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Reliability of harvested rainfall as an auxiliary source of non-potable water

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Abstract

Rainwater harvesting is an ancient practice that involves collecting, storing, and using precipitation to meet on-site water needs. This paper develops and demonstrates guidelines for sizing the capacity of storage tanks to provide a reliable continuous supply of harvested rainwater for residential households. Operation of the rainwater harvesting system is simulated with a stochastic mass balance performed on an Excel spreadsheet. The daily volume of rainwater in an unbounded tank is tracked to determine the maximum accumulated deficit on a monthly basis. Results are summarized in dimensionless charts showing the minimum size of a rainwater storage tank needed to meet water demands at specified levels of reliability.

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Keywords: Rainwater harvesting; urban water supply; reliability; simulation

Nomenclature

A	area of rainfall catchment (L^2)
C	cistern size (L^3)
I	input to the control volume (L^3)
K	dimensionless cistern size

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N	net daily water supply (L^3)
P	daily precipitation (L)
Q	daily residential water demand (L^3)
S	volume of water stored in cistern (L^3)
t	time (T)
α	percentile of cumulative probability distribution function
ε	dimensionless daily drift
μ	mean
σ	standard deviation

1. Introduction

Rainwater harvesting (RWH) is the practice of collecting, storing, and using rainwater for various water needs. The basic idea is to collect rainwater on-site from hard man-made surfaces and subsequently use it to irrigate landscapes, flush fixtures, and meet other indoor non-potable water needs. In 2013, Cincinnati City Council passed Section 1105-08 of the Cincinnati Municipal Code [1] permitting rainwater harvesting "…*as a non-potable auxiliary water source for use in subsurface irrigation, flushing of fixtures and other non-potable uses*…"

Treated drinking water is currently used for these purposes. Drinking water is expensive to produce and distribute so substituting rainwater as a non-potable alternative provides a way to alleviate energy use, conserve water, and reduce greenhouse gas emissions. Reducing the demand for treated water will lead to energy and cost savings at municipal treatment facilities. How can this be accomplished with harvested rainwater? There are currently no guidelines for sizing rainwater harvesting systems to provide a reliable continuous supply for non-potable uses in Cincinnati.

The goal of the research project is to investigate the technical feasibility of using rainwater harvesting as a non-potable auxiliary source of water for various end-users in Cincinnati. The investigation focused specifically on single family homes using rainwater for flushing fixtures but the final design aid can be expanded beyond this example. The main objective of this project is to develop guidelines for sizing the volumetric capacity of the on-site storage tank needed to provide a safe reliable continuous supply of harvested rainwater for residential users

2. Literature Review

Designing rainwater harvesting systems to provide a reliable water supply of water to an urban end-user is a topic of recent discussion. Basinger *et al* [2] developed and demonstrated the Storage and Reliability Estimation Tool (SARET) to investigate the feasibility of using harvested rainwater for flushing toilets, irrigating gardens and topping off air conditioners at multifamily residential buildings. SARET used a nonparametric stochastic generator to simulate rainfall in New York City. The authors found that harvested rainwater could meet the water needs of garden irrigation and air conditioning units with relatively high reliability (80-90%); however, for purposes of toilet flushing, the reliability of using harvested rain as the water source was rather low (10-40%).

Baek and Coles [3] used a water balance simulation model called DAMCAT5 to compare the reliability of harvested rainfall systems versus catchment dam systems for a 60-year period at ten farm sites in semi-arid Western Australia. The authors found that a monthly simulation horizon provided the most realistic estimates of reliability for designing artificial catchment rainfall-runoff systems connected to farm dams in Western Australia.

Sample *et al* [4] formulated a water balance model that simulates a single RWH system in Richmond, Virginia, using as independent design variables the cistern storage volume, roof catchment area, irrigated garden area, indoor non-potable demand, and a storage dewatering goal. The model was used to assess the dual benefits of water supply and runoff reduction across a wide variety of scenarios. Performance curves were developed to evaluate the tradeoff among design variables when system reliability was held constant. Results indicate that land uses associated with larger water demands (e.g., office buildings, commercial sites, high-density residential developments) may achieve greater benefits from RWH systems than lower-density residential lots.

Hanson and Vogel [5] developed storage-reliability-yield relationships for rainwater harvesting systems. The storage equations found through regression analyses include water demand, collection area and local precipitation statistics. Daily rainfall data for 231 locations across the United States were used to simulate the rainwater harvesting systems. Reliability was defined as the fraction of days during which the full demand can be met; results were computed for reliability levels of 80%, 90%, 95%, and 98%.

Malinowski *et al* [6] investigated the energy savings expected from a rainwater harvesting system and from a gray-water reuse operation, both at a national level and a household level. On a national scale, the authors claim that harvested rainwater used for outdoor uses can save up to 3.8 billion kWh and $270 million annually. On a household scale, both metrics are low with potential annual energy savings estimated to be 120 kWh corresponding to an annual cost savings of around $10.

3. Methodology

The rainwater harvesting system considered here has two main elements: (i) a catchment area that collects rainfall and (ii) a large unpressurized vessel or cistern that stores the harvested rain water until it is called by the end-user. In a residential application, typical end-uses of stored rainfall include non-potable water for flushing toilets and for irrigating gardens.

Rainfall is a random sporadic process with intensities, durations and frequencies that cannot be predicted with certainty. The water demand at the residential end-uses also varies from day to day. A large volume of rainfall received in a few hours may have to satisfy non-potable water demands at the household over a period of several days or weeks. The proposed method for sizing RWH systems will be demonstrated in the humid mid-west region of the United States. Hourly rainfall data for the 60-year period 1952 to 2012 were obtained from the National Weather Service gage at the Cincinnati/NKY International Airport [7]. As shown in Table 1, Cincinnati receives significant rainfall throughout the year. The five-month period from March through July tends to be wetter than the other seven months.

Table 1. Daily rainfall (inches[1]) in Cincinnati, Ohio for period 1952-2012.

Statistic	Jan	Feb	Mar	Apr	May	June	July	Aug	Sep	Oct	Nov	Dec
Average	0.095	0.097	0.126	0.131	0.142	0.138	0.130	0.104	0.095	0.095	0.111	0.102
Stan Dev	0.261	0.253	0.318	0.287	0.334	0.351	0.345	0.302	0.291	0.296	0.271	0.259
Maximum	4.03	2.84	5.13	2.50	3.02	3.40	3.93	3.50	3.90	4.30	2.56	2.81
Minimum	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
No rain[2]	0.613	0.617	0.589	0.578	0.614	0.643	0.673	0.714	0.732	0.735	0.646	0.609

[1] Multiply by 0.6233 to convert inches to gallons per square foot.
[2] "No rain" is the probability of having a dry day (no rainfall) during the given month.

The rainwater harvesting design problem is stated as follows: Give a random intermittent supply of rainwater, what size of collection catchment and water cistern are needed to provide a reliable supply of harvested rain water to residential end uses? In this investigation, the catchment area is assumed to be known. The design process will focus on sizing the water cistern to meet non-potable household water demands while meeting a specified level of operating reliability.

3.1. Water Balance

A simple mass balance is the governing principle for this investigation. The mass balance requires that the rate of change in storage in a control volume is equal to the input rate minus the output rate as seen in Equation 1

$$\frac{dS}{dt} = I(t) - Q(t) \tag{1}$$

In the context of a rainwater harvesting system the control volume is the cistern. The input to the cistern is the rainfall harvested from the catchment area and the output from the cistern is the non-potable residential water use. The water balance equation in (1) is discretized using a daily time step (Δt = 1 day) and solved over a monthly time horizon assuming an *unbounded reservoir* (i.e., the cistern does not fill or empty). Starting from an initial volume, S_0, in the cistern at the beginning of the month, the mass balance for day 1 is,

$$S_1 = S_0 + AP_1 - Q_1 \tag{2}$$

Here S_1 is the volume of water in the cistern at the end of day 1, P_1 is the total depth of rainfall on the catchment of area A that enters the cistern during day 1 and Q_1 is the total volume of water that is released from the cistern to satisfy non-potable demand during day 1. In a similar manner, the volume of water in the cistern at the end of day 2 depends on the volume of water carried over from day 1,

$$S_2 = S_1 + AP_2 - Q_2 = S_0 + A\sum_{i=1}^{2} P_i - \sum_{i=1}^{2} Q_i \tag{3}$$

In general, the volume of water in the unbounded cistern at the end of the nth day is

$$S_n = S_0 + A\sum_{i=1}^{n} P_i - \sum_{i=1}^{n} Q_i \tag{4}$$

A plot of S_n versus day of the month depicts the daily fluctuation of rainwater stored in the unbounded cistern and, hence, mimics the behavior of an ideal rainwater harvesting system. The time trajectory of S_n reveals the maximum drawdown of rainwater in the cistern during the simulation period. The maximum drawdown is a direct estimate of the minimum rainwater storage capacity needed to meet the non-potable demand without allowing the cistern to go dry during the month. This simulation approach for reservoir analysis has roots that extend back to sequent peak procedure of Harold Thomas as described by Fiering [8].

The sequent peak simulation exercise is repeated for many trials to generate many independent estimates of the required monthly cistern capacity. These estimates of cistern size are used to compute sample statistics (mean, variance, etc) and to generate a frequency distribution of the required storage capacity of the rainwater cistern for the month in question. Finally, if a reliability target (e.g., 90th or 99th percentile) is specified, then the appropriate cistern size for monthly operation can be selected using a classic frequency approach given in Equation (5),

$$C(\alpha) = \mu_C + \xi(\alpha)\sigma_C \tag{5}$$

Here, for example, $C(\alpha)$ is the α percentile of the probability distribution for cistern size, $\zeta(\alpha)$ is the corresponding "frequency factor" and μ_C and σ_C are, respectively, the sample mean and sample standard deviation of the simulated cistern capacities for the month in question.

3.2. Dimensionless Daily Drift

At most locations, the occurrence of rainfall is not uniform during the year. The average daily rainfall amount and the average daily time between rain events will vary from month to month. Further, the catchment area A and the end-user demand Q may change. As a consequence, it is expected that the sample statistics and the simulated frequency distribution of the minimum required cistern size will also vary from month to month and from location to location. A new frequency analysis would be necessary anytime one of the terms (A, P, Q) in the mass balance changed. It may be possible to formulate the solution in terms of a single dimensionless design algorithm by rescaling the problem in terms of the standard deviation of the net daily water supply at the rainwater harvesting system. Let N be the net daily water supply to the cistern,

$$N = AP - Q \tag{6}$$

Assuming P and Q are independent (reasonable if harvested rainfall is limited to indoor use only), then the mean and variance of the net daily water supply to the cistern are,

$$\mu_N = A\mu_P - \mu_Q \tag{7}$$

$$\sigma_N^2 = A^2\sigma_P^2 + \sigma_Q^2 \tag{8}$$

The dimensionless daily drift of the RWH system is the inverse of the coefficient of variation of N or,

$$\varepsilon = \frac{\mu_N}{\sigma_N} = \frac{A\mu_P - \mu_Q}{\sqrt{A^2\sigma_P^2 + \sigma_Q^2}} \tag{9}$$

The dimensionless drift has a simple intuitive meaning. When the drift is positive, the cistern has a tendency over the long term to gain water; conversely, when the drift is negative, the cistern has a tendency to deplete its supply. The drift plays a central role in formulating the dimensionless design approach.

3.3 Benchmark Case

For the benchmark case, consider a single family residence with a catchment area A=1400 ft^2, a mean daily non-potable demand μ_Q = 100 gallons (per day) with a standard deviation of σ_Q = 20 gallons (per day). This use could represent water needed each day to flush toilets in a household of four. In addition, assume that 100 percent of catchment rainfall is harvested. As the first step to find the required size of the rainwater cistern, Equation (4) is solved using the historical daily precipitation for a given month. For instance, the key steps applied to the month of April 1953 are illustrated in Figures 1a,b,c, below.

Figure 1a is a time series plot of daily rainfall for April 1953 expressed as gallons harvested from the catchment. Rainfall occurred on 17 of 30 days with total daily amounts ranging from a trace at 0.01 inches (8.7 gallons, on April 16 and again on April 27) up to 0.76 inches (663 gallons on April 18). The total monthly rainfall for April 1953 was 2.67 inches (2330 gallons from the 1400 ft^2 catchment in 30 days). This amount is only about 2/3 of the normal average rainfall expected for the month of April in Cincinnati.

Figure 1b is a time series plot of random daily water demands or withdrawals from the rainwater cistern as simulated with an Excel spreadsheet. Withdrawals occurred every day with total daily amounts ranging from 59 gallons (April 5) to 135 gallons on (April 22). The total simulated monthly withdrawal for April 1953 was 3034 gallons or, on average, about 101 gallons per day. This value is very close to the mean non-potable water demand specified in the benchmark statement for the family of four.

Figure 1c is the graphical representation of Equation (4), showing the daily water level fluctuations in the cistern, subject to the sporadic rainfall supply and random rainwater releases depicted in Figures 1a and 1b, respectively. There are two periods with extended drawdowns. The first drawdown runs from April 1 to April 14 during which time the simulated water level in the cistern drops from +91 gallons to -667 gallons for a total accumulated deficit of 758 gallons. This first drawdown recovers during the period of five wet days in mid-April.

The second drawdown runs from April 18 to April 30 during which time the simulated level in the cistern drops from +199 gallons to -703 gallons resulting in a total accumulated deficit of 902 gallons. This result (902 gallons) is the critical maximum drawdown on the cistern for this simulation based on conditions in April 1953. This value of 902 gallons provides one estimate of the minimum required cistern size for the month of April.

Using a different year in the simulation exercise will generate a different estimate of the required cistern size. Hence, the process illustrated in Figures 1a,b,c for identifying the maximum drawdown in the cistern is performed for all 60 years of April rainfall to build an empirical distribution of 60 maximum drawdowns for April. This step is then repeated for all other months to provide a complete picture of monthly drawdowns for the full year.

For example, the distributions of simulated maximum monthly drawdowns for April and October are shown in Figure 2. The drawdown for April ranges from a minimum of 310 gallons to a maximum of over 2600 gallons, while for October the endpoints run from nearly 700 to 2800 gallons. The wide range of simulated drawdowns reflects the high variability and the sporadic nature of the daily rainfall that occurs during any given month.

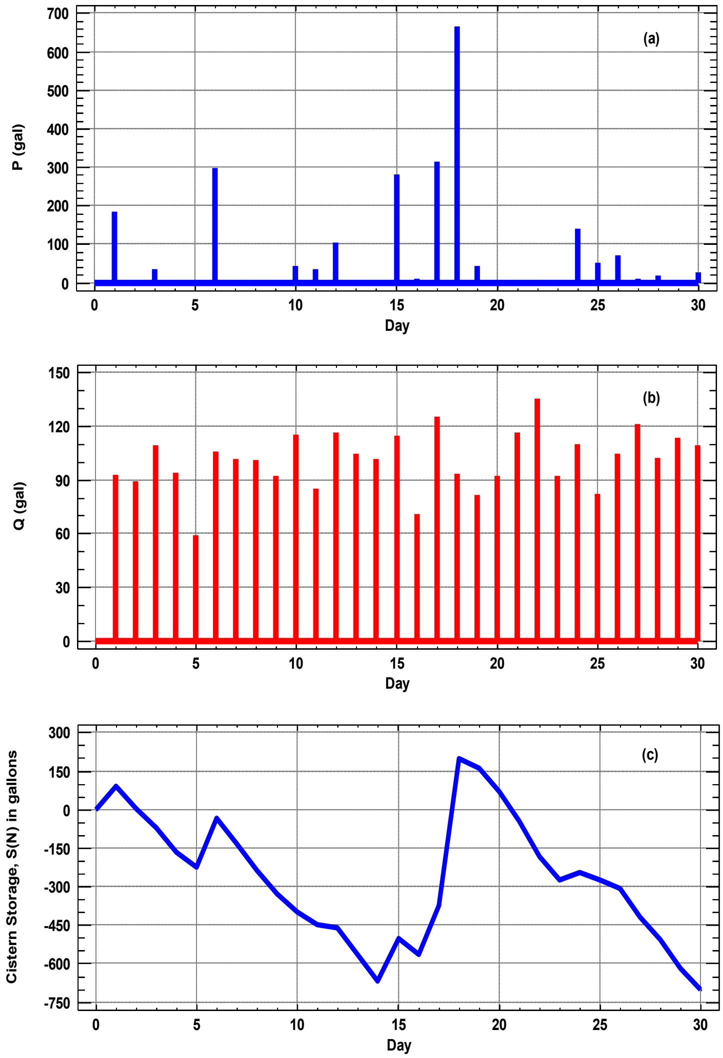

Figure 1. Intermittent daily rainfall (a) and random daily water demands (b) produce sizable fluctuations in the water level at the rainwater cistern (c). Rainfall data are from April 1953 in Cincinnati, Ohio. Note difference in scales on y-axis.

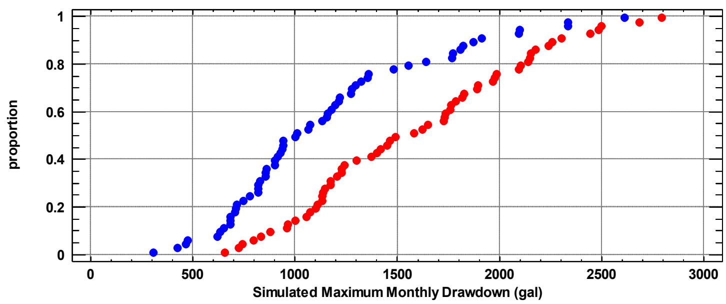

Figure 2. Empirical frequency distribution of 60 simulated maximum drawdowns at rainwater cistern for the months of April (blue) and October (red) in Cincinnati.

Statistics for net daily water supply for the benchmark case are shown for each month of the year in Table 2. The mean and the standard deviation of the net daily supply for each month are found using Equations (7) and (8) with rainfall statistics from Table 1. The maximum and minimum values are extracted from the 60-year simulation for each month. The five months from March through July have rainfall above the annual average and register a positive mean net daily water supply; the other seven months from August through February register a negative mean net daily water supply. The standard deviation of the net daily water supply tends to be larger during the "wet" months (March – July) than the "dry" months (August – February). The largest net daily water supply is 4421 gallons which occurred in March 1964 when over five inches of rain fell in one day.

Statistics for the simulated maximum drawdowns are shown for each month in Table 3. Not surprisingly, the average drawdown for the five wet months is slightly less than the average drawdown for the seven dry months. This simply reflects the fact that the wet months tend to receive greater rainfall amounts and/or experience more rainy days than the dry months do. Hence, wet months are more likely than dry months to deliver rainfall needed to replenish the cistern. As a consequence, wet months are more likely than dry months to reverse an accumulating rainwater deficit.

Table 4 shows the dimensionless daily drift computed using Equation (9) as the ratio of the entries in the first two rows of Table 2. The five wet months have a positive drift while the seven dry months have a negative drift. The dimensionless average and standard deviation of the monthly drawdown are computed as the ratio of the entries in Table 2 to the standard deviation of the net daily water supply given in Table 1. For example, the dimensionless average drawdown for January is 1461/230 = 6.35 and the dimensionless standard deviation for January is 538/230 = 2.34. The results in Table 4 reveal that both the dimensionless average and dimensionless standard deviation of the monthly drawdowns are negatively correlated with the drift. As the drift increases, the average and standard deviation of the drawdown tend to decrease. This behavior is the key to developing generalized dimensionless design charts for estimating maximum monthly drawdown and sizing the RWH cistern.

Table 2. Net daily water supply (gallons) for RWH cistern[1] in Cincinnati for period 1952-2012.

Statistic	Jan	Feb	Mar	Apr	May	June	July	Aug	Sep	Oct	Nov	Dec
μ_N	-17.0	-15.8	9.4	13.4	23.7	20.3	13.4	-9.7	-17.4	-17.7	-3.4	-11.0
σ_N	230	223	280	252	294	308	304	265	256	260	239	228
Maximum	3413	2411	4421	2106	2559	2873	3346	2939	3286	3675	2108	2344
Minimum	-161	-161	-168	-166	-168	-168	-166	-168	-168	-168	-166	-166

[1] Benchmark is a residence with 4 occupants, mean daily rainwater demand of 100 gallons and catchment area A=1400 ft^2.

Table 3. Maximum monthly drawdown (gallons) for RWH cistern[1] in Cincinnati for period 1952-2012.

Statistic	Jan	Feb	Mar	Apr[2]	May	June	July	Aug	Sep	Oct[2]	Nov	Dec
μ_c	1461	1394	1133	1152	1180	1227	1243	1473	1600	1583	1293	1387
σ_c	538	540	475	518	539	457	465	540	613	540	465	510
Maximum	2620	2710	2090	2620	2920	2530	2430	2720	2770	2800	2430	2720
Minimum	580	410	260	310	370	470	500	510	380	660	490	700

[1] Benchmark is a residence with 4 occupants, mean daily rainwater demand of 100 gallons and catchment area A=1400 ft^2.
[2] Full distributions of simulated drawdowns for April and October are plotted in Figure 2.

Table 4. Dimensionless drift and maximum monthly drawdown for RWH cistern[1] in Cincinnati, 1952-2012.

Statistic	Jan	Feb	Mar	Apr	May	June	July	Aug	Sep	Oct	Nov	Dec
Drift, ε	-0.074	-0.071	0.034	0.053	0.081	0.066	0.044	-0.037	-0.068	-0.068	-0.014	-0.048
μ_c/σ_N	6.35	6.25	4.04	4.57	4.01	3.98	4.09	5.56	6.04	6.09	5.41	6.08
σ_c/σ_N	2.34	2.42	1.70	2.06	1.83	1.48	1.53	2.04	2.39	2.08	1.95	2.24

[1] Benchmark is a residence with 4 occupants, mean daily rainwater demand of 100 gallons and catchment area A=1400 ft^2.

3.4 Dimensionless Design Chart

Equation (5) is rescaled by the standard deviation of the net daily water supply to provide a dimensionless design expression,

$$K(\alpha) = \left(C(\alpha)/\sigma_N\right) = \left(\mu_C/\sigma_N\right) + \xi(\alpha)\left(\sigma_C/\sigma_N\right) \tag{10}$$

Here $K(\alpha)$ is the α percentile of the dimensionless cistern capacity and $\zeta(\alpha)$ is the frequency factor of a suitable underlying probability distribution. Simple linear regression using entries in Table 4 gives,

$$\begin{aligned} \left(\mu_C/\sigma_N\right) &= 5.07 - 16.01\varepsilon \qquad \left(R^2 = 0.94\right) \\ \left(\sigma_C/\sigma_N\right) &= 1.97 - 4.41\varepsilon \qquad \left(R^2 = 0.68\right) \end{aligned} \tag{11a,b}$$

These results confirm the previous observation that both the dimensionless average and dimensionless standard deviation of the monthly drawdowns exhibited negative correlation with dimensionless drift. The linear association between drift and average cistern size is especially strong. There are a number of potentially suitable candidates for an underlying probability distribution for cistern size (e.g., normal, log-normal, extreme value, etc). For the sake of expedience and illustrative purposes, the normal distribution is used here. Putting Eqs (11a,b) into Equation (10) [and noting $\xi(0.50) = 0.00, \xi(0.90) = 1.28, \xi(99) = 2.33$ for the normal distribution] gives,

$$\begin{aligned} K(0.50) &= 5.1 - 16\varepsilon \\ K(0.90) &= 7.6 - 22\varepsilon \\ K(0.99) &= 9.7 - 26\varepsilon \end{aligned} \tag{12a,b,c}$$

The trio of dimensionless expressions in Eqs (12a,b,c) are plotted in Figure 3 for values of dimensionless drift ranging from -0.10 to +0.10. To recover the actual cistern size, it is necessary to multiply the dimensionless cistern size K by the standard deviation of the net daily water supply, found as the square root of Equation (8).

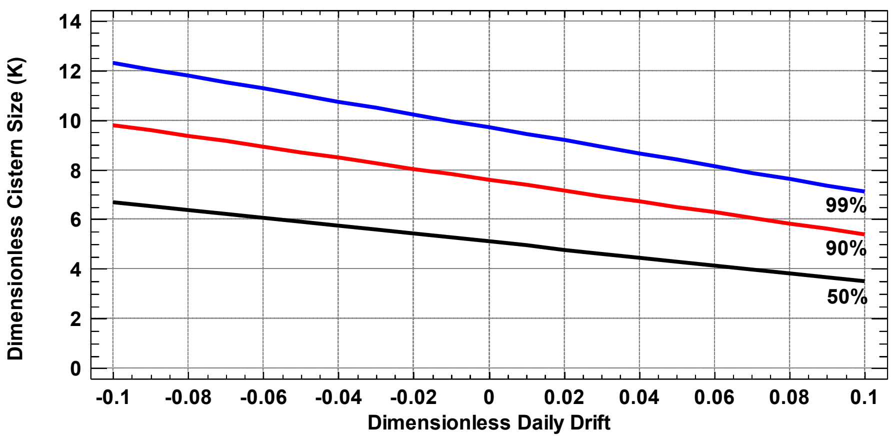

Figure 3. Dimensionless design chart for sizing rainwater harvesting cisterns in Cincinnati.

4. Application

The maximum monthly drawdown is a surrogate for the minimum required cistern needed at a rainwater harvesting operation. Two options have been presented for estimating the maximum monthly drawdown. One is a conventional frequency analysis per Equation (5). The second is a dimensionless approach per Equations 12a,b,c.

4.1 Example 1

The benchmark case is used to estimate the 50th, 90th and 99th percentile of the required cistern size for all months of the year. There are 36 cases, so to illustrate, consider the 90th percentile for January. From Table 3, find μ_c = 1461 gallons and σ_c = 538 gallons; Equation (5) gives C(0.90) = 1461+(1.28×538) = 2150 gallons. From Table 2 find σ_N = 230 gallons and from Table 4 find ε = -0.074. Thus Equation (12b) gives C(0.90) = σ_NK(0.90) = σ_N[7.6-(22)(-0.074)] = 2122 gallons. Results for this January example and the other 35 companion cases are summarized in Table 5. The agreement between the two approaches is reasonably good, but differences in estimated maximum monthly drawdown (i.e., minimum cistern size) tend to increase as reliability grows.

Table 5. Required minimum size (gallons) of RWH cistern for benchmark case[1] in Cincinnati.

Statistic	Jan	Feb	Mar	Apr	May	June	July	Aug	Sep	Oct	Nov	Dec
50th percentile												
Eq. 5	1461	1394	1133	1152	1180	1227	1243	1473	1600	1583	1293	1387
Eq. 12a	1445	1391	1276	1072	1118	1246	1336	1508	1584	1609	1272	1338
% diff	-1.10	-0.22	12.62	-6.94	-5.25	1.55	7.48	2.38	-1.00	1.64	-1.62	-3.53
90th percentile												
Eq. 5	2150	2086	1742	1816	1871	1813	1839	2165	2386	2275	1889	2041
Eq. 12b	2122	2048	1924	1626	1716	1900	2022	2235	2334	2370	1895	1978
% diff	-1.30	-1.82	10.45	-10.46	-8.28	4.80	9.95	3.23	-2.18	4.18	0.32	-3.09
99th percentile												
Eq. 5	2712	2650	2238	2357	2434	2290	2325	2729	3026	2839	2375	2573
Eq. 12c	2677	2578	2454	2082	2210	2438	2584	2823	2939	2984	2400	2496
% diff	-1.29	-2.72	9.65	-11.67	-9.20	6.46	11.14	3.44	-2.88	5.11	1.05	-2.99

[1] Benchmark is a residence with 4 occupants, mean daily rainwater demand of 100 gallons and catchment area A=1400 ft^2.

4.2 Example 2

The utility of the dimensionless design approach is that it can be applied to a wide variety of different RWH scenarios, provided the standard deviation of the net daily water supply and the dimensionless drift are known. To illustrate, consider a large apartment complex in Cincinnati where the average daily non-potable water demand is 3,000 gallons with a 25% coefficient of variation. Find the maximum drawdown (minimum cistern size) to provide water with 50%, 90% and 99% reliability during October assuming catchment areas of 40,000 ft^2, 50,000 ft^2 and 60,000 ft^2. From Table 1, the mean and standard deviation of daily rainfall in October are, μ_p = 0.0592 gal/ft^2 and σ_p = 0.1845 gal/ft^2. Eqs (7) and (8) give μ_N = -632 gallons and σ_N = 7,418 gallons for the 40,000 ft^2 catchment. The corresponding daily drift is ε = (-632/7418) = -0.085. Using this drift value in Eqs (12a,b,c) gives dimensionless drawdowns of $K(50)$ = 6.46, $K(90)$ = 9.47 and $K(99)$ = 11.91. These K values are scaled up by σ_N to yield the desired percentiles of the drawdown in October. Key results for all three catchments are summarized in Table 6. As an independent check, a single Excel run was used to generate an empirical frequency distribution of October drawdowns, similar in nature to Figure 2. From this simulation, point estimates of the drawdown percentiles were found and also posted in Table 6.

Table 6. Maximum drawdown (gallons) on RWH cistern for October in Cincinnati.

Area, A (ft^2)	Drift, ε	σ_N (gallons)	$C(\alpha)=K(\alpha)\sigma_N$ (gallons)			Single Excel Simulation: C (gallons)		
			50th	90th	99th	50th	90th	99th
40,000	-0.085	7,418	47,920	70,250	88,350	48,430	70,020	87,600
50,000	0.000	9,255	47,200	70,340	89,770	44,860	66,100	83,390
60,000	+0.050	11,095	47,700	72,120	93,200	42,320	63,770	81,240

For $\varepsilon<0$ (A=40,000 ft^2), the predicted drawdowns in columns 4-6 and simulated drawdowns in columns 7-9 agree within 1% of each other. For $\varepsilon=0$ (A=50,000 ft^2), the predicted drawdowns exceed the simulated drawdowns by about 6%. The difference grows to about 13% for $\varepsilon>0$ (A=60,000 ft^2). This trend suggests that the right side of all three lines in Figure 3 may need to be lowered slightly. This adjustment can be accomplished by reducing the intercept and/or increasing the magnitude of the slope for each of the three expressions in Equation 12a,b,c. It is of interest to note in Table 6 that the monthly drawdown at a given percentile is not particularly sensitive to the drift. The magnitude of the drawdown is affected mainly by the number of consecutive days without rain. For a given household demand (when the drift is close to zero), the key factor determining the accumulated deficit is the consecutive days without rain, irrespective of the catchment area or the daily drift.

The impact of daily drift on cistern operation is most notable at the monthly mass balance check. Negative drift will tend to deplete cistern storage, while positive drift will tend to replenish cistern storage. The example in Table 6 with $\varepsilon<0$ corresponds to an average daily water shortfall of 632 gallons or about 19,600 gallons over the course of a 31-day month. This represents the average amount of municipal drinking water that must be added to the cistern during October to maintain a stable water level. In contrast, the case with $\varepsilon>0$ corresponds to an average daily water surplus of 553 gallons or about 17,100 gallons over the course of a 31-day month. This is the amount of excess water that would be available, on average, for irrigation or other yard work during October.

5. Conclusions

The maximum monthly drawdown in a rainwater cistern serves as a surrogate for the minimum required size of the cistern needed at a rainwater harvesting operation. Owing to the inherent randomness in the amount and occurrence of daily rainfall, the maximum monthly drawdown at a rainwater cistern varies over a wide range. Two options were presented for estimating the maximum monthly drawdown. Option one is a conventional frequency analysis but requires apriori knowledge of the probability distribution of the maximum monthly drawdowns. Option two is a dimensionless design chart (Figure 3) that is easily implemented with five readily available basic parameters: mean and variance of daily precipitation; mean and variance of daily water demand; catchment area.

This study is a small step in exploring size requirements for RWH cisterns. The emphasis has been on monthly drawdowns. Additional work is needed to extend results to an annual time step and to account for pronounced seasonality in precipitation and/or demands. The time between rain events is likely an important design factor, but it has not been explicitly considered here. Using a rainwater harvesting system to provide water for non-potable uses alleviates the load on water treatment facilities and distribution systems, decreasing energy demand. Life cycle costing is needed to estimate the overall benefit of using a rainwater harvesting system.

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