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## Probing the nature of cosmic acceleration

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### ABSTRACT

The cosmic acceleration is one of the most significant cosmological discoveries over the last century. The two categories of explanation are exotic component (dark energy) and modified gravity. We constrain the two types of model by a joint analysis with perturbation growth and direct  $H(z)$  data. Though the minimal  $\chi^2$  of the  $\Lambda$ CDM is almost the same as that of DGP, in the sense of consistency we find that the dark energy ( $\Lambda$ CDM) model is more favored through a detailed comparison with the corresponding parameters fitted by expansion data.

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### 1. Introduction

The acceleration of the universe is one of the most significant cosmological discoveries over the last century [1]. Various explanations of this acceleration have been proposed, see [2] for recent reviews with fairly complete lists of references of different models. However, although fundamental for our understanding of the universe, its nature remains as a completely open question nowadays.

There are two main categories of proposals. One is that the acceleration is driven by some exotic matter with negative pressure, called dark energy. The other suggests that general relativity fails in the present Hubble scale. The extra geometric effect is responsible for the acceleration. Surely, there are some proposals which mix the two categories. Mathematically, in the dark energy model we present corrections to the right-hand side of Einstein equation (matter part), while the correction terms appear in the left-hand side of Einstein equation (geometric part).

$\Lambda$ CDM model is the most popular and far simple dark energy model, in which vacuum energy with the equation of state (EOS)  $w = -1$  accelerates the universe. From theoretical considerations and by observational implications, people put forward several other candidates for dark energy, such as quintessence ( $-1 < w < -1/3$ ), phantom ( $w < -1$ ), etc. Also there are many possible corrections to the geometric part of the theory. One of the leading modified gravity model is Dvali–Gabadadze–Porrati (DGP) model [3], for a review, see [4]. In the DGP model, the bulk is a flat Minkowski spacetime, but an induced gravity term appears on a

tensionless brane. In this model, gravity appears 4-dimensional at short distances. But, at a distance larger compared to some freely adjustable crossover scale  $r_0$  it is altered through the slow evaporation of the graviton off our 4-dimensional brane world universe into an unseen, yet large, fifth dimension.

We should find the correct, at least exclude the incorrect models in the model sea. The first step is to discriminate between dark energy and modified gravity, whose nature are completely different. To construct a model simulating the accelerated expansion is not very difficult. That is the reason why we have so many different models. Recently, some suggestions are presented that growth function  $\delta(z) \equiv \delta\rho_m/\rho_m$  of the linear matter density contrast as a function of redshift  $z$  can be a probe to discriminate between dark energy and modified gravity [5,6] models. The growth function can break the degenerations between dark energy and modified gravity models which share the same expansion history.

There is an approximate relation between the growth function and the partition of dust matter in standard general relativity [7],

$$f \equiv \frac{d \ln \delta}{d \ln a} = \Omega_m^\gamma, \quad (1)$$

where  $\Omega_m$  is the density partition of dust matter,  $a$  denotes the scale factor, and  $\gamma$  is the growth index. This relation is a perfect approximation at high redshift region. Also, it can be used in low redshift region, see for example [8]. It is shown that the relation (1) is also valid in the case of modified gravity theory [5]. The theoretical value of  $\gamma$  for  $\Lambda$ CDM model is 6/11 [6,8], while, for spatially flat DGP model is 11/16 [5]. The observation data of perturbation growth are listed in Table 1.

It is shown that  $\Lambda$ CDM model is consistent with the current growth data [16]. The data seem to weaken a spatially flat DGP model, whose  $\gamma = 11/16$ . However, it is found that the growth in-

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**Table 1**  
Observed perturbation growth as a function of redshift  $z$ , see also [15]

$z$	$f_{\text{obs}}$	Reference
0.15	$0.51 \pm 0.11$	[9]
0.35	$0.70 \pm 0.18$	[10]
0.55	$0.75 \pm 0.18$	[11]
0.77	$0.91 \pm 0.36$	[12]
1.4	$0.90 \pm 0.24$	[13]
3.0	$1.46 \pm 0.29$	[14]

dex  $\gamma$  is 4/7 in a non spatially flat DGP model [15], which is very closed to the index of  $\Lambda$ CDM. Thus, the DGP model may be still consistent with current growth data.

In this Letter we take a different strategy. The previous works were concentrated on the limit of the growth index and made some approximations on it (often the high  $z$  limit was assumed and an approximation was made at linear order), in which only approximate asymptotic value of  $\gamma$  can be obtained. In fact, the perturbation growth  $f$  is a variable with respect to  $z$ , as displayed in Table 1. By using these growth data we constrain the parameters in  $\Lambda$ CDM model and DGP model, respectively.

The other one which is very useful but not widely used in model constraint data is the set of direct  $H(z)$ .  $H(z)$  is derived by a newly developed scheme to obtain the Hubble parameter directly at different redshift [17], which is based on a method to estimate the differential ages of the oldest galaxies [18]. By using the previously released data [19], Simon et al. obtained a sample of direct  $H(z)$  data in the interval  $z \in (0, 1.8)$  [20], just as the same interval of the data of luminosity distances from supernovae. For the present sample of growth data derived with the assumption of the expansion behaviors of the universe, we will present joint fittings to obtain the constraints on the  $\Lambda$ CDM and DGP, respectively. Then, through comparing with allowed regions by expansion constraint using different observations, including supernovae (SN), cosmic microwave background (CMB), baryon acoustic oscillations (BAO) etc., we examine which model is more self-consistent.

This article is organized as follow: In Section 2 we construct the evolution equation for  $f$  in a very general frame. In Section 3, by using the growth data and  $H(z)$  data we present the parameter constraints of  $\Lambda$ CDM and DGP, respectively. Our conclusion and some discussions appear in Section 4.

## 2. The evolution equation for the growth function $f$

We consider a mixed model in which dark energy drives the universe to accelerate in frame of modified gravity. For FRW universe in modified gravity, the Friedmann equation can be written as,

$$H^2 + \frac{k}{a^2} + h(a, \dot{a}, \ddot{a}) = \frac{8\pi G}{3}(\rho_m + \rho_e), \quad (2)$$

where  $H$  denotes the Hubble parameter,  $h$  comes from the corrections to general relativity.  $\rho_m$  and  $\rho_e$  represent the density of dust matter and the exotic matter, respectively. A dot implies the derivative with respect to cosmic time  $t$ . Comparing with the corresponding Friedmann equation in standard general relativity, we obtain the density of effective dark energy,

$$\rho_{\text{de}} = \rho_e - \frac{3}{8\pi G}h. \quad (3)$$

Here we call  $h$  geometric sector of dark energy. The behavior of the effective dark energy has been separately discussed in some previous works. For example, it is investigated in detail in a modified gravity model where a four-dimensional curvature scalar on the brane and a five-dimensional Gauss–Bonnet term in the bulk are present [21].

For any modified gravity theory, Bianchi identity is a fundamental requirement. Using the continuity equation of the dust matter and the Bianchi identity, we derive,

$$\dot{\rho}_{\text{de}} + 3H\rho_{\text{de}}(1 + w_{\text{de}}) = 0, \quad (4)$$

which yields,

$$w_{\text{de}} = -1 - \frac{1}{3} \frac{d \ln \rho_{\text{de}}}{d \ln a}, \quad (5)$$

where  $w_{\text{de}}$  is the EOS (equation of state) of the effective dark energy.

After the matter decoupling from radiation, for a region well inside a Hubble radius, the perturbation growth satisfies the following equation in standard general relativity [22],

$$\ddot{\delta} + 2H\dot{\delta} - 4\pi G\rho_m\delta = 0. \quad (6)$$

It is found that the perturbation equation is still valid in a modified gravity theory if we replace the Newton constant  $G$  with an effective gravitational parameter  $G_{\text{eff}}$ , which is defined by Cavendish-type experiment [5,23]. (This point may need more studies.)  $G_{\text{eff}}$  may be time-dependent, for example in Bran–Dicke theory and in generalized DGP theory [24].

With the partition functions,

$$\Omega_m = \frac{8\pi G\rho_m}{3H^2}, \quad (7)$$

$$\Omega_{\text{de}} = \frac{8\pi G\rho_{\text{de}}}{3H^2}, \quad (8)$$

$$\Omega_k = -\frac{k}{a^2H^2}, \quad (9)$$

the perturbation equation (6) becomes,

$$(\ln \delta)'' + (\ln \delta)'^2 + \left(2 + \frac{H'}{H}\right)(\ln \delta)' = \frac{3}{2}\alpha\Omega_m, \quad (10)$$

where a prime denotes the derivative with respect to  $\ln a$ ,  $\alpha$  is the strength of the gravitational field scaled by that of standard general relativity,

$$\alpha = \frac{G_{\text{eff}}}{G}. \quad (11)$$

$\Omega_m$  and  $\Omega_k$  redshift as

$$\Omega_m = \frac{\Omega_{m0}(1+z)^3}{\Omega_{m0}(1+z)^3 + \Omega_{k0}(1+z)^2 + \frac{8\pi G\rho_{\text{de}}}{3H_0^2}}, \quad (12)$$

$$\Omega_k = \frac{\Omega_{k0}(1+z)^2}{\Omega_{m0}(1+z)^3 + \Omega_{k0}(1+z)^2 + \frac{8\pi G\rho_{\text{de}}}{3H_0^2}}, \quad (13)$$

where 0 denotes the present value of a quantity. The growth function defined in (1) is just  $(\ln \delta)'$ . Thus (10) generates,

$$f' + f^2 + \left[\frac{1}{2}(1 + \Omega_k) + \frac{3}{2}w_{\text{de}}(\Omega_m + \Omega_k - 1)\right]f = \frac{3}{2}\alpha\Omega_m, \quad (14)$$

where we have used

$$\frac{H'}{H} = -\Omega_k - \frac{3}{2}[\Omega_m + (1 + w_{\text{de}})\Omega_{\text{de}}], \quad (15)$$

and

$$\Omega_m + \Omega_k + \Omega_{\text{de}} = 1. \quad (16)$$

In  $\Lambda$ CDM model, we have  $w_{\text{de}} = -1$  and  $\alpha = 1$ . For the self-accelerating branch of DGP model [15],

$$w_{\text{de}} = \frac{-1 + \Omega_k}{1 + \Omega_m - \Omega_k}, \quad (17)$$

and

$$\alpha = \frac{4\Omega_m^2 - 4(1 - \Omega_k)^2 + 2\sqrt{1 - \Omega_k}(3 - 4\Omega_k + 2\Omega_m\Omega_k + \Omega_k^2)}{3\Omega_m^2 - 3(1 - \Omega_k)^2 + 2\sqrt{1 - \Omega_k}(3 - 4\Omega_k + 2\Omega_m\Omega_k + \Omega_k^2)}. \quad (18)$$

$r_c$  is another important parameter in DGP model, which is defined by the relative strength of five-dimensional gravity to four-dimensional gravity  $r_c = G_5/G$ . Here  $G_5$  is the five-dimensional gravity constant. We define the partition of  $r_c$  as

$$\Omega_{r_c} \equiv 1/(H_0^2 r_c^2). \quad (19)$$

One can derive the following relation from Friedmann equation of DGP model,

$$1 = [\sqrt{\Omega_{m0} + \Omega_{r_c}} + \sqrt{\Omega_{r_c}}]^2 + \Omega_{k0}. \quad (20)$$

With (14) and the observed data of  $f$  in Table 1, we can fit parameters of the models, either dark energy or modified gravity.

### 3. Joint analysis with the growth data and the direct $H(z)$ data

In this section we fit  $\Omega_{m0}$ ,  $\Omega_{k0}$  in  $\Lambda$ CDM model and  $\Omega_{m0}$ ,  $\Omega_{r_c}$  in DGP model with the growth data and direct  $H(z)$  data by  $\chi^2$  statistics, respectively. Before fitting with the two sets of data, we present some discussions about them.

The present growth data in Table 1 are far from being precise. We have a sample consisting of only six points, and the error bars of the growth data are at the same order of the growth data themselves. The reason roots in the method by which we derive the data set.

In the present stage we do not find any absolute probes to the perturbation amplitude. People extract the information of perturbation growth from galaxy clustering data through redshift distortion parameter  $\beta$  observed in the anisotropic pattern of galactic redshifts. We need the galaxy bias factor  $b$  to get the perturbation growth  $f = b\beta$ . The current available galaxy bias can be obtained mainly in two ways. The most popular method is to refer to the simulation results of galaxy formations [9,10,12,14]. At the present stage the simulations we obtained only in frame of  $\Lambda$ CDM model. The second method to get the galaxy bias depends on the CMB normalization [11]. Also,  $\Lambda$ CDM model is involved. Further, to convert from redshift  $z$  to comoving distance one should assume a clear relation between distance and redshift. For instance Tegmark et al. [10] adopt a flat  $\Lambda$ CDM model in which  $\Omega_{m0} = 0.25$ . They also tested that if a different cosmological model is assumed for the conversion from redshift to comoving distance, the measured dimensionless power spectrum is varied very slightly (<1%).<sup>1</sup>

Hence, we see that people obtain data in Table 1 always by assuming a  $\Lambda$ CDM model. Its reliability may decrease when we use it in the scenarios of other models. Fortunately, it is pointed out that this problem can be evaded at least in the DGP model since the expansion history in DGP with proper parameters is very similar to that of  $\Lambda$ CDM [15]. Since the growth data are derived with some assumptions of expansion history, we should fit the model by growth data together with expansion data.

The direct  $H(z)$  data are independent of the data of luminosity distances and reveal some fine structures of  $H(z)$ . They have not been widely used in the constraints on dark energy models up to now. Here we present a joint fitting of  $\Lambda$ CDM and DGP with perturbation growth data and direct  $H(z)$  data.

We show the sample of  $H(z)$  data in Table 2.

**Table 2**  
The direct observation data of  $H(z)$  [20]

$z$	0.09	0.17	0.27	0.40	0.88	1.30	1.43	1.53	1.75
$H(z)$ (km s <sup>-1</sup> Mpc <sup>-1</sup> )	69	83	70	87	117	168	177	140	202
68.3% confidence interval	$\pm 12$	$\pm 8.3$	$\pm 14$	$\pm 17.4$	$\pm 23.4$	$\pm 13.4$	$\pm 14.2$	$\pm 14$	$\pm 40.4$

Table 2 displays an unexpected feature of  $H(z)$ : It decreases with respect to the redshift  $z$  at  $z \sim 0.3$  and  $z \sim 1.5$ , which is difficult to be found in the data of supernovae since the wiggles will be integrated in the data of luminosity distances. A study shows that the model whose Hubble parameter is directly endowed with oscillating ansatz by parameterizations fits the data much better than those of  $\Lambda$ CDM, Int $\Lambda$ CDM, XCDM, IntXCDM, VecDE, IntVecDE [25]. A physical model, in which the phantom field with natural potential, i.e., the potential of a pseudo Nambu-Goldstone Boson (PNGB) plays the role of dark energy, is investigated in [26]. The oscillating behavior of  $H$  appears naturally in this model.

For  $\Lambda$ CDM, in the joint analysis with a marginalization of  $H_0$ ,  $\chi^2$  reads

$$\chi^2(\Omega_{m0}, \Omega_{k0}) = \sum_{i=1}^6 \left[ \frac{f_{\text{obs}}(z_i) - f_{\text{th}}(z_i; \Omega_{m0}, \Omega_{k0})}{\sigma_{f_{\text{obs}}}} \right]^2 + \sum_{i=1}^9 \left[ \frac{H_{\text{obs}}(z_i) - H_{\text{th}}(z_i; \Omega_{m0}, \Omega_{k0})}{\sigma_{f_{\text{obs}}}} \right]^2 + \left( \frac{H_0 - 72}{0.08} \right)^2, \quad (21)$$

where  $f_{\text{obs}}$  denotes the observation value of the growth index, and  $f_{\text{th}}$  represents its theoretical value. We read  $f_{\text{obs}}(z_i)$ ,  $\sigma_{f_{\text{obs}}}$  from Table 1 and calculate  $f_{\text{th}}(z_i; \Omega_{m0}, \Omega_{k0})$  using (14). To get the theoretical value of  $f$  using (14), we need its initial value. Our considerations are as follows. In any dark energy model the universe should behave as the same one in some high redshift region such as  $z = 1000$ , that is, it behaves as standard cold dark matter (SCDM) model, which has been sufficiently tested by observations. In SCDM model we obtain  $f = 1$  by using (1). So we just take  $f = 1$  as the initial value at high enough redshift region. And the theoretical Hubble parameter reads,

$$H_{\text{th}}^2 = H_0^2 [\Omega_{m0}(1+z)^3 + \Omega_{k0}(1+z)^2 + 1 - \Omega_{m0} - \Omega_{k0}]. \quad (22)$$

We take the value of present Hubble parameter  $H_0$  from the HST key project  $H_0 = 0.72 \pm 0.08$  km s<sup>-1</sup> Mpc<sup>-1</sup> [29]. The result is shown in Fig. 1.

In DGP model, traditionally, we often use  $\Omega_{r_c}$  rather than  $\Omega_{k0}$  in fittings. There is no essential difference since they are constrained by (20). In the joint analysis with a marginalization of  $H_0$ ,  $\chi^2$  becomes

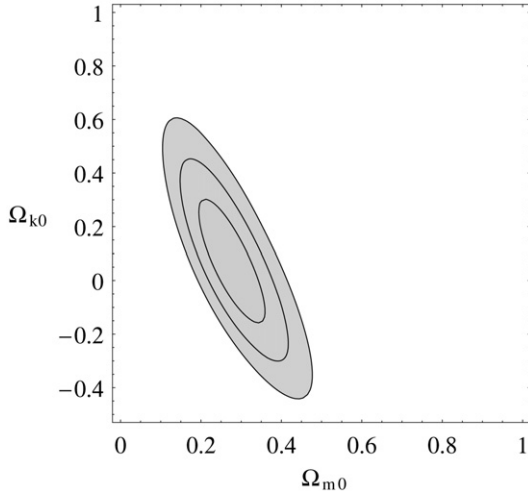
$$\chi^2(\Omega_{m0}, \Omega_{r_c}) = \sum_{i=1}^6 \left[ \frac{f_{\text{obs}}(z_i) - f_{\text{th}}(z_i; \Omega_{m0}, \Omega_{r_c})}{\sigma_{f_{\text{obs}}}} \right]^2 + \sum_{i=1}^9 \left[ \frac{H_{\text{obs}}(z_i) - H_{\text{th}}(z_i; \Omega_{m0}, \Omega_{r_c})}{\sigma_{f_{\text{obs}}}} \right]^2 + \left( \frac{H_0 - 72}{0.08} \right)^2, \quad (23)$$

where

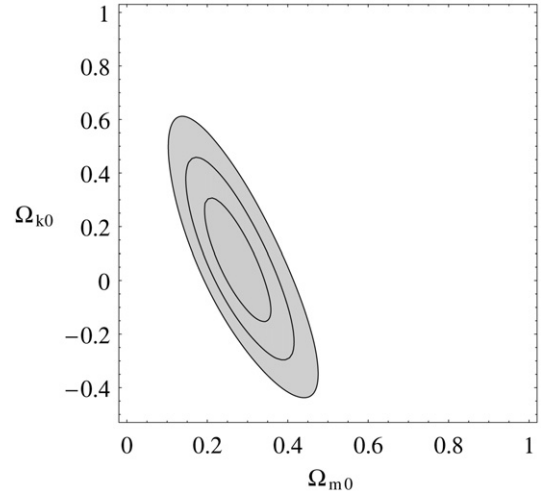
$$H_{\text{th}}^2 = H_0^2 [(\Omega_{m0}(1+z)^3 + \Omega_{r_c})^{1/2} + \Omega_{r_c}^{1/2}]^2 + \Omega_{k0}(1+z)^2]. \quad (24)$$

With the same reason as the case of  $\Lambda$ CDM we take  $f = 1$  as the initial value at high enough redshift. The result is illuminated in Fig. 2.

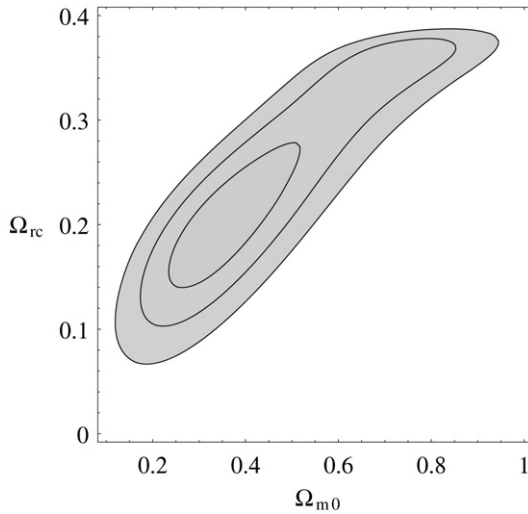
<sup>1</sup> The anonymous referee's comments.



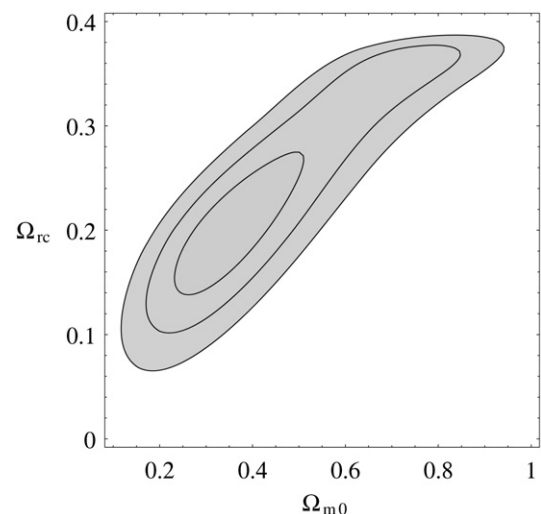
**Fig. 1.** 68%, 95% and 99% confidence contour plot of  $\Omega_{m0}, \Omega_{k0}$  in  $\Lambda$ CDM by the growth data in Table 1 and  $H(z)$  data in Table 2. For 1  $\sigma$  level,  $\Omega_{m0} = 0.275^{+0.0544}_{-0.0549}$ ,  $\Omega_{k0} = 0.065^{+0.159}_{-0.149}$ .



**Fig. 3.** 68%, 95% and 99% confidence contour plot of  $\Omega_{m0}, \Omega_{k0}$  in  $\Lambda$ CDM by the growth data in Table 1 and  $H(z)$  data in Table 2. For 1  $\sigma$  level,  $\Omega_{m0} = 0.270^{+0.0598}_{-0.0531}$ ,  $\Omega_{k0} = 0.080^{+0.146}_{-0.159}$ . The point  $z = 3.0$  in the sample of the growth data is excluded.



**Fig. 2.** 68%, 95% and 99% confidence contour plot of  $\Omega_{m0}, \Omega_{rc}$  in DGP by the growth data in Table 1 and  $H(z)$  data in Table 2. For 1  $\sigma$  level,  $\Omega_{m0} = 0.350^{+0.132}_{-0.0974}$ ,  $\Omega_{rc} = 0.200^{+0.0631}_{-0.0483}$ .



**Fig. 4.** 68%, 95% and 99% confidence contour plot of  $\Omega_{m0}, \Omega_{rc}$  in DGP by the growth data in Table 1 and  $H(z)$  data in Table 2. For 1  $\sigma$  level,  $\Omega_{m0} = 0.345^{+0.1296}_{-0.0940}$ ,  $\Omega_{rc} = 0.198^{+0.0613}_{-0.0471}$ . The point  $z = 3.0$  in the sample of the growth data is excluded.

Observing Table 1 carefully, one may find that the datum at  $z = 3.0$  is odd in some degree. From (1) we see that in  $\Lambda$ CDM  $\Omega_k$  or  $\Omega_{de}$  should be smaller than 0 if we require  $f > 1$ . Hence our present universe will be curvature dominated or becomes an anti-de Sitter (AdS) space, since dust matter redshifts much faster than curvature or vacuum energy. Here we give a simple example of this problem. In the spatially flat  $\Lambda$ CDM model,  $f = 1.46$  yields,

$$\Omega_m^{6/11} = 1.46. \quad (25)$$

Then we derive  $\Omega_m = 2.00$ ,  $\Omega_{de} = -1.00$ . The universe will brake and then start to contract at  $z = 2.17$ , which completely contradicts to the observations of expansion. So we present Fig. 3, which displays the constraint on  $\Omega_{m0}, \Omega_{k0}$  in  $\Lambda$ CDM by  $H(z)$  data and growth data, which only include 5 points. Similarly, we plot Fig. 4, which illuminates the constraint on  $\Omega_{m0}, \Omega_{rc}$  in DGP by  $H(z)$  data and growth data, which only includes 5 points. The datum at  $z = 3.0$  is excluded.

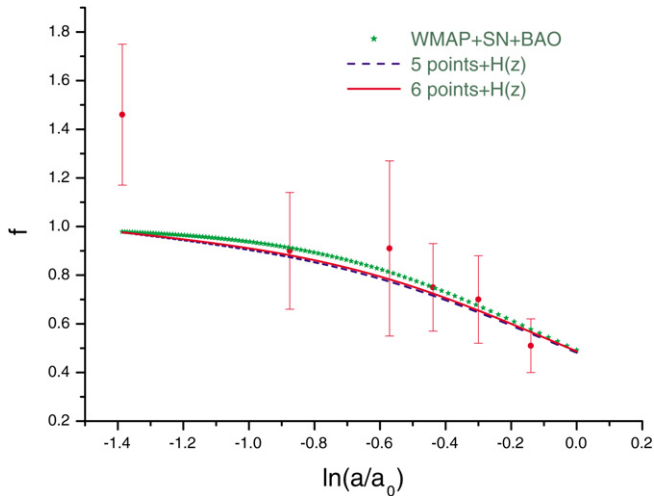
Comparing Fig. 1 with Fig. 3, we find the profiles of the two figures are almost the same, but the minimal  $\chi^2$ ,  $\chi_{\min}^2$  decreases from 12.26 to 9.479. Similarly, comparing Fig. 2 with Fig. 4, we

find the profiles of the two figures are almost the same, but  $\chi_{\min}^2$  decreases from 12.11 to 9.375. Without the point at  $z = 3.0$  the constraints on  $\Omega_{k0}$  of  $\Lambda$ CDM,  $\Omega_{m0}$  and  $\Omega_{rc}$  of DGP become more tighten instead. This is also a signal that the datum  $z = 3.0$  may not be well consistent with other data.

If we only consider  $\chi_{\min}^2$ , we may conclude that DGP is more favored. However,  $\chi_{\min}^2$  is only one point and the difference is tiny between the two models. We need more comparisons with the independent results, especially the permitted parameter internals, fitted by the expansion data, which were thoroughly studied. The latest results are shown as follows. For  $\Lambda$ CDM model,  $\Omega_{m0} = 0.279 \pm 0.008$ ,  $\Omega_{k0} = -0.0045 \pm 0.0065$ , which are derived from the joint analysis of the CMB (five-year WMAP data), the distance measurements from the Type Ia SN, and the Baryon Acoustic Oscillations (BAO) in the distribution of galaxies [27]. For DGP model,  $\Omega_{m0} = 0.28^{+0.03}_{-0.02}$ ,  $\Omega_{rc} = 0.13 \pm 0.01$  (SN(new gold) + CMB + SDSS + gas), and  $\Omega_{m0} = 0.21 \pm 0.01$ ,  $\Omega_{rc} = 0.16 \pm 0.01$  (SN(SNLS) + CMB + SDSS + gas) [28].

For  $\Lambda$ CDM, the result of joint fitting by growth data and  $H(z)$  data  $\Omega_{m0} = 0.275^{+0.0544}_{-0.0549}$ ,  $\Omega_{k0} = 0.065^{+0.159}_{-0.149}$ , almost coin-





**Fig. 5.** The perturbation growth  $f$  with error bars in Table 1 and the best fit curves in  $\Lambda$ CDM model. The best fit result by growth data and  $H(z)$  data inhabits on the red solid curve, the best fit result by growth data except the point  $z = 3.0$  and  $H(z)$  data resides on the blue dashed one, and the best fit result by joint analysis of WMAP, SN and BAO dwells on the green triangle ones. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this Letter.)

cides with the result by expansion data  $\Omega_{m0} = 0.279 \pm 0.008$ ,  $\Omega_{k0} = -0.0045 \pm 0.0065$ . These types of data are well consistent in frame of  $\Lambda$ CDM model.

For DGP, the result of joint fitting by growth data and  $H(z)$  data impose  $\Omega_{m0} = 0.350^{+0.132}_{-0.0974}$ ,  $\Omega_{rc} = 0.200^{+0.0631}_{-0.0483}$ , which is not well consistent with the result by expansion data  $\Omega_{m0} = 0.28^{+0.03}_{-0.02}$ ,  $\Omega_{rc} = 0.13 \pm 0.01$  (SN(new gold) + CMB + SDSS + gas), and  $\Omega_{m0} = 0.21 \pm 0.01$ ,  $\Omega_{rc} = 0.16 \pm 0.01$  (SN(SNLS) + CMB + SDSS + gas). Concretely speaking,  $\Omega_{rc} = 0.200^{+0.0631}_{-0.0483}$  (*growth* +  $H(z)$ ) inhabits beyond  $2\sigma$  level of expansion data  $\Omega_{rc} = 0.13 \pm 0.01$  (SN(new gold) + CMB + SDSS + gas). For the data set SN(SNLS) + CMB + SDSS + gas, the result by growth and  $H(z)$   $\Omega_{m0} = 0.350^{+0.132}_{-0.0974}$  dwells beyond  $3\sigma$  level of  $\Omega_{m0} = 0.21 \pm 0.01$ . Therefore DGP model cannot fit the observations of expansion and growth very well at the same time.

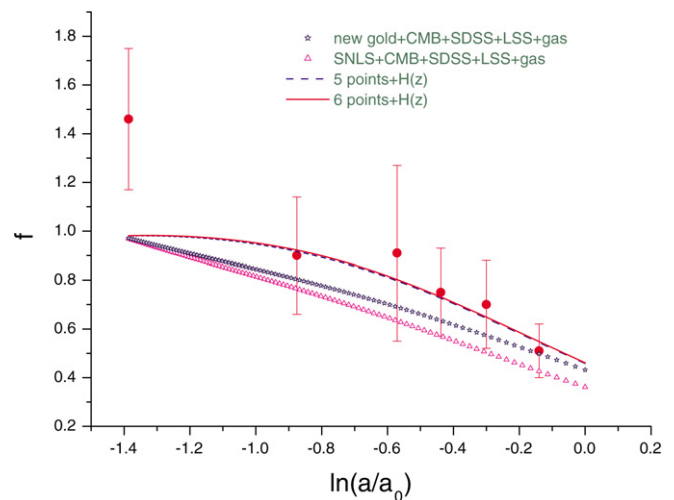
Through the above discussions, we see that the dark energy model is more favored than the DGP model by the present data, and the growth data can be an effective probe to study the nature of the dark energy.

We plot the best fit curves of growth  $f$  by *growth* +  $H(z)$  data and expansion data in  $\Lambda$ CDM, respectively in Fig. 5. Fig. 6 illuminates the best fit curves by *growth* +  $H(z)$  data and expansion data in DGP. It is clear that the gap between the best fit curves of growth data and expansion data is much bigger in DGP model than the gap in  $\Lambda$ CDM model.

#### 4. Conclusions

Perturbation growth is a newly developed method to differentiate between dark energy and modified gravity. In the previous works people concentrate on the approximate analytical value of the perturbation growth index of a model, and then compare with the observations. But, the index is not a constant in the history of the universe. We fit dark energy and modified gravity models by using the exact evolution equation of perturbation growth.

The sample of presently available growth data is quite small and the error bars are rather big. Furthermore, we always assumed  $\Lambda$ CDM model for deriving the growth data. Thus it seems proper to fit a model by jointing the growth data and the expansion data. The direct  $H(z)$  data are new type of data, which can be used to explore the fine structures of the Hubble ex-



**Fig. 6.** The perturbation growth  $f$  with error bars in Table 1 and the best fit curves in DGP model. The best fit result by growth+ $H(z)$  data inhabits on the red solid curve, the best fit result by growth data except the point  $z = 3.0$  and  $H(z)$  data resides on the blue dashed one, the best fit result by joint analysis of SN(new gold), CMB, SDSS, and gas dwells on the navy star curve, and the best fit result by joint analysis of SN(SNLS), CMB, SDSS, and gas is denoted by the pink triangle ones. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this Letter.)

pansion history. We put forward a joint fitting by the growth data and  $H(z)$  data. The results are summarized as follows: For  $\Lambda$ CDM,  $\Omega_{m0} = 0.275^{+0.0544}_{-0.0549}$ ,  $\Omega_{k0} = 0.065^{+0.159}_{-0.149}$ ; For DGP,  $\Omega_{m0} = 0.350^{+0.132}_{-0.0974}$ ,  $\Omega_{rc} = 0.200^{+0.0631}_{-0.0483}$ .

The minimal  $\chi^2$  are 12.26 and 12.11 for  $\Lambda$ CDM and DGP, separately. The permitted parameters of  $\Lambda$ CDM by *growth* +  $H(z)$  data show an excellent consistency with the previous results inferred from expansion data. However, for DGP model the discrepancies of the results of *growth* +  $H(z)$  data and expansion data are at least  $2\sigma$  level. Hence in the sense of consistency,  $\Lambda$ CDM is more favored than DGP.

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#### References

- [1] A.G. Riess, et al., *Astron. J.* 116 (1998) 1009; S. Perlmutter, et al., *Astrophys. J.* 517 (1999) 565.
- [2] E.J. Copeland, M. Sami, S. Tsujikawa, *Int. J. Mod. Phys. D* 15 (2006) 1753, hep-th/0603057.
- [3] G. Dvali, G. Gabadadze, M. Porrati, *Phys. Lett. B* 485 (2000) 208; G. Dvali, G. Gabadadze, *Phys. Rev. D* 63 (2001) 065007.
- [4] A. Lue, *Phys. Rep.* 423 (2006) 1, astro-ph/0510068.
- [5] E.V. Linder, R.N. Cahn, *Astropart. Phys.* 28 (2007) 481, astro-ph/0701317.
- [6] E.V. Linder, *Phys. Rev. D* 72 (2005) 043529, astro-ph/0507263; D. Huterer, E.V. Linder, *Phys. Rev. D* 75 (2007) 023519, astro-ph/0608681; L.M. Wang, P.J. Steinhardt, *Astrophys. J.* 508 (1998) 483, astro-ph/9804015; Y. Wang, arXiv: 0710.3885 [astro-ph]; Y. Wang, arXiv: 0712.0041 [astro-ph]; M. Kunz, D. Sapone, *Phys. Rev. Lett.* 98 (2007) 121301, astro-ph/0612452; S. Wang, L. Hui, M. May, Z. Haiman, *Phys. Rev. D* 76 (2007) 063503, arXiv: 0705.0165; A. Cardoso, K. Koyama, S.S. Seahra, F.P. Silva, arXiv: 0711.2563 [astro-ph]; K. Koyama, *Gen. Relativ. Gravit.* 40 (2008) 421, arXiv: 0706.1557;

- A. Lue, R. Scoccimarro, G.D. Starkman, Phys. Rev. D 69 (2004) 124015, astro-ph/0401515;
- C. Di Porto, L. Amendola, arXiv: 0707.2686 [astro-ph];
- L. Amendola, M. Kunz, D. Sapone, arXiv: 0704.2421 [astro-ph];
- D. Sapone, L. Amendola, arXiv: 0709.2792 [astro-ph];
- D. Polarski, R. Gannouji, Phys. Lett. B 660 (2008) 439, arXiv: 0710.1510.
- [7] P.J.E. Peebles, Large-Scale Structure of the Universe, Princeton Univ. Press, Princeton, 1980.
- [8] L.M. Wang, P.J. Steinhardt, Astrophys. J. 508 (1998) 483, astro-ph/9804015.
- [9] E. Hawkins, et al., Mon. Not. R. Astron. Soc. 346 (2003) 78, astro-ph/0212375; L. Verde, et al., Mon. Not. R. Astron. Soc. 335 (2002) 432, astro-ph/0112161; E.V. Linder, arXiv: 0709.1113 [astro-ph].
- [10] M. Tegmark, et al., SDSS Collaboration, Phys. Rev. D 74 (2006) 123507, astro-ph/0608632.
- [11] N.P. Ross, et al., astro-ph/0612400.
- [12] L. Guzzo, et al., Nature 451 (2008) 541, arXiv: 0802.1944.
- [13] J. da Angela, et al., astro-ph/0612401.
- [14] P. McDonald, et al., SDSS Collaboration, Astrophys. J. 635 (2005) 761, astro-ph/0407377.
- [15] H. Wei, arXiv: 0802.4122 [astro-ph].
- [16] S. Nesseris, L. Perivolaropoulos, Phys. Rev. D 77 (2008) 023504, arXiv: 0710.1092.
- [17] R. Jimenez, L. Verde, T. Treu, D. Stern, Astrophys. J. 593 (2003) 622, astro-ph/0302560.
- [18] R. Jimenez, A. Loeb, Astrophys. J. 573 (2002) 37, astro-ph/0106145.
- [19] R.G. Abraham, et al., GDDS Collaboration, Astron. J. 127 (2004) 2455, astro-ph/0402436;
- T. Treu, M. Stiavelli, S. Casertano, P. Moller, G. Bertin, Mon. Not. R. Astron. Soc. 308 (1999) 1037;
- T. Treu, M. Stiavelli, P. Moller, S. Casertano, G. Bertin, Mon. Not. R. Astron. Soc. 326 (2001) 221, astro-ph/0104177;
- T. Treu, M. Stiavelli, S. Casertano, P. Moller, G. Bertin, Astrophys. J. 564 (2002) L13;
- J. Dunlop, J. Peacock, H. Spinrad, A. Dey, R. Jimenez, D. Stern, R. Windhorst, Nature 381 (1996) 581;
- H. Spinrad, A. Dey, D. Stern, J. Dunlop, J. Peacock, R. Jimenez, R. Windhorst, Astrophys. J. 484 (1997) 581;
- L.A. Nolan, J.S. Dunlop, R. Jimenez, A.F. Heavens, Mon. Not. R. Astron. Soc. 341 (2003) 464, astro-ph/0103450.
- [20] J. Simon, L. Verde, R. Jimenez, Phys. Rev. D 71 (2005) 123001, astro-ph/0412269.
- [21] R.G. Cai, H.S. Zhang, A. Wang, Commun. Theor. Phys. 44 (2005) 948, hep-th/0505186.
- [22] A. Liddle, D. Lyth, The Cosmological Inflation and Large-Scale Structure, Cambridge Univ. Press, Cambridge, 2000.
- [23] K. Koyama, R. Maartens, JCAP 0601 (2006) 016, astro-ph/0511634.
- [24] K. Maeda, S. Mizuno, T. Torii, Phys. Rev. D 68 (2003) 024033, gr-qc/0303039.
- [25] H. Wei, S.N. Zhang, Phys. Lett. B 644 (2007) 7, astro-ph/0609597.
- [26] H. Zhang, Z.-H. Zhu, JCAP 0803 (2008) 007.
- [27] E. Komatsu, et al., WMAP Collaboration, arXiv: 0803.0547 [astro-ph].
- [28] M.S. Movahed, M. Farhang, S. Rahvar, astro-ph/0701339.
- [29] W. Freedman, et al., Astrophys. J. 553 (2001) 47.