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A Real-Time and High-Precision Algorithm for Frequency Estimation by Fusing Multi-segment Signals

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Abstract

In the frequency estimation of multi-segment signals, the error caused by discontinuity among different segments cannot be ignored. By correcting the phase difference of each segment, the results can be enormously improved, but the computational complexity of the algorithm grows on the same time. To resolve this problem, a new fusion algorithm based on spectrum zooming is proposed. The new algorithm performs spectrum zoom analysis to reduce the computational complexity while the precision is guaranteed, which is suitable to those real-time applications. Performance of the algorithm is demonstrated and analyzed through computer simulations.

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Keywords: frequency estimation; information fusion; spectrum zoom

1. Introduction

Signal frequency estimation is often applied in the area of radar and communications technology, especially under conditions of low signal-to-noise ratio (SNR) and short duration. The spectrum leakage of a short-duration signal is severe through windowing theorem, which reduces the precision of the estimation. There have been three kinds of methods to resolve the problem. The first kind [1-3] is to correct the spectrum with sorts of mathematical models, which have high precision, but they're sensitive to SNR. The second kind [4, 5] is to improve the hardware, e.g. increasing sampling frequency and

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reducing noise of instruments, these methods usually cost a lot. The last kind [6-8] is to fuse multi-segment signals, which can improve the performance of estimation and have no need of raising the cost.

Existing algorithms to fuse multi-segment signals include spectrum averaging, phase accumulation [6] and phase correction [7, 8]. The first method is to calculate the spectrum of each segment by utilizing Fourier transform (FT), and then obtain the average value of all the segments' spectrum. This method is sensitive to noise and has low precision because it doesn't consider the phase discontinuity among different segments. The second method is to normalize sampling start time of each segment with that of the first segment, which would reduce the spectral spreading caused by discontinuity of the phase. Compared to the first method, this method has better performance, but it can only be used in some special applications. The third method is to correct the phase of each segment to achieve high precision, and it has good anti-noise performance, but the computational complexity is also much higher than the other two.

This paper proposes a modified algorithm based on the third method mentioned above. Compared to the original algorithm, the new one has lower complexity and the precision is guaranteed, this advantage is required in some real-time applications.

2. Algorithm Principals

In [7] the range of the signal frequency is supposed to be known, but in some practical situations it is not, and it requires large computation if we perform accurate spectrum analysis in a wide frequency band. To solve this problem, the method of combing rough estimation with fine estimation is performed. The rough estimation has little computation, and large frequency spacing, while it has low precision. And then the fine estimation is performed to improve the precision.

2.1. Rough Estimation with Phase Correction

Suppose there is an M-segment signal x_0, the frequency is f_1, sampling frequency is f_s, the length of each segment is *D*, considering the m^{th} segment, and the total length of the first (*m*-1) segments is *B*(*m*)=(*m*-1)*D*, suppose the initial phase is $\theta(m)$, and the signal model is:

$$x_0[B(m)+a]=\cos[\theta(m)+2\pi f_1 a/f_s] \quad (1)$$

Where $a\in[1,D]$, and $x_0[B(m)+a]=\cos[\theta(m)+2\pi f_1 a/f_s]$ represents the sampling value of x_0, the discontinuity of the phase means the *M* segment signals cannot be connected to form a continuous signal, and the spectral spreading will affect the estimation precision. But if we correct the phase, the result would be better. The steps of phase correction are as follows:

First, we perform FT to each segment of the signal to obtain

$$F_1(m,k)=\sum_{a=1}^{D}x_0[B(m)+a]e^{-j2\pi a f_a(k)/f_s} \quad (2)$$

Then correct the phase differences of $F_1(m,k)$, and sum the spectrums up, to obtain

$$F_2(k)=\sum_{m=1}^{M}[e^{-jU_0(m,k)}F_1(m,k)] \quad (3)$$

Where $U_0(m,k)$ is phase correction factor, and is given by the following equation

$$U_0(m,k)=[\theta(m)-\theta(1)]+2\pi B(m)[f_a(k)-f_1]/f_s \quad (4)$$

The item $\theta(m)-\theta(1)$ is the initial phase difference between the first and the m^{th} segment, and $2\pi B(m)[f_a(k)-f_1]/f_s$ represents the phase difference caused by frequency error.

In practice, the obtained signal is polluted by noise and interferences, suppose the noise and interferences to be Y, and equation (1) becomes

$$F_1^{'}(m,k)=\sum_{a=1}^{D}\{\cos[\theta(m)+2\pi f_1 a/f_s]+Y\}e^{-j2\pi a f_a(k)/f_s} \tag{5}$$

Obviously, accurate estimations of f_1 and $\theta(m)$ can't be obtained from Eq. (5) due to the noise and interruptions, hence $U_0(m,k)$ can't be used directly. Here we utilize searching sequences, enact the sequence to be $f_b(r)$, where $r\in[1,R]$, and calculate the estimation of $\theta(m)$ corresponds to $f_b(r)$, named $\theta_1(r,m)$. Then we substitute $f_b(r)$ and $\theta_1(r,m)$ into $U_0(m,k)$, obtaining modified phase correction factor $U(r,m,k)$ as the estimation of $U_0(m,k)$ corresponds to $f_b(r)$.

In summary, the practical calculating steps are as follows:

(1) Calculate the initial phase of each segment corresponding to $f_b(r)$. Suppose the nearest value to f_b among f_a is $f_a[h(r)]$, where $h(r)\in[1,K]$, and the phase is calculated by

$$\theta_1(r,m)=angle\{F_1^{'}[m,h(r)]\} \tag{6}$$

Where angle(t) means the phase of variable t.

(2) Calculate the $R\times M\times K$ phase correction factor matrix U. The element in position (r, m, k) of U can be calculated by

$$U(r,m,k)=\theta_1(r,m)-\theta_1(r,1)-2g_2B(m)-g_3 \tag{7}$$

Where

$$g_2=\pi[f_b(r)-f_a(k)]/f_s \tag{8}$$

$$g_3=\pi(D+1)\{f_b(r)-f_a[h(r)]\}/f_s \tag{9}$$

(3) Replace $U_0(m,k)$ with $U(r,m,k)$, $F_1(m,k)$ with $F_1^{'}(m,k)$, and substitute them into equation (3) to obtain spectrum $F_3(r,k)$, frequency corresponding to the maximum of $|F_3(r,k)|$ is the estimated value of f_1.

2.2. Fine Estimation

After estimating the frequency roughly, spectrum zoom analysis is performed around the estimated frequency. Referring to Zoom-FFT theory [9], another FT in a smaller frequency range and with higher spectral resolution is performed. Though FT is performed twice, the total computational complexity is still less than that of an accurate fourier analysis because each time requires little computation. This advantage is more obvious in high frequency applications, which will be analyzed in section 2.3.

In Zoom-FFT theory, we can only analyze a low-pass band of the spectrum because of the character of FFT, so modulation and low-pass filtering are necessary there, which would increase computational complexity of the algorithm, and the Gibbs effect of low-pass filter will affect the precision of estimation [10]. The phase correction algorithm proposed in this paper doesn't utilize FFT, but discrete-time FT (DTFT) instead. Because DTFT can analyze any part of the spectrum, modulation and low-pass filtering are avoided, which would reduce computational complexity and guarantee the precision.

2.3. Analysis

Suppose the frequency range of Fourier analysis without spectrum zooming is $[0,f_s]$, and the resolution is Δf_n, then N-point Fourier calculation is needed, where $N=f_s/\Delta f_n$.

If perform fourier analysis with spectrum zooming, let $A_1=f_s/B$, and $A_2=N/M$, where B is the bandwidth of spectrum zooming, A_1 is spectrum zooming factor, M is the number of fourier points of rough estimation, and A_2 is computational factor.

In rough estimation, M-point fourier calculation is needed to obtain f_a, and the resolution is

$$\Delta f_{sM} = f_s / M \tag{10}$$

And then perform spectrum zoom analysis in the range of $[f_a - B/2, f_a + B/2]$. Let the number of fourier points be M again, then the total computation of $2M$-point fourier calculation is needed, and the resolution becomes

$$\Delta f_m = (A_2 / A_1) \times \Delta f_n \tag{11}$$

From the above we can see that the computational complexity of the algorithm with spectrum zooming is $2 / A_1$ of that without, and the resolution is A_2 / A_1 of that without.

3. Simulations

In this section, Monte-Carlo simulations are performed to illustrate advantages of this algorithm, we compare it with the algorithm without spectrum zooming. Let $f_1 = 100MHz$, and perform simulations in conditions of different SNR and different signal length respectively. In each condition, 50 experiments are performed.

3.1. Different SNR

Mean values of each algorithm under different SNR are shown in Fig. 1. From the result we can see their performances are fairly close. To compare performance and efficiency of the two algorithms in detail, let $SNR = -6dB$, and the results are shown in table 1, by comparison we can see that the algorithm with spectrum zooming consumes less computation, and the precision has been improved.

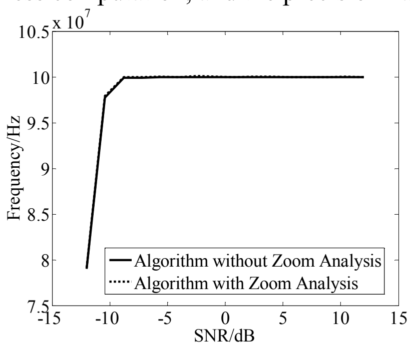

Fig. 1 Mean values of each algorithm under different SNR

Table 1 Comparison of the two algorithms under condition of SNR= -6dB

Algorithm	CPU time	Mean of estimation (*MHz*)	Idea value (*MHz*)
Without zoom analysis	6.1686e+003	99.88	100
With zoom analysis	11.2813	99.9968	100

3.2. different signal length

Fig. 2(a) and (b) respectively illustrates mean values and the ratio of time consumption of the two algorithms with different signal length. By comparison we can see that the algorithm with spectrum zooming has more accurate mean values and its efficiency is over four hundred times of that without.

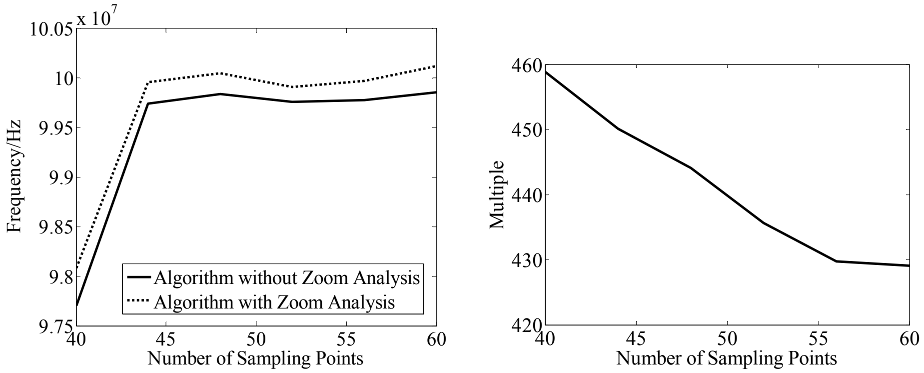

Fig. 2 (a) Mean values of each algorithm with different signal length, (b) The ratio of CPU time of the two algorithms

4. Conclusions

A modified frequency estimation algorithm based on multi-segment signal fusion is proposed. The innovation is that it combines phase correction with spectrum zooming. Phase correction is utilized to improve the precision of estimation. To reduce the computational complexity, estimation is performed twice. First a rough estimation is performed with low resolution, and then spectrum zooming to improve the precision. Simulation experiments show that the new algorithm has largely improved the efficiency while precision is guaranteed.

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