the tested specimen, the air-entry suction is about 106 kPa and the residual degree of saturation is about 8.2%. The SWRC is clearly step-phased with the boundary effect zone and the transition zone well identified. The residual zone is however not so obvious.

In the boundary effect zone, the soil is almost saturated and the water content is nearly independent of suction, while the shear strength develops linearly with suction. In the transition zone, the water content decreases drastically as the suction increases, and the shear strength varies nonlinearly with suction. In the residual zone, the water content has the residual value. For sandy and silty soils, the shear strength is independent of suction; by contrast for clayey soils, it increases slightly with suction increase [8].

3.3.2 Fitting the SWRC

Based on the suction and the corresponding degree of saturation in Table 5, a best fitting relationship can be established using the logarithmic model proposed by Fredlund and Xing [23]:

\[ \theta(\psi, a, n, m) = C(\psi) / \left[ \ln(2.718 + (\psi / a)^n) \right]^m \]  \hspace{1cm} (12)

\[ C(\psi) = \frac{-\ln(1 + \psi / \psi_r)}{\ln[1 + (1 000 000 / \psi_r)]} + 1 \]  \hspace{1cm} (13)

\[ a = \psi_i \]  \hspace{1cm} (14)

\[ m = 3.67 \ln \frac{\theta C(\psi)}{\theta_i} \]  \hspace{1cm} (15)

\[ n = \frac{1.31^{m+1} - 3.72 s^*}{m C(\psi_i)} \]  \hspace{1cm} (16)

\[ s^* = \frac{s}{\theta_i} - \frac{1.31^n (\psi_i + \psi_r) \ln(1 + 1 000 000 / \psi_r)}{\theta_i} \]  \hspace{1cm} (17)

\[ s = \frac{\theta_i}{\ln(\psi / \psi_i)} \]  \hspace{1cm} (18)

where \( a \) is equal to the air-entry suction; \( n \) is the tangent to the curve at the inflexion point (in the transition zone); \( m \) is related to the residual water content; \( C(\psi) \) is a correction function; \( \psi_i \) is the suction corresponding to a given water content, \( \psi_r \) is the suction corresponding to the residual water content, and \( \psi_f \) is the intercept of the tangent line of the transition zone and the matric suction axis.

Based on the test results, the following values of the parameters in Eq.(18) can be found: \( \theta_i = 83\% \), \( \psi_i = 300 \) kPa, \( \psi_r = 1 \) GPa, \( s = 2.36 \), \( \psi_r = 3 \) MPa. Then from Eq.(17), we can find \( s^* = 0.095 \). In the same way, the parameters in Eq.(12) can be found from Eqs.(13)–(16): \( a = 300 \), \( m = 0.34 \), \( n = 1.22 \).

Finally, using Eq.(12) and the parameters listed above, the SWRC can be fitted (Fig.2).

3.3.3 Unsaturated triaxial shear strength

The determined relationships between the axial strain and the deviatoric stress under net confining pressures of 100, 200 and 400 kPa are illustrated in Fig.3. It can be observed in this figure that the shapes of the stress-strain curves are clearly step-phased. At the beginning, the strain increases very rapidly with the increase of deviatoric stress. Then, after a certain strain value, the slope of the curve declines. It also suggests that when the confining pressure increases from 100, 200 to 400 kPa, the stress-strain relationship
of the specimen changes from strain-softening to strain-hardening. Moreover, for a same confining pressure, the deviatoric stress increases with suction. For a same suction, the deviatoric stress increases with the increase of confining pressure.

Based on the testing results, the three-dimensional Mohr-Coulomb’s failure envelopes can be plotted (Fig.4) using Fredlund’s model.

Figure 4 shows that within the low suction range, the unsaturated shear strength increases with suction. Furthermore, it appears that suction mainly influences the effective cohesion $c'$, while the internal friction angle $\phi'$ is relatively constant. The unsaturated shear strength changes linearly with suction and $\phi'$ is almost constant and equal to 17.6°.

3.3.4 Comparisons between prediction and measurement

Based on the SWRC determined and the saturated shear strength parameters $c'$, $\phi'$, Eqs.(3)–(11) were employed to predict the unsaturated shear strength of the soil. The prediction curves and the measured results are presented in Fig.5.

Figure 5 shows that the model proposed by Vanapalli et al. [8] does not give the best fitting to the weakly expansive soil tested. The predicted curves of other models show much greater difference with the experimental results. This may be because that different prediction equations suit different soils.

The results predicted by Kayadelen et al. and Lamborn’s equations [6, 7] give the largest deviation from the measured results. This is probably because that Lamborn’s model only considers the micro-mechanical properties of the soil. However, as mentioned above, the tested weakly expansive soil not
only possesses a good structure, but also contains oxide nodules of Fe and Mn, which can contribute to the increase in shear strength. Kayadelen’s model, which is only suitable for the prediction of unsaturated shear strength of fine-grained soils at low suctions, cannot give a good prediction because the tested soil is a clayey one.

To some extent, the prediction models of Vanapalli et al. [8–10] can predict the unsaturated shear strength of the weakly expansive soil correctly. But there are certain differences between the predicted results and the measured ones. This is probably because that the SWRC has no clear residual zone in the model proposed by Vanapalli et al. [8], and therefore the residual water content in the SWRC is difficult to be determined. As far as the Oberg and Sallfors’ equations are concerned, as they are only suitable for sandy and muddy soils, it is normal to give unsatisfactory results for the tested soil.

In the prediction model of Khalili and Khabbaz [10], \( \chi = \left( \frac{u_a - u_e}{u_a - u_w} \right)^{0.55} \), where the index 0.55 was obtained by fitting the shear test results of 13 fine-grained soils and one kaolin soil. As the weakly expansive soil tested here has different mechanical properties, although the prediction model properly matches with the second experimental point (a suction of 200 kPa), the general trend of the predicted curve clearly diverges from the measured one.

Therefore, for the engineering purpose, the prediction equation should be carefully selected based on the soil properties when calculating the unsaturated shear strength of undisturbed expansive soil using its SWRC.

4 Conclusions

The water retention properties and shear strength behavior of an unsaturated weakly expansive soil were studied experimentally. Based on the results obtained, various models for prediction of unsaturated shear strength were evaluated. The following conclusions can be drawn from this study.

(1) The SWRC of the undisturbed expansive soil indicates that the air-entry suction and the residual degree of saturation of the tested soil are 106 kPa and 8%, respectively. The boundary effect zone and the transition zone can be identified on the desorption curve, but the residual zone is not clear.

(2) When the net confining pressure varies from 100 to 400 kPa, the shear behavior changes from strain-softening to strain-hardening. Moreover, within the low suction range, the suction mainly influences the effective cohesion \( c' \), whereas \( \phi' \) remains almost constant. The unsaturated shear strength changes linearly with suction; \( \phi' \) is a constant value of about 17.6°.

(3) Comparisons between the experimental results and the predictions show that the models proposed by Vanapalli et al. [8–10] can predict the unsaturated shear strength of the expansive soil satisfactorily. The prediction by Vanapalli’s model seems to be the best one since it is relatively close to the experimental results. It appears clearly that for the engineering purpose, the prediction equation should be carefully selected based on the soil properties when calculating the unsaturated shear strength of undisturbed expansive soil using its SWRC.

References


[10] Khalili N, Khabbaz M H. A unique relationship for \( \chi \) for the determination of the shear strength of unsaturated soils. Geotechnique,


