Quantum birth of a hot universe

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Abstract

We consider the quantum birth of a hot FRW universe from a vacuum-dominated quantum fluctuation with admixture of radiation and strings which corresponds to quantum tunnelling from a discrete energy level with a non-zero temperature. The presence of strings with the equation of state $p = -\varepsilon/3$ mimics a positive curvature term which makes it possible, in the case of a negative deficit angle, the quantum birth of an open and a flat universe. In the pre-de-Sitter domain radiation energy levels are quantized. We calculate the temperature spectrum and estimate the range of the model parameters restricting temperature fluctuations by the observational constraint on the CMB anisotropy. For the GUT scale of initial de Sitter vacuum the lower limit on temperature at the start of classical evolution is close to the values as predicted by reheating theories, while the upper limit is far from the threshold for a monopole rest mass.

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1. Introduction

Quantum cosmology treats quantum-mechanically the universe as a whole and describes it by a wave function ψ (for review see [1]). The full formalism of quantum geometrodynamics was introduced in 1967 by DeWitt and applied to a dust-filled closed Friedmann–Robertson–Walker (FRW) universe with a curvature-generated potential to find a discrete system of energy levels [2]. In 1969 Misner extended this approach to anisotropic cosmological models [3].

DeWitt has calculated the energy levels in the case of zero cosmological term and with the boundary condition $\psi(0) = 0$, which corresponds to quantization in the well with infinite walls. In 1972 Kalinin and Melnikov considered the FRW closed model with a non-zero cosmological term $\Lambda g_{\mu\nu}$, and found that adding $\Lambda g_{\mu\nu}$ results in transformation of an infinite well into a finite barrier [4].

A year later Fomin [5] and Tryon [6] put forward the idea that a closed universe can be born as a quantum object from nothing due to the uncertainty principle. In 1975 a nonsingular model was proposed for a FRW universe arising from a quantum fluctuation in de Sitter vacuum [7]. A more detailed consideration of the origin of a universe in the quantum tunnelling event has been done in the late 70s and early 80s

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[8–11]. The possibility of multiple birth of causally disconnected universes from de Sitter background noticed in [7], was investigated by Gott III for the case of an open FRW universe [13].

In the framework of the standard scenario, the quantum birth of the universe is followed by decay of the de Sitter vacuum ultimately resulting in a hot expanding universe [7,12,14]. The hot model has been proved by the discovery of the Cosmic Microwave Background (CMB) [15], first predicted by Gamow [16] who was also the author of the tunnel effect in quantum mechanics [17] basic for the quantum tunnelling of a universe.

The wave function of the universe satisfies the Wheeler–DeWitt equation [2,18]

$$\widehat{H}\psi = 0 \tag{1}$$

analogous to the Schrödinger equation. To put a universe into the quantum mechanical context, one has to specify boundary conditions for the wave function ψ. In quantum mechanics the boundary conditions are related to the exterior of an isolated quantum system. In case of a universe there is no exterior, and the boundary conditions must be formulated as an independent physical law [19]. This question has been debated in the literature for about 15 years. The recent summary of these debates can be found in [19]. At present there exist three approaches to imposing boundary conditions on the wave function of the universe: the Hartle–Hawking wave function [20], the Vilenkin (tunnelling) wave function [21], and the Linde wave function [22].

The birth of a closed world from nothing (favored by that its total energy is zero [5,6]) starts from arising of a quantum fluctuation, and the probability of tunnelling describes its quantum growth on the way to the classically permitted region beyond the barrier confined by the values of the scale factor $a = 0$ and $a = a_0$, which implies that the eigenvalue of the Wheeler–DeWitt operator is fixed at the energy value $E = 0$ [21].

In this Letter we address the question of quantum birth of a universe with non-zero temperature. We apply the approach proposed by Vilenkin for a quantum birth of a universe from nothing [21] to the case of quantum birth from a state with non-zero quantized energy.

In the presence of radiation in an initial fluctuation, its energy density in the quantized Friedmann equation

written in terms of conformal time, plays the role of an energy E in the Wheeler–DeWitt equation [19,23,24].

Quantization of energy levels in the conformal time has been investigated by Kuzmichev for the case of a closed FRW universe filled with a scalar field and radiation and considered as a quantum system in the curvature generated well [25]. This model describes the evolution of the universe as a succession of transitions to progressively higher energy levels in the well, so that the presently observable Universe is considered as the quantum system in a highly excited state in accordance with the basic idea suggested by Hartle and Hawking in 1983 [20].

In the present Letter we consider the quantum birth of a universe from a vacuum-dominated quantum fluctuation with admixture of radiation and strings or some other quintessence with the equation of state $p = -\varepsilon/3$. This corresponds to quantum birth of a closed, flat or open universe by tunnelling from a discrete energy level with a non-zero temperature.

In the literature the quantum birth of an open and flat universes has been typically considered in the context of anti-de-Sitter space–time [26,27]. In our model the nonzero probability of quantum birth in this case is related to the presence of strings with a negative deficit angle which mimics the curvature term in producing a potential appropriate for quantum tunnelling [28,29].

2. Model

The FRW quantum universe is described by the minisuperspace model with a single degree of freedom, and the Wheeler–DeWitt equation reads [2,10]

$$\frac{d^2\psi}{da^2} - V(a)\psi = 0, \tag{2}$$

where

$$V(a) = \frac{1}{l_{\text{Pl}}^4}\left(ka^2 - \frac{8\pi G\varepsilon a^4}{3c^4}\right), \tag{3}$$

a is the scale factor, $k = 0, \pm 1$ is the curvature parameter.

In the Friedmann equations the total energy density may be written in the form [23]

$$\varepsilon = \varepsilon_{\text{vac}} \sum_{q=0}^{6} B_q \left(\frac{a_0}{a} \right)^q. \tag{4}$$

The coefficients B_q refer to contributions of different kinds of matter. Here we chose normalizing scale a_0 as the de Sitter horizon radius connected with the vacuum energy density ε_{vac} by

$$a_0^2 = \frac{3c^4}{8\pi G \varepsilon_{\text{vac}}}. \tag{5}$$

which leads to $B_0 = 1$. The parameter q is connected by

$$q = 3(1 + \alpha) \tag{6}$$

with the parameter α in the equation of state

$$p = \alpha \varepsilon. \tag{7}$$

For the most frequently used equations of state the parameter q takes the values [23]:

$q = 0$ ($\alpha = -1$) for the de Sitter vacuum,

$q = 1$ $\left(\alpha = -\frac{2}{3} \right)$ for domain walls,

$q = 2$ $\left(\alpha = -\frac{1}{3} \right)$ for strings,

$q = 3$ ($\alpha = 0$) for dust,

$q = 4$ $\left(\alpha = \frac{1}{3} \right)$ for radiation or ultrarelativistic gas,

$q = 5$ $\left(\alpha = \frac{2}{3} \right)$ for perfect gas,

$q = 6$ ($\alpha = 1$) for ultrastiff matter.

Matter with a negative pressure has been recently included into quintessence which is a time-varying spatially inhomogeneous component of matter content satisfying the equation of state $p = -\alpha \varepsilon$ with $0 < \alpha < 1$ [30].

Separating a scale-factor-free term in the potential (3), we reduce the Wheeler–DeWitt equation to the Schrödinger form

$$-\frac{\hbar^2}{2m_{\text{Pl}}} \frac{d^2\psi}{da^2} + (U(a) - E)\psi = 0 \tag{8}$$

with the energy E given by

$$E = \frac{B_4}{2} \left(\frac{a_0}{l_{\text{Pl}}} \right)^2 E_{\text{Pl}} \tag{9}$$

and related to the contribution of radiation to the total energy density. Eq. (8) describes a quantum system with the energy E related to radiation, in the potential created by other components of matter content.

We consider an initial vacuum-dominated quantum fluctuation with admixture of radiation and strings (or some other quintessence with the equation of state $p = -\varepsilon/3$). In this case the potential takes the form

$$U(a) = \frac{E_{\text{Pl}}}{2l_{\text{Pl}}^2} \left((k - B_2)a^2 - \frac{a^4}{a_0^2} \right). \tag{10}$$

Imposing the boundary condition on the wave function at $a = 0$, we follow DeWitt who adopted $\psi(0) = 0$ for a quantized FRW universe [2]. At infinity we adopt the Vilenkin boundary condition which prescribes the presence only of the outgoing mode of a wave function [21].

The quantization of energy in the well (a Lorentzian domain of the pre-de-Sitter universe) is given in the WKB approximation by the Bohr–Sommerfeld formula   [31]

$$2 \int_0^{a_1} \sqrt{2m_{\text{Pl}}(E_n - U)} \, da = \pi \hbar \left(n + \frac{1}{2} \right), \tag{11}$$

where a_1 is defined by $U(a_1) = E_n$. The potential (10) is shown in Fig. 1. It has a maximum

$$U_m = \frac{(k - B_2)^2}{8} \left(\frac{a_0}{l_{\text{Pl}}} \right)^2 E_{\text{Pl}} \tag{12}$$

at

$$a_m = a_0 \sqrt{\frac{k - B_2}{2}} \tag{13}$$

and zeros at

$$a_3 = a_0 \sqrt{k - B_2} \tag{14}$$

and $a = 0$, where a potential has a minimum $U_{\text{min}} = 0$.

Let us note here that for a vacuum energy scale $E_{\text{GUT}} \sim 10^{15}$ GeV, $a_0/l_{\text{Pl}} \sim 10^8$. Later we verify the validity of the WKB approximation more accurately restricting the model parameter $k - B_2$ by the observable upper limit on the value of the CMB anisotropy $\Delta T/T$.
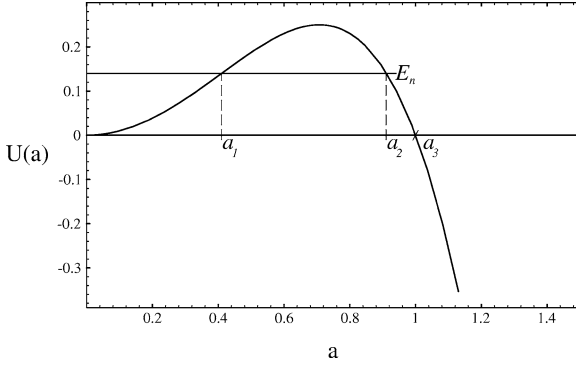
Fig. 1. Initial quantum fluctuation as a quantum system in the well (a Lorentzian domain of a pre-de-Sitter universe).

Turning points $a_{1,2}$ where $U(a) = E_n$, are given by

$$a_{1,2}^2 = a_m^2 \left(1 \pm \sqrt{1 - \frac{E_n}{U_m}} \right). \tag{15}$$

Approximating the potential near the maximum by

$$U = U_m + \frac{1}{2} \frac{d^2 U}{da^2} \bigg|_{a=a_m} (a - a_m)^2, \tag{16}$$

and calculating the spectrum with the Bohr–Sommerfeld formula (11) we get the general model restriction on the quantum number n

$$n + \frac{1}{2} < \frac{(k - B_2)^{3/2}}{\pi \sqrt{2}} \left(\frac{a_0}{l_{Pl}} \right)^2. \tag{17}$$

By Eqs. (4) and (9) we connect the energy E_n with the energy density of radiation

$$\varepsilon_\gamma = 2\varepsilon_{vac} \frac{E_n}{E_{Pl}} \left(\frac{l_{Pl}}{a_0} \right)^2 \left(\frac{a_0}{a} \right)^4 \tag{18}$$

which is related to the temperature $\Theta = kT$ as [32]

$$\varepsilon_\gamma = \frac{\pi^2}{30 \hbar^3 c^3} N(\Theta) \Theta^4, \tag{19}$$

where $N(\Theta)$ counts the total number of effectively massless degrees of freedom (species with $E \ll \Theta$) which grows with temperature and at the GUT scale is estimated within the range [14]

$$N(\Theta) \sim 10^2 - 10^4. \tag{20}$$

From Eq. (19) we get the quantized temperature

$$\Theta = \left(\frac{45}{2\pi^3 N(\Theta)} \right)^{1/4} \left(\frac{l_{Pl}}{a} \right) \left(\frac{E_n}{E_{Pl}} \right)^{1/4} E_{Pl}. \tag{21}$$

The upper limit for the temperature, which follows from the basic restriction $E_n < U_{max}$, does not depend on the value of the parameter $k - B_2$ and is given by

$$\Theta_{max} = \left(\frac{45}{4\pi^3 N(\theta)} \right)^{1/4} \left(\frac{l_{Pl}}{a_0} \right)^{1/2} E_{Pl}. \tag{22}$$

For a_0 corresponding to the GUT scale vacuum, with $N(\Theta)$ from the range (20), the maximal possible value of temperature is estimated as $\Theta_{max} \approx (0.25$–$0.08) E_{GUT}$ which is far from the monopole rest energy $E_{mon} \sim 10^{16}$–10^{17} GeV [14].

Near the minimum the potential (10) is approximated by a harmonic oscillator

$$U = \frac{E_{Pl}}{2} \frac{a^2}{l_{Pl}^2} (k - B_2), \tag{23}$$

which gives the reasonable approximation up to the inflection point $a_{infl} = a_m / \sqrt{3}$, where $U_{infl} = (5/9) U_m$.

In the region where the potential can be approximated by (23), the energy spectrum is given by

$$E_n = E_{Pl} \sqrt{k - B_2} \left(n + \frac{1}{2} \right), \tag{24}$$

and the quantum number n is restricted by the condition $E_n < U_{infl}$ which gives

$$n + \frac{1}{2} < \frac{5}{72} \left(\frac{a_0}{l_{Pl}} \right)^2 (k - B_2)^{3/2}. \tag{25}$$

For this range of the quantum numbers n the temperature (21) reduces to

$$\Theta = \left(\frac{45}{2\pi^3 N(\Theta)} \right)^{1/4} \left(\frac{l_{Pl}}{a_0} \right) \frac{(n + \frac{1}{2})^{1/4}}{(k - B_2)^{3/8}} E_{Pl}. \tag{26}$$

We put here the value of the scale factor $a = a_2$ with which a system starts a classical evolution beyond the barrier, to make evident which is the dependence of the temperature on the model parameter $k - B_2$.

The lower limit on the temperature corresponds to the lowest level of the energy spectrum. This is $n = 1$ for the case of the adopted boundary condition $\psi(0) = 0$, while the lowest energy possible in principle, $E_0 = \hbar\omega/2$, corresponds to $n = 0$. The values of the temperature for these two values of n differs by the factor $3^{1/4}$, and an absolute lower limit for a temperature related to a zero-point energy $\hbar\omega/2$, is given by

$$\Theta_{min} = \left(\frac{45}{4\pi^3 N(\Theta)} \right)^{1/4} \left(\frac{l_{Pl}}{a_0} \right) (k - B_2)^{-3/8} E_{Pl}. \tag{27}$$

The model parameter $k - B_2$ can be evaluated by the observational upper bound on the CMB anisotropy [33]

$$\Delta T / T \simeq 10^{-5}. \tag{28}$$

In the context of the inflationary paradigm anisotropy of the relic radiation originates from vacuum fluctuations during inflationary stage [14]. The value of $\Delta T / T$ which arises before decay of de Sitter vacuum, remains to be the same at the end of recombination which is estimated in today observations [34].

In our model anisotropy $\Delta T / T$ at the start of a classical evolution is related to the width of a quantized energy level of a system inside a well. Indeed, quantum tunnelling means not a process occurring in real time (penetration "occurs" within the Euclidean domain where the time coordinate is imaginary), but a nonzero probability to find a quantum system beyond the barrier where initial quantum fluctuation starts a classical evolution, so that no physical process affecting $\Delta T / T$ can occur "in the course of tunnelling". As a result classical evolution starts with the value of $\Delta T / T$ related to the level width in the well which survives till the end of recombination (any additional anisotropy appearing in the course of vacuum decay and later, is proportional to $N_\gamma^{-1/2}$, where N_γ is the number of photons which can only grow in processes of decay). The energy E_n and the value of the scale factor a_2 (see Fig. 1) affect $\Delta T / T$ at the beginning of the classical evolution which starts with those parameters as the initial values.

For the system at the quantum level E_n the temperature fluctuations ΔT originate from the natural width of a level, ΔE_n, and from ΔT due to statistical fluctuations in the photon ensemble.

Statistical fluctuations in the temperature of the ultrarelativistic gas give [32]

$$\left(\frac{\Delta T}{T} \right)_{\text{st}} = \left(\frac{15}{2\pi^2 N(\Theta)} \right)^{1/2} \left(\frac{\hbar c}{\Theta} \right)^{3/2} V^{-1/2}. \tag{29}$$

Putting $V = E_n / \varepsilon_\gamma$ and Θ from Eq. (21) we get

$$\left(\frac{\Delta T}{T} \right)_{\text{st}} = \frac{1}{2} \left(\frac{45}{2\pi^3 N(\Theta)} \right)^{1/8} \left(\frac{E_{\text{Pl}}}{E_n} \right)^{3/8} \left(\frac{l_{\text{Pl}}}{a} \right)^{1/2}. \tag{30}$$

The general constraint (17) and the observational constraint (28) restrict the model parameter $k - B_2$ by

$$k - B_2 > (2.6\text{–}4.7) \times 10^{-6}. \tag{31}$$

Two values correspond to the range (20) for $N(\Theta)$. This gives rough estimate by the order of magnitude, since the number of massless degrees of freedom $N(\Theta)$ is estimated roughly up to two orders of magnitude [14].

The natural width $\Gamma_n = \Delta E_n$ can be evaluated by the level width Γ_n for a harmonic oscillator [31]

$$\Delta E_n = \Gamma_n = \frac{2\alpha}{3} \frac{\hbar \omega}{E_{\text{Pl}}} \hbar \omega n. \tag{32}$$

From Eq. (21) we get

$$\frac{\Delta T}{T} = \frac{1}{4} \frac{\Delta E_n}{E_n}.$$

This gives the anisotropy due to natural width

$$\left(\frac{\Delta T}{T} \right)_n = \frac{\alpha}{6} \sqrt{k - B_2} \frac{n}{n + \frac{1}{2}}, \tag{33}$$

where α is the fine structure constant, which at the GUT scale is estimated within the range [35]

$$\alpha \sim \frac{1}{25} - \frac{1}{40}. \tag{34}$$

The observational constraint (28) puts an upper limit on the model parameter $k - B_2$

$$\sqrt{k - B_2} \leqslant \frac{6}{\alpha} 10^{-5}. \tag{35}$$

For α from the range (34) this gives

$$k - B_2 < (2.3\text{–}5.8) \times 10^{-6}. \tag{36}$$

The qualitative estimates (31), (36) allows us to conclude that the observational constraint (28) restricts the value of the model parameter $k - B_2$ in rather narrow range around 10^{-6}, which leads to some preliminary predictions concerning quantum birth of a hot universe.

For some value of $k - B_2$ from admissible range, say, $k - B_2 \simeq 3 \times 10^{-6}$, three cases are possible:

(i) A closed universe, $k = 1$, $B_2 \simeq (1 - 3 \times 10^{-6})$, born in the presence of strings with positive deficit angle (or other quintessence with the equation of state $p = -\varepsilon/3$) whose density is comparable to the vacuum density ε_{vac}.

(ii) An open universe, $k = -1$, $B_2 \simeq -(1 + 3 \times 10^{-6})$, born due to the presence of strings with the negative deficit angle, strings density $\varepsilon_{\rm str}$ is comparable to $\varepsilon_{\rm vac}$.

(iii) Most plausible case—a flat universe, $k = 0$, $B_2 \simeq -3 \times 10^{-6}$, arising from an initial vacuum-dominated fluctuation with the small admixture of strings with negative deficit angle, $\varepsilon_{\rm str} \ll \varepsilon_{\rm vac}$.

The Friedmann equations governing the classical evolution of a universe after tunnelling, read

$$\dot{a}^2 = \frac{8\pi G a^2}{3c^2}(\varepsilon_{\rm vac} + \varepsilon_\gamma) - (k - B_2)c^2,$$

$$\ddot{a} = -\frac{4\pi G}{3c^2}(-2\varepsilon_{\rm vac} + \varepsilon_\gamma + 3p_\gamma), \quad (37)$$

where the dot denotes differentiation with respect to the synchronous time.

The general constraint $E_n < U_{\rm max}$ restricts the radiation energy density ε_γ by

$$\frac{\varepsilon_\gamma}{\varepsilon_{\rm vac}} < \frac{1}{16}, \quad (38)$$

so that the situation at the beginning of the classical evolution is plausible for inflation: strings (as any matter with the equation of state $p = -\varepsilon/3$) do not contribute to the acceleration, while the de Sitter vacuum ($p = -\varepsilon_{\rm vac}$) provides a huge initial expansion.

For the vacuum of GUT scale $E_{\rm GUT} \sim 10^{15}$ GeV the temperature at the beginning of classical evolution, is estimated within the range

$$0.4 \times 10^{13} \text{ GeV} \leqslant \Theta \leqslant 0.3 \times 10^{15} \text{ GeV}. \quad (39)$$

The lower limit on the temperature is close to the values predicted by reheating theories [36]. The upper limit is far from the monopole rest energy, so that the problem of the monopole abundance seems to not appear in this model.

Now let us estimate the probability of quantum birth of a hot universe with the parameters restricted by (28). The penetration factor is given by the Gamow formula

$$D = \exp\left(-\frac{2}{\hbar}\left|\int_{a_1}^{a_2} \sqrt{2m_{\rm Pl}(E - U)}\, da\right|\right). \quad (40)$$

For the potential (10) in the range of n satisfying (25), this gives

$$D = \exp\left\{-\frac{2}{3}\left(\frac{a_0}{l_{\rm Pl}}\right)^2 (k - B_2)^{3/2} + (2n + 1) + I\right\}, \quad (41)$$

where $I < 10^{-2}(2n + 1)$.

Near the maximum of the potential (10) the penetration factor is calculated using the approximation (16) which gives

$$D_1 = \exp\left\{-\frac{\pi}{4\sqrt{2(k - B_2)}} \times \left|\frac{(k - B_2)^2}{4} - B_4\right|\left(\frac{a_0}{l_{\rm Pl}}\right)^2\right\}. \quad (42)$$

Comparing the penetration factors (41) and (42) with taking into account restrictions on $k - B_2$ and B_4, we see that more probable is quantum birth of a universe from the levels with quantum numbers n from the range (25) corresponding to the harmonic oscillator wing of the potential (10).

Formulae (41)–(42) evidently satisfy the WKB approximation since $(a_0/l_{\rm Pl})^2 \sim 10^{16}$ for the GUT scale $E_{\rm GUT} \sim 10^{15}$ GeV, while the model parameter $k - B_2$ is restricted by (31) and (36). For the values of this parameter compatible with observational constraint (28), the probability of tunnelling is estimated as

$$D_{\text{from a level}} \sim \exp\left(-\frac{2}{3} \times 10^7\right), \quad (43)$$

while probability of quantum birth of a universe from nothing is estimated for the same scale $E_{\rm GUT}$ as [21, 23,29]

$$D_{\text{from nothing}} \sim \exp\left(-\frac{2}{3} \times 10^{16}\right). \quad (44)$$

3. Conclusions

The main conclusion is the existence of the lower limit on the temperature of a universe born in a tunnelling event. A quantum fluctuation giving rise to a quantum universe cannot in principle have zero temperature, because its zero-level energy has non-zero value given by Eq. (19) for $n = 0$, which is a zero-point vacuum mode $\hbar\omega/2$. Minimal zero-level energy puts a lower limit on a temperature of a universe

arising as a result of quantum tunnelling, which is close to the values predicted by reheating theories.

The upper limit on the temperature for the GUT scale vacuum is far from the monopole rest mass, so problem of monopole abundance does not arise in this model.

The probability of a quantum birth from a level of non-zero energy is much bigger than the probability of a quantum birth from nothing at the same energy scale.

The model predicts the quantum birth of the GUT-scale hot universe with the temperature consistent with reheating theories, and temperature fluctuations compatible with the observed CMB anisotropy. The model does not predict the monopole abundance for the universe born from a level of quantized temperature. Quantum cosmology proves thus to be able to make proper predictions concerning direct observational consequences.

Acknowledgements

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