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The generalized Gerasimov–Drell–Hearn sum rule for deuteron electrodisintegration

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Abstract

The generalized Gerasimov–Drell–Hearn sum rule $I_{\gamma^* d}^{\mathrm{GDH}}(Q^2)$ for deuteron electrodisintegration $d(e,e')np$ as function of the squared four-momentum transfer Q^2 is evaluated by explicit integration. The calculation is based on a conventional nonrelativistic framework using a realistic NN-potential and including contributions from meson exchange currents, isobar configurations and leading order relativistic terms. Good convergence is achieved. The prominent feature is a deep negative minimum, $I_{\gamma^* d}^{\mathrm{GDH}} = -9.5$ mb, at low $Q^2 \approx 0.2\ \mathrm{fm}^{-2}$ which is almost exclusively driven by the nucleon isovector anomalous magnetic moment contribution to the magnetic dipole transition to the ${}^1S_0$-state. Above $Q^2 = 20\ \mathrm{fm}^{-2}$ the integral $I_{\gamma^*}^{\mathrm{GDH}}(Q^2)$ approaches zero rapidly.

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1. Introduction

The Gerasimov–Drell–Hearn (GDH) sum rule for real photons [1,2] relates the square of the anomalous magnetic moment of a particle to the energy weighted integral I_γ^{GDH} from threshold up to infinity over the beam-target spin asymmetry, i.e., the difference of the total photoabsorption cross sections for circularly polarized photons on a target with spin parallel and antiparallel to the spin of the photon,

$$I_\gamma^{\mathrm{GDH}} = 4\pi^2\kappa^2\frac{e^2}{M^2}S = \int_0^\infty \frac{d\omega^{\mathrm{lab}}}{\omega^{\mathrm{lab}}}\left(\sigma_\gamma^P(\omega^{\mathrm{lab}}) - \sigma_\gamma^A(\omega^{\mathrm{lab}})\right), \tag{1}$$

with mass M, charge eQ, anomalous magnetic moment κ and spin S of the particle. Furthermore, $\sigma_\gamma^{P/A}(\omega^{\mathrm{lab}})$ denote the total absorption cross sections for circularly polarized photons of energy ω^{lab} on a target with spin parallel and antiparallel to the photon spin, respectively. The anomalous magnetic moment is defined by the total magnetic moment operator of the

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particle

$$\vec{M} = (Q+\kappa)\frac{e}{M}\vec{S}. \tag{2}$$

Previously, this sum rule has been evaluated for the deuteron by explicit integration up to an energy of 550 MeV including the contributions from the photodisintegration and single pion production channels [3,4]. While for photodisintegration convergence was achieved yielding a negative contribution of -413 μb, the incoherent pion production contributions had not converged and a substantial positive contribution was still missing, as is needed to balance the negative result from photodisintegration in order to yield the small positive sum rule prediction $I^{\mathrm{GDH}}_{\gamma,d} = 0.65$ μb from the deuteron's small anomalous magnetic moment $\kappa_d = -0.143$.

It is the aim of the present Letter to report on a first evaluation of the contribution of the electrodisintegration channel, i.e., $d(e,e')np$, to the generalized GDH integral $I^{\mathrm{GDH}}_{\gamma^* d}(Q^2)$ for the deuteron by explicit integration up to a maximum excitation energy of 1 GeV.

2. The generalized GDH sum rule

The spin asymmetry of the deuteron for real photons is related to the vector target asymmetry τ^c_{10} of the total photoabsorption cross section [9], i.e.,

$$\sigma^P_\gamma(\omega^{\mathrm{lab}}) - \sigma^A_\gamma(\omega^{\mathrm{lab}}) = \sqrt{6}\sigma^0_\gamma(\omega^{\mathrm{lab}})\tau^c_{10}(\omega^{\mathrm{lab}}), \tag{3}$$

where σ^0_γ denotes the unpolarized total photoabsorption cross section. This spin asymmetry can be related to the transverse form factor F'^{10}_T of the inclusive electrodisintegration cross section which appears for a vector polarized deuteron target in conjunction with a longitudinally polarized electron beam.

The general inclusive cross section for deuteron electrodisintegration including polarization degrees of freedom is governed by a set of ten inclusive form factors, namely two longitudinal F_L and F^{20}_L, four transverse F_T, F^{20}_T, F^{2-2}_{TT}, and F'^{10}_T, and four longitudinal-transverse interference form factors F^{1-1}_{LT}, F^{2-1}_{LT}, F'^{1-1}_{LT}, and F'^{2-1}_{LT}, of which F^{1-1}_{LT} and F'^{2-1}_{LT} vanish below pion threshold due to time reversal invariance. Explicitly, the inclusive cross section reads [5]

$$\begin{aligned}
\sigma_e\big(h, P^d_1, P^d_2\big)
&\equiv \frac{d\sigma}{dk^{\mathrm{lab}}_2\, d\Omega^{\mathrm{lab}}_e}\\
&= 6c\big(k^{\mathrm{lab}}_1, k^{\mathrm{lab}}_2\big)\\
&\quad\times\Big\{\rho_L F_L + \rho_T F_T - P^d_1 \rho_{LT} F^{1-1}_{LT}\sin\phi_d d^1_{10}(\theta_d)\\
&\qquad + P^d_2\big[\big(\rho_L F^{20}_L + \rho_T F^{20}_T\big)d^2_{00}(\theta_d)\\
&\qquad\qquad - \rho_{LT} F^{2-1}_{LT}\cos\phi_d d^2_{10}(\theta_d)\\
&\qquad\qquad + \rho_{TT} F^{2-2}_{TT}\cos 2\phi_d d^2_{20}(\theta_d)\big]\\
&\qquad + h P^d_1\big[-\rho'_T F'^{10}_T d^1_{00}(\theta_d)\\
&\qquad + \rho'_{LT} F'^{1-1}_{LT}\cos\phi_d d^1_{10}(\theta_d)\big]\\
&\qquad - h P^d_2 \rho'_{LT} F'^{2-1}_{LT}\sin\phi_d d^2_{10}(\theta_d)\Big\},
\end{aligned} \tag{4}$$

where incoming and scattered electron momenta are denoted by k^{lab}_1 and k^{lab}_2, respectively, $c(k^{\mathrm{lab}}_1, k^{\mathrm{lab}}_2)$ and $\rho^{(\prime)}_\alpha$ ($\alpha \in \{L, T, LT, TT\}$) denote kinematical factors, h the degree of longitudinal electron polarization. Furthermore, $P^d_{00} = 1$, and P^d_1 and P^d_2 describe vector and tensor polarization of the deuteron, respectively, and the spherical angles (θ_d, ϕ_d) characterize the deuteron orientation axis. The various form factors are functions of E_{np}, the c.m. final state excitation energy, and of $q^{\mathrm{c.m.}}$, the three-momentum transfer in the c.m. system.

At the photon point, $Q^2 = (\vec{q})^2 - \omega^2 = 0$, the purely transverse form factors are related to the various contributions of the general total photoabsorption cross section of deuteron photodisintegration, namely to the unpolarized total cross section $\sigma^{\mathrm{tot}}_\gamma$ and to the beam and target asymmetries for polarized photons and deuterons as defined in [9]. In detail one has for $Q^2 = 0$

$$\begin{aligned}
\sigma^{\mathrm{tot}}_\gamma &= \frac{M_d}{W_{np} q^{\mathrm{c.m.}}} F_T, &\qquad \tau^0_{20} &= \frac{F^{20}_T}{F_T},\\
\tau^c_{10} &= \frac{F'^{10}_T}{F_T}, &\qquad \tau^l_{22} &= \frac{F^{2-2}_{TT}}{F_T},
\end{aligned} \tag{5}$$

where the invariant mass of the final np system is denoted by $W_{np} = E_{np} + 2M$ with M for the nucleon mass.

Thus the spin asymmetry for real photons in (3) corresponds to the vector target asymmetry for longitudinally polarized electrons of the above inclusive cross section as defined by [5]

$$A^V_{ed}(\theta_d,\phi_d) = \frac{1}{4hP^d_1\sigma^0_e} \times \big[\sigma_e\big(h,P^d_1,P^d_2\big)-\sigma_e\big(-h,P^d_1,P^d_2\big) - \sigma_e\big(h,-P^d_1,P^d_2\big)+\sigma_e\big(-h,-P^d_1,P^d_2\big)\big], \tag{6}$$

yielding for $(\theta_d,\phi_d)=(0,0)$, i.e., deuteron orientation axis parallel to $\vec{q}$,

$$A^V_{ed}(0,0) = \frac{6c(k_1^{\mathrm{lab}},k_2^{\mathrm{lab}})}{\sigma^0_e}\rho'_T F'^{10}_T, \tag{7}$$

with $\sigma^0_e=\sigma_e(0,0,0)$ as unpolarized inclusive cross section.

Therefore, we introduce as spin asymmetry for transverse virtual photons

$$\sigma^P_{T,\gamma^*}\big(\omega^{\mathrm{lab}}\big)-\sigma^A_{T,\gamma^*}\big(\omega^{\mathrm{lab}}\big)=\sqrt{6}\,\frac{M_d}{W_{np}q^{\mathrm{c.m.}}}F'^{10}_T, \tag{8}$$

which coincides at the photon point with Eq. (3). Correspondingly, we take as extension of the GDH integral from real to virtual photons the definition [6,7]

$$I^{\mathrm{GDH}}_{\gamma^*d}\big(Q^2\big) = \sqrt{6}\int_{\omega^{\mathrm{lab}}_{\mathrm{th}}}^{\infty}\frac{d\omega^{\mathrm{lab}}}{\omega^{\mathrm{lab}}}\frac{M_d}{W_{np}q^{\mathrm{c.m.}}} \times F'^{10}_T\big(E_{np},q^{\mathrm{c.m.}}\big)g\big(\omega^{\mathrm{lab}},Q^2\big), \tag{9}$$

where M_d denotes the deuteron mass. Here E_{np} or equivalently $W_{np}=E_{np}+2M$ and $q^{\mathrm{c.m.}}$ are functions of ω^{lab} and Q^2

$$W_{np}\big(\omega^{\mathrm{lab}},Q^2\big)=\sqrt{M_d^2-Q^2+2M_d\omega^{\mathrm{lab}}},$$
$$q^{\mathrm{c.m.}}\big(\omega^{\mathrm{lab}},Q^2\big)=\frac{M_d}{W_{np}}\sqrt{Q^2+\big(\omega^{\mathrm{lab}}\big)^2}. \tag{10}$$

The factor $g(\omega^{\mathrm{lab}},Q^2)$ in (9) takes into account the fact, that the generalization of the GDH integral is to a certain extent arbitrary. The only restriction for this factor is the condition that at the photon point $Q^2=0$ one has

$$g\big(\omega^{\mathrm{lab}},0\big)=1, \tag{11}$$

and that

$$\lim_{\omega^{\mathrm{lab}}\to\infty} g\big(\omega^{\mathrm{lab}},Q^2\big)\Big|_{Q^2=\mathrm{const}}<\infty \tag{12}$$

remains finite. As simplest extension we choose here $g(\omega^{\mathrm{lab}},Q^2)\equiv 1$.

Transforming (9) into an integral over E_{np} using

$$\omega^{\mathrm{lab}}=\frac{1}{2M_d}\big(W^2_{np}+Q^2-M^2_d\big) = \frac{1}{2M_d}\big((E_{np}+2M)^2+Q^2-M^2_d\big), \tag{13}$$

one obtains

$$I^{\mathrm{GDH}}_{\gamma^*d}\big(Q^2\big) = 2\sqrt{6}M_d\int_0^\infty dE_{np}\,\frac{F'^{10}_T(E_{np},q^{\mathrm{c.m.}})}{(W^2_{np}+Q^2-M^2_d)q^{\mathrm{c.m.}}}, \tag{14}$$

where now $q^{\mathrm{c.m.}}$ has to be considered as a function of E_{np} and Q^2, i.e.,

$$q^{\mathrm{c.m.}}\big(E_{np},Q^2\big) = \frac{1}{2W_{np}} \times\sqrt{\big((W_{np}-M_d)^2+Q^2\big)\big((W_{np}+M_d)^2+Q^2\big)}. \tag{15}$$

3. Results for electrodisintegration

The generalized GDH integral of (14) has been evaluated by explicit integration up to a maximum excitation energy $E_{np}=1$ GeV. The evaluation of F'^{10}_T is based on an expansion into transverse electric and magnetic multipole matrix elements according to [5]

$$F'^{10}_T = 16\pi^2\sum_{LL'j\mu}(-)^j\begin{pmatrix} L' & L & 1\\ 1 & -1 & 0\end{pmatrix} \times\begin{Bmatrix} L' & L & 1\\ 1 & 1 & j\end{Bmatrix}e^{-2\rho^j_\mu} \times\Re\mathrm{e}\big[\big(E^{L'}(\mu j)+M^{L'}(\mu j)\big)^* \times\big(E^L(\mu j)+M^L(\mu j)\big)\big], \tag{16}$$

where μ labels the possible final partial waves of given total angular momentum j in the Blatt–Biedenharn parametrization [8], and ρ_μ^j its inelasticity which is zero below pion threshold. Note, that due to parity conservation one has in (16) either electric or magnetic contributions for a given multipolarity L and state μj.

The calculation is based on a nonrelativistic framework as is described in detail in Refs. [9,11] but with inclusion of the leading order relativistic contributions. In the current operator we distinguish the one-body currents with Siegert operators (N), explicit meson exchange contributions (MEC) beyond the Siegert operators, essentially from π- and ρ-exchange, contributions from isobar configurations of the wave functions (IC), calculated either in the impulse approximation [10] or in a coupled channel approach for the most dominant $N\Delta$-configuration [11], and leading order relativistic contributions (RC). Bound and scattering states are obtained from a realistic NN-potential for which we have chosen the Bonn r-space and q-space (B) models [12] and the Argonne V_{18} potential [13]. The final state interaction (FSI) is taken into account for all multipoles up to $L = 6$ whereas for the higher multipoles FSI can safely be neglected and plane waves are used.

In Fig. 1 the transverse spin asymmetry $\sigma^P_{T,\gamma^*} - \sigma^A_{T,\gamma^*}$ as function of E_{np} for various values of Q^2 are shown. The prominent and most interesting feature, which one readily notes, is the resonance like structure right above np-break-up threshold around $E_{np} = 70$ KeV. It stems essentially from the isovector M1-transition to the antibound ${}^1S_0$-state located at this energy, which is well known from photo- and electrodisintegration to dominate the cross section near threshold. Up to several MeV above threshold the leading contributions come essentially alone from the $(L = 1)$-multipoles while the higher multipoles give a negligible contribution only. Restriction to $L = 1$ yields from (16) explicitly

$$
\begin{aligned}
F'^{10}_T = {} & -\frac{8\pi^2}{3\sqrt{6}} \\
& \times \bigl(2\bigl|\mathrm{M}^1(2,0)\bigr|^2 + \bigl|\mathrm{M}^1(1,1)\bigr|^2 + \bigl|\mathrm{M}^1(3,1)\bigr|^2 \\
& \quad - \bigl|\mathrm{M}^1(2,2)\bigr|^2 - \bigl|\mathrm{M}^1(4,2)\bigr|^2 \\
& \quad + 2\bigl|\mathrm{E}^1(3,0)\bigr|^2 + \bigl|\mathrm{E}^1(2,1)\bigr|^2 + \bigl|\mathrm{E}^1(4,1)\bigr|^2 \\
& \quad - \bigl|\mathrm{E}^1(1,2)\bigr|^2 - \bigl|\mathrm{E}^1(3,2)\bigr|^2\bigr).
\end{aligned}
\tag{17}
$$

The E1-transitions leading to ${}^1P_1$ and ${}^3P_j$ $(j = 0, 1, 2)$ states and which are most important in the inclusive cross section, do not play a significant role in the spin asymmetry in this energy region. The reason for this feature is that the isoscalar transition to ${}^1P_1$ is largely suppressed, while the triplet ${}^3P_j$ contributions to (16) almost cancel each other. The cancellation would be complete if spin–orbit and tensor forces could be neglected, because in this case the matrix elements are simply related by angular momentum recoupling coefficients. Thus, at low energies only M1-transitions remain, essentially to ${}^1S_0$ and ${}^3S_1$ states. The ${}^1S_0$ contribution is dominant because of the large isovector part of the M1-operator arising from the large isovector anomalous magnetic moment of the nucleon. It is particularly strong close to break-up threshold at about 70 KeV, where the ${}^1S_0$ state is resonant. This feature is seen in Fig. 2 where this matrix element is displayed for various constant values of Q^2. Since this state can only be reached by the antiparallel spin combination one finds a strong negative spin asymmetry and thus a negative contribution to the GDH integral. The overwhelming predominance of the M1-transition into the ${}^1S_0$-state is demonstrated in Fig. 3 where a comparison of the spin asymmetry between calculations with all multipoles, with all M1-multipoles and with the M1-transition into the ${}^1S_0$-state alone is displayed. The latter two coincide completely and also the calculation including all multipoles shows only above $E_{np} \approx 1$ MeV a small deviation.

Besides this low energy feature which, however, becomes less and less pronounced with increasing Q^2 above $Q^2 = 1\ \mathrm{fm}^{-2}$, one notes the evolution of the quasi-free peak as a distinct negative minimum in both the spin asymmetry as well as in the leading M1-matrix element located at $E_{np}/\mathrm{MeV} \approx 10Q^2/\mathrm{fm}^{-2}$ (see lower panels of Figs. 1 and 2). However, its size decreases rapidly with increasing Q^2. The rapid fall-off of the spin asymmetry with increasing energy E_{np} ensures furthermore that the generalized GDH-integral converges sufficiently fast in view of the additional energy weighting. In fact, convergence is achieved if one integrates up to an energy E_{np} roughly 100 MeV above the quasi-free peak.

The resulting $I^{\mathrm{GDH}}_{\gamma^* d}(Q^2)$ is shown in Fig. 4. For $Q^2 \to 0$ the integral approaches $I^{\mathrm{GDH}}_{\gamma d}$ for real pho-

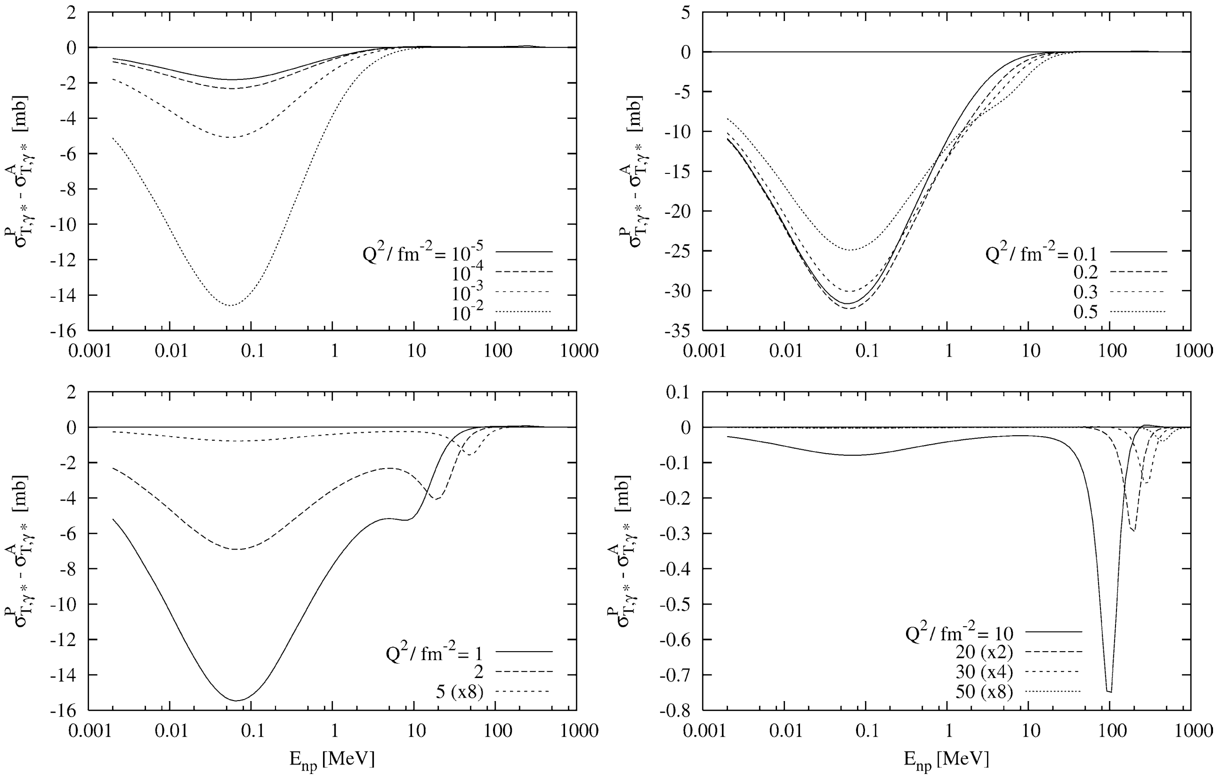

Fig. 1. Transverse spin asymmetry $\sigma^P_{T,\gamma^*} - \sigma^A_{T,\gamma^*}$ of deuteron electrodisintegration $d(e,e')np$ as function of E_{np} for various constant four-momentum transfers Q^2. The calculation is based on the Argonne V_{18} potential [13] and includes all interaction and relativistic effects.

tons. A pronounced minimum is readily seen around $Q^2 \approx 0.2\ \text{fm}^{-2}$ reflecting the deepest minimum of the spin asymmetries in Fig. 1 for this value of Q^2. The left panel shows the influence of the various interaction effects from MEC, IC and RC. Near the minimum, the largest effect arises from MEC, increasing the depth by about 10%, and to a smaller extent from IC while their influences in the other regions of Q^2 is quite small. Relativistic contributions are substantial near the photon point as has been noted already for photodisintegration [3]. But at higher Q^2 they are quite tiny. The bottom panel of Fig. 4 shows a comparison of $I^{\text{GDH}}_{\gamma^* d}(Q^2)$ for three realistic potential models, the Bonn r-space, the Bonn q-space (B) [12] and the Argonne V_{18} [13] models. Obviously, the potential model variation is quite small compared to the interaction effects. In view of the fact, that for real photons $I^{\text{GDH}}_{\gamma,d}$ is driven by the nucleon anomalous magnetic moments, we have also evaluated $I^{\text{GDH}}_{\gamma^* d}(Q^2)$ for vanishing anomalous moments. The resulting integral, also shown in the bottom panel of Fig. 4, is quite tiny, which underlines the fact that also the generalized GDH-integral is driven by the nucleon anomalous magnetic moments.

4. Summary and conclusions

The beam-target spin asymmetry of deuteron electrodisintegration for transverse virtual photons and the associated generalized Gerasimov–Drell–Hearn integral have been evaluated. The spin asymmetry for constant four momentum transfer exhibits as function of the final state excitation energy E_{np} a very interesting low energy property, a pronounced negative minimum around $E_{np} = 70$ KeV, which is deepest for $Q^2 \approx 0.2\ \text{fm}^{-2}$. It is dominated by a single magnetic dipole transition to the $^1S_0$-scattering state and al-

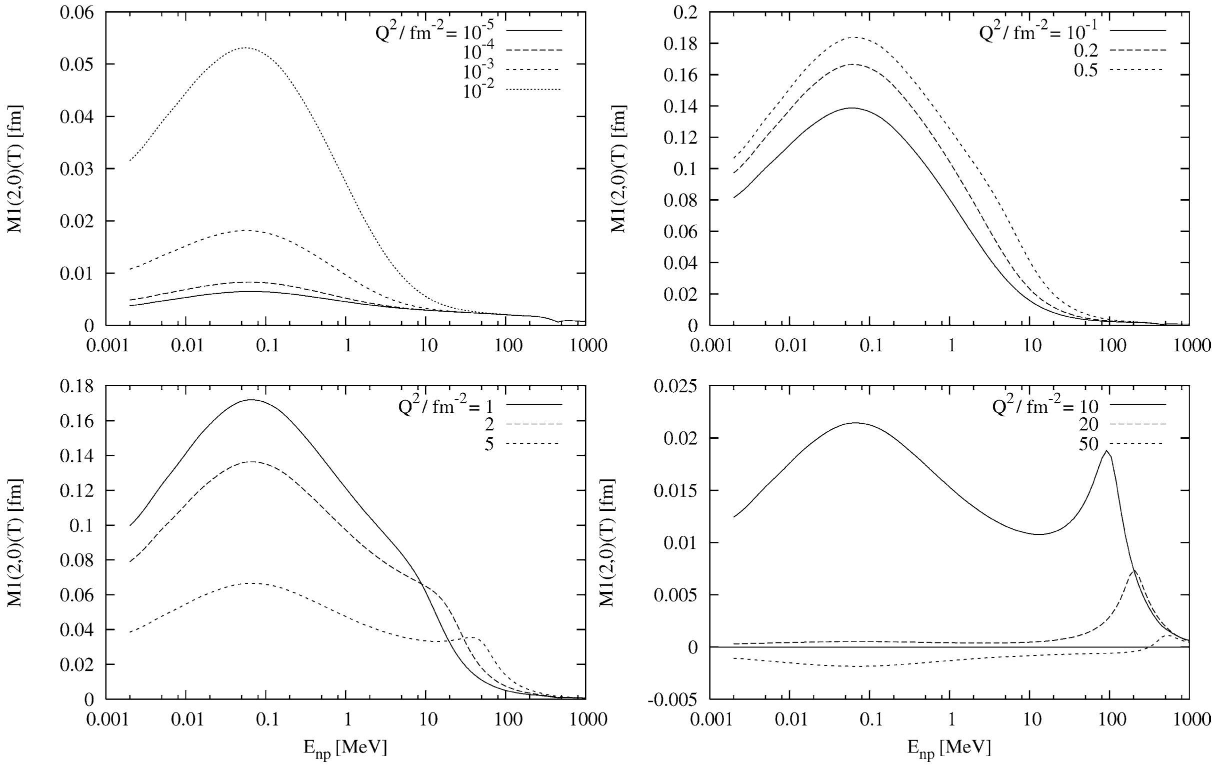

Fig. 2. Magnetic M1$(2,0)$-matrix element into the ${}^1S_0$-state for deuteron electrodisintegration $d(e,e')np$ as function of E_{np} for various constant four-momentum transfers Q^2 for Argonne V_{18} potential [13].

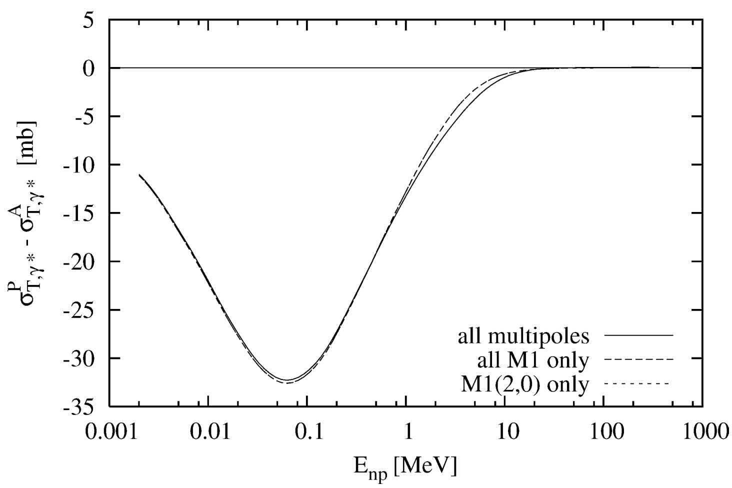

Fig. 3. Transverse spin asymmetry $\sigma^P_{T,\gamma^*} - \sigma^A_{T,\gamma^*}$ of deuteron electrodisintegration $d(e,e')np$ as function of E_{np} for $Q^2 = 0.2\ \mathrm{fm}^{-2}$ calculated including all multipoles (solid), all M1-multipoles only (dashed) and the M1-transition into the ${}^1S_0$-state alone (dotted).

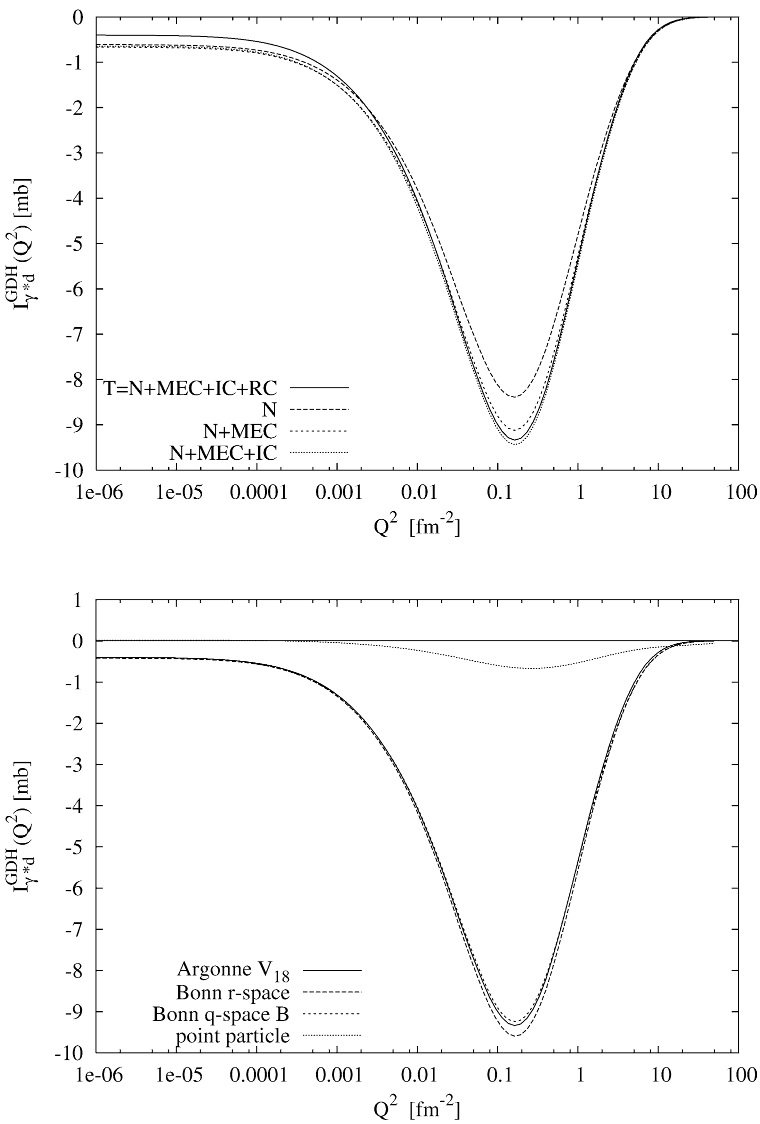

Fig. 4. Generalized Gerasimov–Drell–Hearn integral as function of Q^2 for deuteron electrodisintegration $d(e,e')np$. Top panel: separate current contributions from normal nonrelativistic theory (N) and successively added meson exchange currents (MEC), isobar configurations (IC), and relativistic contributions (RC). Bottom panel: results of the complete calculation (T) for different potential models and for vanishing anomalous nucleon magnetic moments (labeled "point particle").

most completely governed by the nucleon anomalous magnetic moment. All other multipoles play an insignificant role. At higher excitation energies the spin asymmetry tends rapidly to zero, so that the generalized GDH-integral converges fast, already at a few hundreds of MeV. The minimum in the spin asymmetry leads to a corresponding negative minimum of $I^{\mathrm{GDH}}_{\gamma^* d}(Q^2)$ around $Q^2 = 0.2\ \mathrm{fm}^{-2}$. An experimental check of these predictions for both the spin asymmetry as well as for the GDH-integral would provide an additional significant test of our present understanding of low energy behavior of few-body nuclei. Furthermore, in view of this low energy property, an independent evaluation in the framework of effective field theory would be very interesting.

It remains as a task for future theoretical research to evaluate the spin asymmetry and the GDH-integral for the other possible channels, like coherent and incoherent single pion as well as two-pion electroproduction.

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