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Flavor violating Z' from $SO(10)$ SUSY GUT in High-Scale SUSY

Junji Hisano [a,b,c], Yu Muramatsu [b], Yuji Omura [a,*], Masato Yamanaka [a]

[a] Kobayashi-Maskawa Institute for the Origin of Particles and the Universe (KMI), Nagoya University, Nagoya 464-8602, Japan
[b] Department of Physics, Nagoya University, Nagoya 464-8602, Japan
[c] Kavli IPMU (WPI), University of Tokyo, Kashiwa, Chiba 277-8583, Japan

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ABSTRACT

We propose an $SO(10)$ supersymmetric grand unified theory (SUSY GUT), where the $SO(10)$ gauge symmetry breaks down to $SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1)_X$ at the GUT scale and $U(1)_X$ is radiatively broken at the SUSY-braking scale. In order to achieve the observed Higgs mass around 126 GeV and also to satisfy constraints on flavor- and/or CP-violating processes, we assume that the SUSY-breaking scale is $O(100)$ TeV, so that the $U(1)_X$ breaking scale is also $O(100)$ TeV. One big issue in the $SO(10)$ GUTs is how to realize realistic Yukawa couplings. In our model, not only **16**-dimensional but also **10**-dimensional matter fields are introduced to predict the observed fermion masses and mixings. The Standard-Model quarks and leptons are linear combinations of the **16**- and **10**-dimensional fields so that the $U(1)_X$ gauge interaction may be flavor-violating. We investigate the current constraints on the flavor-violating Z' interaction from the flavor physics and discuss prospects for future experiments.

1. Introduction

The Grand Unified Theories (GUTs) are longstanding hypotheses, and continue to fascinate us because of the excellent explanation of mysteries in the Standard Model (SM). The GUTs unify not only the gauge groups but also quarks and leptons, and reveal the origin of the structure of the SM, such as the hypercharge assignment for the SM particles.

The gauge groups in the SM are $SU(3)_c \times SU(2)_L \times U(1)_Y$ ($\equiv G_{SM}$). The minimal candidate for the unified gauge group is $SU(5)$, which was originally proposed by Georgi and Glashow [1]. There, quarks and leptons belong to **10**- and $\bar{\mathbf{5}}$-dimensional representations in $SU(5)$, and the SM Higgs doublet is embedded into **5**, introducing additional colored Higgs particle. One big issue is the unification of the SM gauge coupling constants, and it could be realized in the supersymmetric (SUSY) extension. It is well-known that the minimal $SU(5)$ SUSY GUT realizes the gauge coupling unification around 2×10^{16} GeV, if SUSY particle masses are around 1 TeV [2].

Another candidate for the unified gauge group would be $SO(10)$. It is non-minimal, but it would be an attractive extension because the $SO(10)$ GUT explains the anomaly-free conditions in the SM. Furthermore, all leptons and quarks, including the right-handed neutrinos, in one generation may belong to one **16**-dimensional representation in the minimal setup [3].

On the other hand, the GUTs face several problems, especially because of the experimental constraints. One stringent constraint is from nonobservation of proton decay [1,4]. While the GUT scale in the SUSY GUT may be high enough to suppress the proton decay induced by the so-called X-boson exchange, the dimension-five operator generated by the colored Higgs exchange is severely constrained. Another stringent constraint is from the observed fermion masses and mixings. The $SU(5)$ GUT predicts a common mass ratio of down-type quark and charged lepton in each generation. Furthermore, in the $SO(10)$ GUT, the up-type, down-type quarks, and charged lepton in each generation would have common mass ratios if the all matter fields in one generation are embedded in one **16**-dimensional representation. The predictions obviously conflict with the observation, and the modifications should be achieved by, for instance, higher-dimensional operators [5], additional Higgs fields [6] and additional matter fields [7].

In this letter, we propose an $SO(10)$ SUSY GUT model, where the realistic fermion masses and mixings may be achieved by introducing extra **10**-dimensional matter fields. The SM quarks and leptons come from **10**- and **16**-dimensional fields, and especially, the right-handed down-type quarks and left-handed leptons in the SM are given by the linear combinations of **10**- and

* Corresponding author.
E-mail addresses: hisano@eken.phys.nagoya-u.ac.jp (J. Hisano), mura@eken.phys.nagoya-u.ac.jp (Y. Muramatsu), yujiomur@eken.phys.nagoya-u.ac.jp (Y. Omura), yamanaka@eken.phys.nagoya-u.ac.jp (M. Yamanaka).

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Table 1
Charge assignment for matter fields, which belong to **16** and **10** representations in $SO(10)$ gauge symmetry. Charge assignment for G_{SM} is denoted as $(SU(3)_c, SU(2)_L, U(1)_Y)$. $U(1)_X$ gauge coupling constant is normalized as $g_X = g/\sqrt{40}$ at GUT scale, where g is $SO(10)$ gauge coupling constant.

16	Q_L	U_R^c	E_R^c	$\hat{L}_L$	$\hat{D}_R^c$	N_R^c
$SU(5)\times U(1)_X$		$(\mathbf{10}, -1)$		$(\bar{\mathbf{5}}, 3)$		$(\mathbf{1}, -5)$
G_{SM}	$(\mathbf{3}, \mathbf{2}, \frac{1}{6})$	$(\bar{\mathbf{3}}, \mathbf{1}, -\frac{2}{3})$	$(\mathbf{1}, \mathbf{1}, 1)$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$	$(\bar{\mathbf{3}}, \mathbf{1}, \frac{1}{3})$	$(\mathbf{1}, \mathbf{1}, 0)$

10	L'_L	D'^c_R	$\overline{L'}_L$	$\overline{D'^c_R}$
$SU(5)\times U(1)_X$	$(\bar{\mathbf{5}}, -2)$		$(\mathbf{5}, 2)$	
G_{SM}	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$	$(\bar{\mathbf{3}}, \mathbf{1}, \frac{1}{3})$	$(\mathbf{1}, \mathbf{2}, \frac{1}{2})$	$(\mathbf{3}, \mathbf{1}, -\frac{1}{3})$

16-dimensional fields. We assume that $SO(10)$ gauge symmetry breaks down to $G_{SM} \times U(1)_X$ around 10^{16} GeV according to the nonzero vacuum expectation values (VEVs) of $SO(10)$ adjoint fields. Thus, the low-energy effective theory is a $U(1)_X$ extension of the SUSY SM with extra matters. The additional gauge symmetry will survive up to the SUSY scale, but we could expect that it is radiatively broken, as the electroweak (EW) symmetry breaking in the minimal supersymmetry Standard Model (MSSM).

We assume that SUSY particles in the SUSY SM, except for gauginos, reside around 100 TeV, in order to realize the observed 126 GeV Higgs mass and also to satisfy constraints on flavor-and/or CP-violating processes. This type of setup is called the high-scale SUSY [8]. In the high-scale SUSY, the gauge coupling unification is rather improved when only the gaugino masses are around 1 TeV [9], and the dangerous dimension-five proton decay is suppressed [10]. On the other hand, since $\tan\beta$ (the ratio of the VEVs of the two Higgs doublets in the SUSY SM) is close to one, it is difficult to explain the large hierarchy between top and bottom quarks when all the matter fields are embedded into only **16** representational representations. In our model, the introduction of **10**-representational matter fields makes it possible to explain the large hierarchy. In the high-scale SUSY, the UV theory of the SM need not be the MSSM. The $U(1)_X$ extension of the SUSY SM with extra matters is an alternative model, motivated by the $SO(10)$ SUSY GUTs.

The mass of the Z' boson associated with the gauged $U(1)_X$ may be $O(100)$ TeV so that it may be viable in the searches for flavor violations. The right-handed down-type quarks and left-handed leptons in the SM are given by linear combinations of the parts of **10**- and **16**-dimensional fields. Thus, that generically leads flavor-violating Z' interaction and crucial promises against flavor experiments. We will see that tree-level Flavor Changing Neutral Currents (FCNC) induced by the Z' boson are generated and they largely contribute to the flavor violation processes: for instance, $\mu \to 3e$, μ–e conversion in nuclei, and $K^0 - \overline{K}^0$ and $B^0_{d/s} - \overline{B}^0_{d/s}$ mixings.

This paper is organized as follows. We introduce our setup of the $SO(10)$ SUSY GUT model in Section 2. We see not only how to break $SO(10)$, but also how to realize realistic fermion masses and mixings. The conventional seesaw mechanism, in which the Majorana masses for the right-handed neutrinos are much higher than the EW scale, could not work, since the $U(1)_X$ gauge symmetry forbids the Majorana masses. We show our solution according to the so-called inverted hierarchy [11] in Section 2.1. The small parameters could be controlled with the global $U(1)_{PQ}$ symmetry there. In Section 2.2, we discuss the tree-level FCNCs corresponding to the realistic fermion masses and mixings. Section 3 is devoted to the flavor physics induced by the Z' interaction. Section 4 is conclusion and discussion.

2. Setup of $SO(10)$ SUSY GUT

The $SO(10)$ gauge group has been considered to unify the three gauge symmetry in the SM. In the simple setup, the SM matter fields are also unified into **16**-dimensional representation in the each generation, and the number of Yukawa couplings for the fermions masses is less than in the SM. When the SM Higgs field belongs to **10**-dimensional field $\mathbf{10}_H$, the only Yukawa couplings are

$$W_{\text{min}} = h_{ij}\mathbf{16}_i\mathbf{16}_j\mathbf{10}_H \tag{1}$$

where $i, j = 1, 2, 3$ are defined. This assumption is too strict to explain the observed fermion masses and mixings, even if we include radiative corrections. The observed mass hierarchies are different in the up-type and down-type quarks, and the CKM mixing will be vanishing without other Yukawa couplings.

Now, we introduce a **10**-dimensional matter field in the each generation in addition to **16**-dimensional matter fields. Three $SO(10)$-singlet matter fields S_i are also introduced to achieve the realistic masses of neutrinos. The matter fields $\mathbf{10}_i$ and $\mathbf{16}_i$ are decomposed as the ones in Table 1. For convenience, the assignment of $SU(5) \times U(1)_X$ is also shown in Table 1.

Let us show the superpotential relevant to the Yukawa couplings for the matter fields in our model;

$$\begin{aligned} W_Y = {} & h_{ij}\mathbf{16}_i\mathbf{16}_j\mathbf{10}_H + f_{ij}\mathbf{16}_i\overline{\mathbf{16}}_H S_j + g_{ij}\mathbf{10}_i\mathbf{16}_j\mathbf{16}_H \\ & + \mu_{BL}\mathbf{16}_H\overline{\mathbf{16}}_H + \mu_H\mathbf{10}_H\mathbf{10}_H + \mu_{10\,ij}\mathbf{10}_i\mathbf{10}_j \\ & + \mu_{S\,ij}S_i S_j. \end{aligned} \tag{2}$$

Here, the **16**- and $\overline{\mathbf{16}}$-dimensional Higgs fields $\mathbf{16}_H$ and $\overline{\mathbf{16}}_H$ are introduced to break the $U(1)_X$ gauge symmetry in $SO(10)$. We assume that the mass parameters μ_{BL}, μ_{10} and μ_H are around SUSY scale (m_{SUSY}) and μ_S is much smaller to realize the tiny neutrino masses. It may be important to pursue the origin of the mass scales. In Section 2.1, we show that the global $U(1)_{PQ}$ symmetry may control their mass scales.

We assume that two $SO(10)$ adjoint Higgs fields, $\mathbf{45}_H$ and $\mathbf{45}'_H$, develop nonzero VEVs so that the $SO(10)$ gauge symmetry breaks down to $G_{SM} \times U(1)_X$ at the GUT scale [12]. The low-energy effective theory is the $U(1)_X$ extension of the SUSY SM with **10**- and **16**-dimensional matter fields. The G_{SM}-singlet fields charged under $U(1)_X$, Φ and $\overline{\Phi}$, which are originated from $\mathbf{16}_H$ and $\overline{\mathbf{16}}_H$, should be included there. Φ and $\overline{\Phi}$ would develop the nonzero VEVs as $\langle\Phi\rangle = v_\Phi$ and $\langle\overline{\Phi}\rangle = \overline{v_\Phi}$ around m_{SUSY}, and the $U(1)_X$ symmetry is spontaneously broken. For simplicity, we assume that the other fields in $\mathbf{16}_H$ and $\overline{\mathbf{16}}_H$ have masses at the GUT scale. If they stay at the low energy spectrum, the gauge coupling constants at the GUT scale is not perturbative.

The superpotential in the $U(1)_X$ extension of the SUSY SM is given as follows,

$$W_Y^{eff} = h_{u\,ij} Q_{L\,i} U^c_{R\,j} H_u + (h_{u\,ij} + \epsilon_{d\,ij}) Q_{L\,i} \hat{D}^c_{R\,j} H_d + (h_{u\,ij} + \epsilon_{e\,ij}) \hat{L}_{L\,i} E^c_{R\,j} H_d + g_{ij} \Phi (\overline{D'^c_{R\,i}} \hat{D}^c_{R\,j} + \overline{L'_{L\,i}} \hat{L}_{L\,j}) + \mu_{10\,ij} (\overline{D'^c_{R\,i}} D'^c_{R\,j} + \overline{L'_{L\,i}} L'_{L\,j}) + h_{ij} \hat{L}_{L\,i} N^c_{R\,j} H_u + f_{ij} \overline{\Phi} N^c_{R\,i} S_j + \mu_{S\,ij} S_i S_j + \mu_{BL} \overline{\Phi} \Phi + \mu_H H_u H_d. \quad (3)$$

The effective Yukawa couplings will be deviated from the ones in Eq. (2), because of the higher-order terms involving $\mathbf{45}_H$ and $\mathbf{45}'_H$.[1] h_u is Yukawa coupling for up-type quark including effect of higher-dimensional operators. ϵ_d and ϵ_e describe the size of higher-dimensional operators for the down-type quarks and charged leptons, which suppressed by $\langle \mathbf{45}_H \rangle / \Lambda$ and $\langle \mathbf{45}'_H \rangle / \Lambda$.

After the $U(1)_X$ symmetry breaking, the chiral superfields $\hat{D}^c_{R\,i}$ and $D'^c_{R\,i}$ ($\hat{L}_{L\,i}$ and $L'_{L\,i}$) mix each other, and we find the massless modes which correspond to the SM right-handed down-type quarks and left-handed leptons. $g_{ij} v_\Phi$ and $\mu_{10\,ij}$ give the mass mixing between $\hat{D}^c_{R\,i}$ and $D'^c_{R\,i}$ ($\hat{L}_{L\,i}$ and $L'_{L\,i}$). Eventually, the relevant Yukawa couplings for quarks and charged leptons are described as

$$W_Y^{SSM} = h_{u\,ij} Q_{L\,i} U^c_{R\,j} H_u + Y_{d\,ij} Q_{L\,i} D^c_{R\,j} H_d + Y_{e\,ij} L_{L\,i} E^c_{R\,j} H_d + \widetilde{\mu}_{ij} (\overline{D^c_{R\,hi}} D^c_{R\,h\,j} + \overline{L_{L\,hi}} L_{L\,h\,j}). \quad (4)$$

$D^c_{R\,i}$, $D^c_{R\,h\,i}$, $L_{L\,i}$ and $L_{L\,h\,i}$ are the chiral superfields of right-handed down-type quarks and left-handed leptons in the mass bases defined by

$$\begin{pmatrix} \hat{\psi} \\ \psi' \end{pmatrix} = U_\psi \begin{pmatrix} \psi \\ \psi_h \end{pmatrix} = \begin{pmatrix} \hat{U}_\psi & \hat{U}_{\psi\,h} \\ \hat{U}'_\psi & \hat{U}'_{\psi\,h} \end{pmatrix} \begin{pmatrix} \psi \\ \psi_h \end{pmatrix}, \quad (5)$$

where ψ denotes D^c_R and L_L. ψ and ψ_h are massless modes which correspond to the SM matters and the superheavy modes with masses $O(m_{SUSY})$, respectively. U_ψ is the 6×6 unitary matrix, and $\hat{U}_\psi$, $\hat{U}_{\psi\,h}$, $\hat{U}'_\psi$ and $\hat{U}'_{\psi\,h}$ satisfy not only the unitarity condition for U_ψ but also the following relations,

$$0 = g_{ik} v_\Phi (\hat{U}_\psi)_{kj} + \mu_{10\,ik} (\hat{U}'_\psi)_{kj}, \quad (6)$$

$$\widetilde{\mu}_{ij} = g_{ik} v_\Phi (\hat{U}_{\psi\,h})_{kj} + \mu_{10\,ik} (\hat{U}'_{\psi\,h})_{kj}. \quad (7)$$

Using the couplings in Eq. (3), the Yukawa coupling constants for the SM down-type quarks and charged leptons in Eq. (4) are described as

$$(Y_d)_{ij} = (h_{u\,ik} + \epsilon_{d\,ik}) (\hat{U}_{D^c_R})_{kj}, \; (Y_e)_{ij} = (\hat{U}^T_{L_L})_{ik} (h_{u\,kj} + \epsilon_{e\,kj}). \quad (8)$$

In general, the up-type quark Yukawa coupling constants $h_{u\,ij}$ are given by

$$h_{u\,ij} = \frac{m_{u\,i}}{v \sin\beta} \delta_{ij}. \quad (9)$$

$v \sin\beta$ ($v \cos\beta$) is the VEV of the neutral component of H_u (H_d) and $m_{u\,i}$ are the up-type quark masses. We define the diagonalizing matrices V_{CKM} and V_{e_R} for $(Y_d)_{ij}$ and $(Y_e)_{ij}$ as below:

$$(Y_d)_{ij} = \frac{1}{v \cos\beta} (V^*_{CKM})_{ij} m_{d\,j}, \; (Y^T_e)_{ij} = \frac{1}{v \cos\beta} (V^*_{e_R})_{ij} m_{e\,j}, \quad (10)$$

where $m_{d\,i}$ and $m_{e\,i}$ are the down-type quark and the charged lepton masses. Note that we take the flavor basis that the right-handed down-type quarks and left-handed charged leptons are in the mass eigenstates. Then V_{CKM} is the CKM matrix and V_{e_R} satisfies $V_{e_R} = V_{CKM}$ in the $SU(5)$ limit.

The size of higher-dimensional terms is depicted by ϵ_d and ϵ_e and expected to be small, compared to the third generation, $h_{u\,33} = m_t / (v \sin\beta)$. According to Eq. (8), $(\hat{U}_\psi)_{ij}$ could be described by the observables as,

$$(m_{u\,i} \delta_{ik} + \epsilon_{d\,ik} v \sin\beta) (\hat{U}_{D^c_R})_{kj} = \tan\beta (V^*_{CKM})_{ij} m_{d\,j},$$

$$\left(m_{u\,i} \delta_{ik} + \epsilon^T_{e\,ki} v \sin\beta \right) (\hat{U}_{L_L})_{kj} = \tan\beta (V^*_{e_R})_{ij} m_{e\,j}. \quad (11)$$

If $\epsilon_{d\,11} v \sin\beta$ is sufficiently smaller than m_u, the $(1, j)$ elements of $\hat{U}_{D^c_R}$ are too large to satisfy the unitary condition for U_ψ. In order to achieve the consistency, the extra term $\epsilon_{d\,11} v \sin\beta$ should be larger than $O(\tan\beta (V_{CKM})_{13} m_b)$.

2.1. Neutrino mass

Let us briefly mention the neutrino sector in our model. W_Y^{eff} in Eq. (3) includes neutral particles after the EW symmetry breaking. They reside in the neutral components of $SU(2)_L$ doublets $\{\hat{L}_{L\,i}, L'_{L\,i}, \overline{L'_{L\,i}}\}$ and the singlets $\{N_{R\,i}, S_i\}$. Let us decompose $\hat{L}_{L\,i}$, $L'_{L\,i}$ and $\overline{L'_{L\,i}}$ as the charged and neutral ones: $\hat{L}^T_{L\,i} = (\hat{\nu}_{L\,i}, \hat{E}_{L\,i})$, $L'^T_{L\,i} = (\nu'_{L\,i}, E'_{L\,i})$ and $\overline{L'_L}^T_i = (\overline{\nu'_{L\,i}}, \overline{E'_{L\,i}})$. The mass matrix for the neutral particles in the basis of $(\hat{\nu}_{L\,i}, N^c_{R\,i}, \nu'_{L\,i}, \overline{\nu'_{L\,i}}, S_i)$ is

$$M_\nu = \begin{pmatrix} 0 & h_{ij} v \sin\beta & 0 & g_{ij} v_\Phi & 0 \\ h_{ij} v \sin\beta & 0 & 0 & 0 & f_{ij} \overline{v_\Phi} \\ 0 & 0 & 0 & \mu_{10\,ij} & 0 \\ g_{ij} v_\Phi & 0 & \mu_{10\,ij} & 0 & 0 \\ 0 & f_{ij} \overline{v_\Phi} & 0 & 0 & \mu_{S\,ij} \end{pmatrix}. \quad (12)$$

When we admit the large hierarchy between μ_S and the other elements, the neutrino mass matrix (m_ν) is given by

$$(m_\nu)_{ij} = (h f^{-1} \mu_S f^{-1} h)_{ij} \left(\frac{v \sin\beta}{\overline{v_\Phi}} \right)^2, \quad (13)$$

following Ref. [11]. For instance, $\overline{v_\Phi} = O(100)$ TeV and $v \sin\beta = O(100)$ GeV lead $O(1)$-eV neutrino masses, if μ_S is $O(1)$ MeV and h and f are $O(1)$. The masses of the other neutral elements are $O(m_{SUSY})$, and the phenomenology has been well investigated in Ref. [11].

One may wonder why μ_S is so tiny and $\mu_{10,BL,H}$ are $O(m_{SUSY})$. We show one mechanism to explain the large mass hierarchy. In order to induce the dimensional parameters in Eq. (2) effectively, let us assign the global $U(1)_{PQ}$ symmetry to the matter and Higgs fields as in Table 2. The global $U(1)_{PQ}$ symmetry, under which the SM fields are charged anomalously, has been proposed motivated by the strong CP problem [13]. We introduce $SO(10)$-singlet fields, P and T, whose $U(1)_{PQ}$ charges are fixed to allow the $c_{PQ} P^3 T$ term in the superpotential. Assuming canonical Käller potential and their soft SUSY breaking terms, the scale potential for P and T is derived from the superpotential as

$$V_{PQ} = \left| \frac{c_{PQ}}{\Lambda} P^3 \right|^2 + \left| \frac{c_{PQ}}{\Lambda} P^2 T \right|^2 + m_P^2 |P|^2 + m_T^2 |T|^2. \quad (14)$$

m_P^2 and m_T^2 are the soft SUSY breaking masses, and they could be estimated as m^2_{SUSY}. The mass squared would be driven to the

Table 2
Charge assignment of global $U(1)_{PQ}$ symmetry.

	16_i	10_H	16_H	$\overline{16}_H$	10_i	S_i	P	T
$SO(10)$	**16**	**10**	**16**	$\overline{\mathbf{16}}$	**10**	**1**	**1**	**1**
$U(1)_{PQ}$	1	−2	−1/3	5/3	−2/3	−8/3	−2/3	2

[1] In general, the other parameters such as μ_S and μ_{10} would be effectively modified by the higher-dimensional operators as well. We disregard these extra corrections to the parameters because they are not essential in this discussion.

negative value due to the radiative corrections, so that the negative mass squared leads the nonzero VEVs of P and T,

$$\langle T\rangle \sim \langle P\rangle \sim \sqrt{\Lambda |m_{SUSY}|}, \tag{15}$$

and breaks $U(1)_{PQ}$ spontaneously. This leads a light scalar, so-called axion, corresponding to the Nambu–Goldstone boson. As discussed in Ref. [14], it is favorable that the $U(1)_{PQ}$ symmetry breaking scale is around 10^{12} GeV, to explain the correct relic density of dark matter. That is, Λ should be almost the Planck scale ($O(10^{19})$ GeV), when m_{SUSY} is $O(100)$ TeV, for instance.

On the other hand, the $U(1)_{PQ}$ charge assignment for the other chiral superfields forbids dimensional parameters like μ_S and $\mu_{10,BL,H}$. Using higher dimensional parameters, μ_S and $\mu_{10,BL,H}$ are effectively generated after $U(1)_{PQ}$ breaking:

$$\mu_{10} = \frac{\langle P\rangle\langle T\rangle}{\Lambda},\ \mu_{BL} = \frac{\langle P\rangle^2}{\Lambda},\ \mu_H = \frac{\langle T\rangle^2}{\Lambda},\ \mu_S = \frac{\langle P\rangle\langle T\rangle^3}{\Lambda^3}, \tag{16}$$

ignoring the dimensionless couplings in front of the higher-order couplings. The above estimation tells that $\mu_{10,BL,H} = O(m_{SUSY})$ and $\mu_S = m_{SUSY} \times O(m_{SUSY}/\Lambda)$. If $m_{SUSY} \ll \Lambda$ is satisfied, very small μ_S, compared to m_{SUSY}, is predicted, and could realize the observed light neutrino masses, as we discussed above.

2.2. Flavor violating gauge interaction

As we see above, the SM right-handed down-type quarks and left-handed leptons are given by the linear combinations of quarks and leptons in **10**- and **16**-dimensional matter fields, respectively. Since the fields in **10** and **16** representations carry different $U(1)_X$ charges, the SM fields may have flavor-dependent $U(1)_X$ interaction.

Let us see it more explicitly. The $U(1)_X$ gauge interactions of right-handed down-type quarks and left-handed leptons are described in the interaction basis as

$$\mathcal{L}_g = -ig_X(3\bar{\hat{\varphi}}_i \not{Z}' \hat{\varphi}_i - 2\overline{\varphi'}_i \not{Z}' \varphi'_i), \tag{17}$$

where the factors 3 and -2 are $U(1)_X$ charges for the fermionic components $\hat{\varphi}_i$ and φ'_i of the chiral superfields $\hat{\psi}_i$ and ψ'_i. Z' is the $U(1)_X$ gauge boson and g_X is defined as $g_X = g/\sqrt{40}$ at GUT scale, where g is the $SO(10)$ gauge coupling constant. We have obtained the mass eigenstates for the fermions in Eqs. (5) and (8). Using the unitary matrix U_ψ, we define the flavor-violating couplings A^φ_{ij} for the SM fermions as

$$\mathcal{L}_g = -ig_X\overline{\varphi}_i\left(5(\hat{U}^\dagger_\psi \hat{U}_\psi)_{ij} - 2\delta_{ij}\right)\not{Z}'\varphi_j \equiv -ig_X A^\varphi_{ij}\overline{\varphi}_i \not{Z}'\varphi_j, \tag{18}$$

where φ is the fermion component of the chiral superfield ψ in the mass base and denotes right-handed down-type quark (d^c_R) and left-handed lepton (l_L).

Here we discuss the size of flavor violating couplings A^φ_{ij}. According to Eq. (11), $(\hat{U}_{D^c_R})_{ij}$ and $(\hat{U}_{L_L})_{ij}$ are depicted by the observables in the SM. The flavor violating couplings A^φ_{ij} depend on the parameters, ϵ_d and ϵ_e. They are required to satisfy the unitary condition for U_ψ, as discussed in Eqs. (11). In other words, they should be sizable in some elements, compared to $h^u_{ij} = m_{u\,i}/v\cos\delta_{ij}$, in order to break the GUT relation and to realize realistic mass matrices. Assuming $\epsilon_{d\,ij} = \epsilon_i\delta_{ij}$, at least $\epsilon_1 \gtrsim O(10^{-5})$ is required to compensate for the small up quark mass.

Let us show one example to demonstrate the size of the flavor violating coupling $A^{d^c_R}_{ij}$. Assuming $\epsilon_1 \gtrsim O(10^{-5})$ and $\epsilon_2 = \epsilon_3 = 0$, $A^{d^c_R}_{ij}$ is approximately estimated as

$$A^{d^c_R}_{ij} \approx \frac{5\tan^2\beta m_{d\,i} m_{d\,j}}{|\epsilon_1 v\sin\beta|^2}(V_{CKM})_{1i}(V^*_{CKM})_{1j} - 2\delta_{ij}. \tag{19}$$

Setting the extra parameter to $\epsilon_1 = 5\times 10^{-4}$, $A^{d^c_R}_{ij}$ is estimated as

$$\left(A^{d^c_R}_{ij}\right) \approx \begin{pmatrix} -1.9 & 0.6 & 0.3 \\ 0.6 & 1.6 & 2.2 \\ 0.3 & 2.2 & -0.3 \end{pmatrix}. \tag{20}$$

We find that all elements of the flavor violating couplings are $O(1)$, so that we need careful analyses of their contributions to flavor physics, even if the Z' boson is quite heavy.

Note that the alignment of $A^{l_L}_{ij}$ differs from the one of $A^{d^c_R}_{ij}$, because of the different mass spectrum between charged leptons and down-type quarks. In any case, however, the size of $A^{l_L}_{ij}$ would be also $O(1)$, because of the small electron mass. The detail analysis on the relation between the FCNCs and the realistic mass spectrum will be given in Ref. [15]. In Section 3, we introduce the flavor constraints relevant to our model and scan the current experimental bounds and future prospects in flavor physics.

2.3. Gauge coupling unification

Before phenomenology, let us briefly comment on the gauge coupling unification and the predicted Z' coupling (g_X). As well-known, the MSSM miraculously achieves the unification of the three SM gauge couplings, if at least gaugino masses are close to the EW scale. We assume the SUSY mass spectrum, where gauginos reside around the TeV-scale and the other SUSY particle masses are around 100 TeV. It is shown in Ref. [9] that the unification of the gauge coupling constants is improved compared with the MSSM with the SUSY particle masses $O(1)$ TeV.

Once we determine the $SO(10)$ gauge coupling at the GUT scale according to the gauge coupling unification, the $U(1)_X$ gauge coupling $g_X(\mu)$ is derived with the renormalization group equation at the one-loop level as

$$4\pi\alpha_X^{-1}(\mu) = 4\pi\alpha_G^{-1}\times 40 + b_X \ln\left(\frac{\Lambda_G^2}{\mu^2}\right), \tag{21}$$

where $\alpha_X = g_X^2/(4\pi)$ and $\alpha_G = g^2(\Lambda_G)/(4\pi)$ are defined and Λ_G is the unification scale. b_X is fixed by the number of $U(1)_X$-charged particles from μ to Λ_G. In our scenario, right-handed neutrinos, additional three **10**s of $SO(10)$, and the $U(1)_X$ breaking Higgs fields as well as MSSM particles contribute to b_X between m_{SUSY} and Λ_G, so that they lead $b_X = 426$. At the scale $\mu = 100$ TeV, g_X is estimated as

$$g_X(100\ \text{TeV}) = 0.073, \tag{22}$$

where the GUT scale and the gauge coupling with $m_{SUSY} = 100$ TeV are given by

$$\Lambda_G = 8.7\times 10^{15}\ \text{GeV},\ \alpha_G = 0.062. \tag{23}$$

Note that the introduction of additional matter fields increases the gauge coupling constant at the GUT scale α_G. Furthermore, heavier gaugino masses than the EW scale decrease the GUT scale Λ_G. This means that the proton decay rate may be enhanced in our model [9,16], and could be tested at the future proton decay searches.

3. Flavor physics

As discussed in Subsection 2.2, the tree-level FCNCs involving the Z' boson may be promised in our model. The flavor changing couplings denoted by A^{φ}_{ij} could be $O(1)$ in the all elements, as we see in Eq. (20). Here, we sketch the relevant constraints on the flavor-violating Z' interactions and give prospects for future experiments.

In our model, the SUSY SM Higgs doublets are charged under $U(1)_X$, so that their nonzero VEVs contribute to the Z' mass ($m_{Z'}$) as well as the SM gauge bosons. The $U(1)_X$ charges of Higgs doublets are ± 2 respectively, and then the mass mixing between Z and Z' is generated by the VEVs as well. The mixing angle between Z and Z' is approximately estimated as

$$\sin\theta \simeq 4\frac{g_X}{g_Z}\frac{m_Z^2}{m_{Z'}^2}, \tag{24}$$

where g_Z is the gauge coupling of Z boson and m_Z is the Z boson mass. $\sin\theta$ is about 3.4×10^{-7} when Z' mass and coupling are fixed at $m_{Z'} = 100$ TeV and $g_X = 0.073$. Since the mixing is quite small as long as the Z' mass is $O(100)$ TeV, we treat with Z and Z' as the fields in the mass basis and discuss the mixing effect up to $O(\theta^2)$.

The gauge interactions of Z and Z' and SM fermions are given by

$$\mathcal{L} = -i\left(g_Z\cos\theta J^{\mu}_{\rm SM} + g_X\sin\theta J^{\mu}_{\rm GUT}\right)Z_\mu - i\left(g_X\cos\theta J^{\mu}_{\rm GUT} - g_Z\sin\theta J^{\mu}_{\rm SM}\right)Z'_\mu, \tag{25}$$

where $J^{\mu}_{\rm SM}$ is the SM weak neutral current, and $J^{\mu}_{\rm GUT}$ is defined by

$$J^{\mu}_{\rm GUT} = A^{l_L}_{ij}\overline{l}_{Li}\gamma^\mu l_{Lj} - A^{d^c_R}_{ij}\overline{d}_{Ri}\gamma^\mu d_{Rj} + \overline{e}_{Ri}\gamma^\mu e_{Ri} - \overline{q}_{Li}\gamma^\mu q_{Li} + \overline{u}_{Ri}\gamma^\mu u_{Ri}. \tag{26}$$

The fermions in $J^{\mu}_{\rm GUT}$ describe the fermionic components of the MSSM chiral superfields in the mass base denoted by the capital letters. The neutral current $J^{\mu}_{\rm GUT}$ may significantly contribute to flavor violating processes: $B^0_{d/s}$–$\overline{B}^0_{d/s}$ and K^0–$\overline{K}^0$ mixings, flavor-violating decays, and μ–e conversion in nuclei. Below, we summarize the constraints relevant to the Z' interaction, and discuss the predictions in flavor physics. Note that we ignore contribution from SUSY flavor violating processes, because the sfermion masses are $O(100)$ TeV.

3.1. Flavor violating decays of leptons

First, let us discuss the contributions to flavor violating decays of leptons. There are two types of flavor violating decays in the presence of Z' FCNCs: one is three-body flavor violating decays $l_j \to l_i l_k l_k$ and the other is radiative flavor violating decays $l_j \to l_i\gamma$. With the Z' FCNCs, the three-body flavor violating decays occur at the tree level, while the radiative flavor violating decays occur at the loop level. The radiative flavor violating decays have smaller rates by $O(10^{-3})$ than the tree-level decays. If flavor violating interactions stem from both left- and right-handed lepton (quark) sector, there might be a strong enhancement in radiative flavor violating decays via a chirality flip on an internal heavy fermion [17]. In our model, however, there exists no such an enhancement because only left-handed lepton (right-handed quark) have the flavor violating interactions. Hence we focus on the three-body flavor violating decays.

Let us discuss the $\mu \to 3e$ process. The current upper bound on the branching ratio of $\mu \to 3e$ is 1.0×10^{-12} [18] and future experimental limit is expected to be 1.0×10^{-16} [19]. In this model the branching ratio of $\mu \to 3e$ is evaluated as follows,

$$\mathrm{BR}(\mu\to 3e) = 1.1\times 10^{-15}\left(\frac{g_X}{0.073}\right)^4\left(\frac{100\ \mathrm{TeV}}{m_{Z'}}\right)^4\left|A^{l_L}_{12}\right|^2 \times\left\{1+0.63\left|1-0.93A^{l_L}_{11}\right|^2\right\}. \tag{27}$$

This is below the current experimental bound as long as $m_{Z'}$ is $O(100)$ TeV. It is also important to emphasize that BR($\mu \to 3e$) in our scenario has an additive structure in last bracket, and our prediction may yield to the stringent bound. If we assume $m_{Z'} = 100$ TeV and $A^{l_L}_{11} = -2$, the Mu3e experiment will cover $\left|A^{l_L}_{12}\right| \lesssim 0.1$.

We also evaluate the branching ratios of other lepton flavor violating decays, and we find that they are also much below the current experimental upper bounds.

3.2. μ–e conversion in nuclei

The flavor violating coupling $A^{l_L}_{12}$ also gives rise to the μ–e conversion process. The SINDRUM-II experiment, which searched for the μ–e conversion signal with the Au target, gave the upper limit on the branching ratio: BR($\mu^-\mathrm{Au}\to e^-\mathrm{Au}$) $< 7\times 10^{-13}$ [20]. The DeeMe [21] and the COMET-I [22] will launch soon and they aim to reach to $O(10^{-15})$ for the branching ratio with different targets. Furthermore, COMET-II and Mu2e [23] are planed to improve the sensitivity up to $O(10^{-17})$.[2]

In our model, the branching ratio for the Au target is predicted as [24]

$$\mathrm{BR}(\mu^-\mathrm{Au}\to e^-\mathrm{Au}) = 2.2\times 10^{-13}\left(\frac{g_X}{0.073}\right)^4\left(\frac{100\ \mathrm{TeV}}{m_{Z'}}\right)^4 \times\left(A^{l_L}_{12}\right)^2\left|1+0.58A^{d^c_R}_{11}\right|^2, \tag{28}$$

which is close to the current upper bound at the SINDRUM-II. The branching ratio for the Al target, which is a candidate target of COMET, Mu2e, and PRISM experiments, is evaluated as

$$\mathrm{BR}(\mu^-\mathrm{Al}\to e^-\mathrm{Al}) = 6.3\times 10^{-14}\left(\frac{g_X}{0.073}\right)^4\left(\frac{100\ \mathrm{TeV}}{m_{Z'}}\right)^4 \times\left(A^{l_L}_{12}\right)^2\left|1+0.61A^{d^c_R}_{11}\right|^2. \tag{29}$$

The branching ratios for the other materials could be estimated as $O(10^{-13})$ as well, so that we expect that our model could be proved in the future experiments.

3.3. Neutral meson mixing

The Z' FCNCs contribute to the mass splitting and CP violation in neutral meson systems. The UTfit Collaboration analyzes the experimentally allowed ranges for the effective couplings of 4-Fermi interactions [25]. We obtain the limits on the Z' interaction as follows:

$$-9.8\times 10^{-3} < \left(\frac{g_X}{0.073}\right)^2\left(\frac{100\ \mathrm{TeV}}{m_{Z'}}\right)^2\mathrm{Im}[(A^{d^c_R}_{12})^2] < 1.6\times 10^{-2}, \tag{30}$$

$$\left(\frac{g_X}{0.073}\right)^2\left(\frac{100\ \mathrm{TeV}}{m_{Z'}}\right)^2\left|\mathrm{Re}[(A^{d^c_R}_{12})^2]\right| < 3.4, \tag{31}$$

[2] It is discussed that the sensitivity might be improved to $O(10^{-(18-19)})$ in the PRISM experiment [22].

$$\left(\frac{g_X}{0.073}\right)^2 \left(\frac{100\ \text{TeV}}{m_{Z'}}\right)^2 \left|(A_{13}^{d_R^c})^2\right| < 81, \tag{32}$$

$$\left(\frac{g_X}{0.073}\right)^2 \left(\frac{100\ \text{TeV}}{m_{Z'}}\right)^2 \left|(A_{23}^{d_R^c})^2\right| < 3.9 \times 10^3. \tag{33}$$

The measurement of K^0–$\overline{K}^0$ oscillation is a strong probe on both real and imaginary part of $(A_{12}^{d_R^c})^2$. Especially, the CP violation gives a sever constraint on the FCNC as we see in Eq. (30), so that the Z' mass has to be heavier than a few PeV, if $A_{12}^{d_R^c}$ possesses $O(1)$ CP phase.

4. Conclusion and discussion

We have proposed an $SO(10)$ SUSY GUT, where the $SO(10)$ gauge symmetry breaks down to $SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1)_X$ at the GUT scale and $U(1)_X$ is radiatively broken at the SUSY-breaking scale. In order to achieve the observed Higgs mass around 126 GeV and also to satisfy constraints on flavor-and/or CP-violating processes, we assume that the SUSY-breaking scale is $O(100)$ TeV, so that the $U(1)_X$ breaking scale is also $O(100)$ TeV. In order to realize realistic Yukawa couplings, not only **16**-dimensional but also **10**-dimensional matter fields are introduced. The SM quarks and leptons are linear combinations of the **16**- and **10**-dimensional fields so that the $U(1)_X$ gauge interaction may be flavor violating. We investigate the current constraints on the flavor violating Z' interaction from the flavor physics and discuss prospects for future experiments. Our model could be tested in the flavor experiments, especially searches for the μ–e conversion processes, even if the Z' mass is $O(100)$ TeV.

In this paper, we did not mention the GUT mass hierarchy problem such as the doublet–triplet splitting problem. In fact, there is another mass hierarchy between the singlet of $\mathbf{16}_H$ and the other components of $\mathbf{16}_H$ in our model. The Z' mass is given by the VEV of the singlet, while other components reside around the GUT scale. We need more careful study on physics at the GUT scale to complete our discussion.

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