ELSEVIER
SCIENCE DIRECT
PHYSICS LETTERS B
Physics Letters B 598 (2004) 197–202
www.elsevier.com/locate/physletb

Diagnostic decays of the $X(3872)$

Eric S. Swanson

Department of Physics and Astronomy, University of Pittsburgh, Pittsburgh, PA 15260, USA

Received 24 June 2004; accepted 29 July 2004

Available online 20 August 2004

Editor: H. Georgi

Abstract

The unusual properties of the $X(3872)$ have led to speculation that it is a weakly bound state of mesons, chiefly $D^0\bar{D}^{0*}$. Tests of this hypothesis are investigated and it is proposed that measuring the $3\pi J/\psi$, $\gamma J/\psi$, $\gamma\psi'$, $\bar{K}K^*$, and $\pi\rho$ decay modes of the X will serve as a definitive diagnostic of the molecule hypothesis.

1. Introduction

The discovery of a new charmonium state, the $X(3872)$, in B decays [1–4][1] has caused some interest because the X mass and its narrow width ($\Gamma <$ 2.3 MeV, 90% C.L.) do not agree well with quark model expectations. The most likely candidates are 1D or 2P charmonium states; however, as Table 1 illustrates, the 1D states tend to lie below 3872 while the 2P states are somewhat above. The masses quoted in this table are given by a simple nonrelativistic quark model with perturbative spin orbit and tensor interactions and a smeared hyperfine interaction [5]. The 'model range' column of this table is an average of several theoretical predictions [6].

This situation has led to speculation that the X could be a novel charmonium state, such as a hybrid [7], or a molecule [8]. The former scenario is viewed as unlikely because lattice computations of the lightest charmonium hybrid yield masses around 4400 MeV [9]. The molecular hypothesis is bolstered by the proximity of the X to the $D^0\bar{D}^{0*}$ threshold at 3871.2(7) MeV. Furthermore, the charged $D\bar{D}^*$, $\rho J/\psi$, and $\omega J/\psi$ channels all lie within 7 MeV of the X mass. Thus it is natural to expect a strong admixture of these states in the X (assuming it is indeed a resonance and not a threshold enhancement). This scenario has been examined in Ref. [10] where a 6 coupled channel model with quark exchange and pion exchange interactions was diagonalised to determine the structure of the X. The conclusion was that the only viable molecular candidate is $J^{PC} = 1^{++}$ and is

E-mail address: swansone+@pitt.edu (E.S. Swanson).

[1] The reported masses are $3872 \pm 0.6 \pm 0.5$ (Belle); $3871.3 \pm 0.7 \pm 0.4$ (CDF); $3871.8 \pm 3.1 \pm 3.0$ (D0); and 3873.4 ± 1.4 (BaBar). The world average is 3871.9 ± 0.5 MeV.

0370-2693
doi:10.1016/j.physletb.2004.07.059

Table 1
Quark model $c\bar{c}$ masses

State	Expt (MeV)	Mass (MeV)	Model range (MeV)
1^3D_3		3806	3841(22)
1^3D_2		3800	3827(24)
1^3D_1	3770	3785	3803(25)
1^1D_2		3799	3821(32)
2^3P_2		3972	3990(25)
2^3P_1		3925	3957(28)
2^3P_0		3852	3903(40)
2^1P_1		3934	3964(20)

dominantly $D^0\bar{D}^{0*}$ with important but small admixtures of $\omega J/\psi$ and $\rho J/\psi$. This state was dubbed the $\hat{\chi}_{c1}$. In this scenario, the decay $X \to \pi^+\pi^- J/\psi$ is permitted by the isospin symmetry violating $\rho J/\psi$ component[2] of the X. However, the $\omega J/\psi$ component can decay via $3\pi J/\psi$ and detecting the X in this mode will be an important test of this scenario. We remark that isospin violating decay modes are a generic feature of molecular states with binding energies comparable to the mass splittings of its constituent neutral and charged mesons.

Recent experimental effort has significantly narrowed the range of options available for the structure of the X. The Belle Collaboration [11] has measured the ratio of partial widths

$$\frac{\Gamma(X \to \gamma\chi_{c1})}{\Gamma(X \to \pi^+\pi^- J/\psi)} < 0.89, \ 90\% \text{ C.L.} \tag{1}$$

and stated that this contradicts expectations for the ψ_2 identification of the X.[3] Similarly, there is a weak signal in $\gamma\chi_{c2}$, indicating that the ψ_3 identification is unlikely. Belle also reports that the helicity angle distribution of the J/ψ in the X final state rules out $J^{PC} = 1^{+-}$. BaBar have not seen the X in the $\eta J/\psi$ mode [4] which is consistent with the molecular interpretation of the X. BES reports that the X is not seen in initial state radiation production [12], implying that it is inconsistent with $J^{PC} = 1^{--}$. Finally, CLEO has not detected the X in untagged two photon production [13], which indicates that $J^{PC} = 0^{\pm+}$ and $2^{\pm+}$ are disallowed.

Together, these observations imply that the only viable nonexotic quantum numbers below $J = 3$ are $J^{PC} = 1^{++}$. The remainder of this Letter therefore concentrates on the $2^3P_1 c\bar{c} = \chi'_{c1}$ and molecular $\hat{\chi}_{c1}$ assignments for the X. Distinguishing these assignments is a task for experiment coupled with reliable theoretical expectations. The importance of measuring the $3\pi J/\psi$ mode arising from the short range $\omega J/\psi$ component of the $\hat{\chi}_{c1}$ has already been mentioned. Here we examine the utility of a novel annihilation decay and radiative decays of the X as diagnostic tools.

Radiative decays of the molecular $\hat{\chi}_{c1}$ to charmonium may occur via vector meson dominance in the $\rho J/\psi$ or $\omega J/\psi$ components of the X. Thus $\gamma J/\psi$ is the only possible final state available to this mechanism. An additional mechanism has the light quarks coupling to the final state photon from the neutral and charged $D\bar{D}^*$ components of the $\hat{\chi}_{c1}$. This diagram permits coupling to a variety of charmonia in the final state. Specific computations, presented in the next section, reveal that measurements of the rates $X \to \gamma J/\psi$ and $X \to \gamma\psi''$ will definitively distinguish the charmonium from the molecular options for the $X(3872)$.

The annihilation process $X \to K\bar{K}^*$ is studied in Section 3. This decay of the molecular state may occur via single gluon exchange but is strongly suppressed by the weakly bound nature of the $\hat{\chi}_{c1}$. The multigluon exchange process which contributes to $c\bar{c} \to K\bar{K}^*$ is also suppressed but comparisons with typical charmonia indicate that charmonium rates for this reaction should be three orders of magnitude larger than that for the molecular state. A novel feature of the molecular decay is that production of charged kaons is driven by the dominant neutral $D\bar{D}^*$ component of the $\hat{\chi}$ and therefore should be larger than decay into neutral kaons.

2. Radiative decays of the X

The primary mechanisms for radiative decays of the $\hat{\chi}_{c1}$ are via vector meson dominance (Fig. 1) and light quark annihilation (Fig. 2). The vector meson dominance (VMD) process proceeds via the $\rho J/\psi$ and $\omega J/\psi$ components of the $\hat{\chi}_{c1}$ and thus contributes to $\Gamma(X \to \gamma J/\psi)$. It is clear that the amplitude must be

[2] Isospin violation is driven by the 8 MeV difference between the neutral and charged $D\bar{D}^*$ channels. The resulting prediction of production of $\pi\pi$ via the ρ is supported by the observed $\pi\pi$ spectrum [1,4].

[3] This assumes that $\Gamma(\psi_2 \to \pi^+\pi^- J/\psi) \approx \Gamma(\psi(3770) \to \pi^+\pi^- J/\psi) \approx 100$ keV.

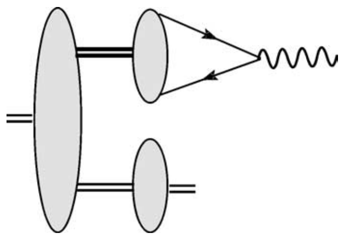

Fig. 1. Vector meson dominance diagram.

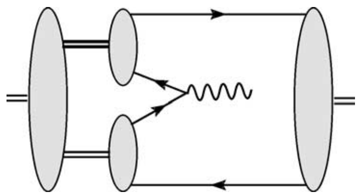

Fig. 2. Annihilation diagram.

proportional to the light vector meson wavefunction at the origin and to the $\hat{\chi}$ wavefunction for the channel in question evaluated at the recoil momentum. The specific result is

$$\Gamma_{\rm VMD} = \frac{4}{27}\alpha \frac{qE_\psi}{m_\chi}\left|\psi_\omega(r=0)\right|^2 \times \left(Z_{\omega\psi}\phi_{\omega\psi}(q) + 3Z_{\rho\psi}\phi_{\rho\psi}(q)\right)^2. \quad (2)$$

Here it is assumed that the ρ and ω wavefunctions are identical. The factor Z_α^2 is the probability of finding the channel α in the molecular state. For a weakly bound state these probabilities are 76% ($D^0\bar{D}^{0*}$), 10% ($D^+\bar{D}^{-*}$), 11% ($\omega J/\psi$), and 0.8% ($\rho J/\psi$) [10] (plus 2% in D-waves). Of course relative phases of these components must be accounted for in Eq. (2). The photon momentum is denoted q. Finally, the factor of 3 appearing in this expression is a reflection of the famous VMD ratio $\rho:\omega:\phi = 9:1:2$.

The annihilation contribution to radiative decay illustrated in Fig. 2 proceeds via the $D^0\bar{D}^{0*}$ + c.c. and $D^-\bar{D}^{+*}$ + c.c. components of the $\hat{\chi}_{c1}$. In evaluating this diagram I have found it convenient to assume simple harmonic wavefunctions (SHO) for the J/ψ, D, and D^* mesons. These wavefunctions are specified in terms of an SHO width parameter, β (I also set $\beta_D = \beta_{D^*}$). This assumption permits analytic integration over eight of the nine required dimensions. The remaining integral over the radial $\hat{\chi}$ wavefunction is performed numerically. The final result for the width due to annihilation is

$$\Gamma_{\rm ANN} = \frac{4}{27}\alpha \frac{qE_\psi}{m_\chi} e^{\frac{-q^2}{2\beta_D^2}}\left(\eta_{00} - \frac{1}{2}\eta_{+-}\right)^2, \quad (3)$$

where the factors η_α are a convolution of the radial $\hat{\chi}$ wavefunction in the α channel with the intermediate D and D^* wavefunctions and the final state charmonium wavefunction. Computations with SHO wavefunctions reveal that these factors are zero unless the final charmonium state is an S-wave. While this is not true for more realistic wavefunctions (such as Coulomb + linear) I choose not to evaluate the integrals in this case since the results will be small and will depend on model details. Relevant S-wave η factors are given by the following expressions:

$$\eta_\alpha(1S) = \frac{\sqrt{8}}{\pi^{3/4}}\left(\frac{\beta_\psi}{2\beta_\psi^2 + \beta_D^2}\right)^{3/2} \times \int d^3p\, Z_\alpha \phi_\alpha(p) \exp\left(-\frac{\rho^2 p^2}{2\beta_\psi^2 + \beta_D^2}\right) \quad (4)$$

and

$$\eta_\alpha(2S) = \frac{4}{\sqrt{3}\pi^{3/4}}\left(\frac{\beta_\psi}{2\beta_\psi^2 + \beta_D^2}\right)^{3/2} \times \int d^3p\, Z_\alpha \phi_\alpha(p) \exp\left(-\frac{\rho^2 p^2}{2\beta_\psi^2 + \beta_D^2}\right) \times \left(\frac{3\beta_\psi^2}{2\beta_\psi^2 + \beta_D^2} - \frac{4\rho^2 p^2}{\beta_\psi^2}\frac{(\beta_\psi^2 + \beta_D^2)^2}{(2\beta_\psi^2 + \beta_D^2)^2}\right). \quad (5)$$

These expressions depend on the quark mass ratio $\rho = m_c/(m_c + m_u)$.

The annihilation and vector meson dominance amplitudes are evaluated with $\hat{\chi}$ wavefunctions computed in Ref. [10] and typical quark model parameters $\beta_\psi = 0.67$ GeV, $\beta_D = \beta_{D^*} = 0.4$ GeV, $m_c = 1.6$ GeV, and $m_u = 0.33$ GeV. The vector wavefunction at the origin is taken to be $\psi_\omega(0) = 0.976\ \text{GeV}^{3/2}$ as determined by numerically integrating the Schrödinger equation with a Coulomb + linear + smeared hyperfine potential. This compares favorably with the SHO result of 1.17 $\text{GeV}^{3/2}$. The final rate for $X \to \gamma J/\psi$ is dominated by the VMD and cross terms in the width and is presented in Table 2. The rate for $X \to \gamma\psi''$ and $\gamma\psi_2$ are zero for SHO wavefunctions since they only proceed via the annihilation diagram. Finally $X \to \gamma\psi'$

Table 2
E1 decays of the $X(3872)$

Mode	m_f (MeV)	q (MeV)	$\Gamma[c\bar{c}]$ (keV) [B&G]	$\Gamma[c\bar{c}]$ (keV) [A]	$\Gamma[c\bar{c}]$ (keV) [B]	$\Gamma[\hat{\chi}_{c1}]$ (keV)
$\gamma J/\psi$	3097	697	11	71	139	8
$\gamma\psi'(2^3S_1)$	3686	182	64	95	94	0.03
$\gamma\psi''(1^3D_1)$	3770	101	3.7	6.5	6.4	0
$\gamma\psi_2(1^3D_2)$	3838	34	0.5	0.7	0.7	0

is also driven by the annihilation diagram and is very small.

A variety of predictions for radiative decays of the χ'_{c1} (assuming a mass of 3872 MeV) are also presented in Table 2. The fourth column summarises the results of Barnes and Godfrey [6]. These rates are computed in the impulse, nonrelativistic, zero recoil, and dipole approximations. The results for $X \to \gamma J/\psi$ are particularly sensitive to model details.[4] This is examined in columns five and six which present the results of two additional computations. The first, labelled [A], employs the same approximations of Barnes and Godfrey but uses meson wavefunctions computed with a simple, but accurate, Coulomb + linear + smeared hyperfine potential. It is apparent that the $\gamma J/\psi$ rate is very sensitive to wavefunction details. Furthermore, one may legitimately question the use of the zero recoil and dipole approximations for the $\gamma J/\psi$ mode since the photon momentum is so large in this case. The sixth column (model [B]) dispenses with these approximations, and one finds a relatively large effect for $\gamma J/\psi$.

Table 2 makes it clear that computations of the $\gamma J/\psi$ radiative transition of the χ'_{c1} are very sensitive to model details.[5] The result of Barnes and Godfrey is similar to that computed here for the molecular $\hat{\chi}$ state but is much smaller than models A and B. Furthermore, the rates for $\gamma\psi''$ and $\gamma\psi_2$ are very small for a molecular X (at the order of eV) and quite small for a charmonium X. Perhaps the most robust diagnostic is the $\gamma\psi'$ decay mode. For a molecular $\hat{\chi}$ this can only proceed via the annihilation diagram of Fig. 2 and hence is very small. Clearly a measurement of the $\gamma J/\psi$ and $\gamma\psi'$ decay modes of the $X(3872)$ will provide compelling clues to its internal structure.

The figures in the last column of Table 1 correspond to a $\hat{\chi}$ state with a binding energy of 1 MeV. Changing the short range cutoff in the pion-exchange interaction of Ref. [10] modifies the binding energy and, hence, the characteristics of the $\hat{\chi}$. This affects decay rates, for example a binding energy of 4 MeV increases the $\gamma J/\psi$ rate by roughly 20% and has minimal impact on the $\gamma\psi'$ rate.

3. Wavefunction suppressed decays: $X \to K\bar{K}^*$ and $\pi\rho$

Hidden flavour changing transitions of the X are interesting because they occur via $c\bar{c}$ annihilation at the origin for molecular states and via multigluon intermediate states for charmonia. One may hope that these very different mechanisms will provide additional clues to the nature of the X.

The lowest order diagram in the strong coupling in the molecular case is illustrated in Fig. 3. Here the dashed line refers to instantaneous gluon exchange. The operator giving rise to this diagram is taken to be

$$V_{oge} = \frac{1}{2}\int d^3x\, d^3y\, \psi^\dagger(\mathbf{x})T^a\psi(\mathbf{x}) \times K(\mathbf{x}-\mathbf{y})\psi^\dagger(\mathbf{y})T^a\psi(\mathbf{y}), \tag{6}$$

where T^a is a colour generator and K is a kernel given by

$$K(r) = -\frac{\alpha_s}{r} + \frac{3}{4}br. \tag{7}$$

The constants appearing here are the strong coupling and the string tension, respectively. This operator com-

[4] This rate is zero for SHO wavefunctions.

[5] There is an additional error induced by arbitrarily changing the quark model χ' mass to 3872 MeV. This is made clear through the observation that the dipole formula for the width scales as q^3 whereas the momentum space formula scales as q.

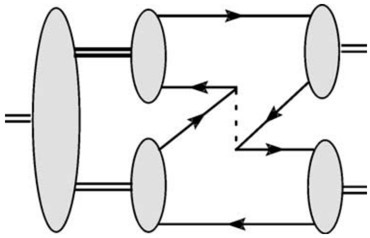

Fig. 3. Wavefunction suppressed annihilation diagram.

prises a relativistic extension of the quark model and hence the computations presented here are on somewhat less firm ground than the radiative transitions of the previous section. However, note that the form used here is consistent with the Hamiltonian of QCD in Coulomb gauge.

It is likely that this diagram is much larger than the analogous transverse gluon exchange diagram because the latter contains an intermediate $u\bar{u}g$ or $d\bar{d}g$ hybrid (bound state time ordered perturbation theory is employed throughout this Letter) and hence is suppressed by a large energy denominator.

The initial state of Fig. 3 is the charged or neutral $D\bar{D}^*$ component of the $\hat{\chi}$; this permits the annihilation of the charmed quarks and the production of an $s\bar{s}$ pair, which materialize in the $K\bar{K}^*$ final state. Notice that the production of charged kaons can only proceed via the neutral, and dominant, $D\bar{D}^*$ component of the $\hat{\chi}$, while neutral kaons arise from the charged $D\bar{D}^*$ component. Thus one expects $\Gamma(X \to K^+\bar{K}^{-*}) \gg \Gamma(X \to K^0\bar{K}^{0*})$ for molecular X while $\Gamma(X \to K^+\bar{K}^{-*}) \approx \Gamma(X \to K^0\bar{K}^{0*})$ for charmonium X.

The computation of Fig. 3 assumes SHO K, K^*, D and D^* meson wavefunctions as above. The result is

$$\Gamma\left(\hat{\chi}_{c1} \to K^- K^{+*}\right) = \frac{1}{8} q \frac{E_K E_{K^*}}{m_\chi} \left| \frac{C_F}{2N} \left(6b + 8\alpha_s \frac{\beta_K^2 \beta_D^2}{\beta_K^2 + \beta_D^2} \right) \times \frac{1}{m_c m_s} \frac{\beta_K \beta_D}{\beta_K^2 + \beta_D^2} \eta_{00}(q) \right|^2, \tag{8}$$

where $C_F = (N^2 - 1)/(2N)$, N is the number of colours, and η_α is a factor defined by

$$\eta_\alpha(q) = \int_0^\infty dp \frac{p}{q} \phi_{\alpha R}(p) \left[\exp\left(-\frac{(p-q)^2}{\beta_D^2 + \beta_K^2} \right) - \exp\left(-\frac{(p+q)^2}{\beta_D^2 + \beta_K^2} \right) \right]. \tag{9}$$

The radial wavefunction appearing in this equation is defined by $\phi_\alpha = Y_{\ell m} \phi_{\alpha R}$.

Evaluating Eq. (8) with typical quark model parameters $b = 0.18\ \text{GeV}^2$, $\alpha_s = 0.5$, and $\beta_K = 0.4$ GeV yields

$$\Gamma\left(\hat{\chi}_{c1} \to K^+ K^{-*}\right) = 4.4\ \text{eV}. \tag{10}$$

The rate to neutral kaons proceeds through the smaller charged $D\bar{D}^*$ component of the $\hat{\chi}$ and is given by

$$\Gamma\left(\hat{\chi}_{c1} \to \bar{K}^0 K^{0*}\right) = 0.8\ \text{eV}. \tag{11}$$

Comparable rates for P-wave charmonia are only known for the χ_{c0} and are

$$\Gamma\left(\chi_{c0} \to K^+ K^-\right) = 95(28)\ \text{keV},$$
$$\Gamma\left(\chi_{c0} \to K_s^0 K_s^0\right) = 32(14)\ \text{keV}. \tag{12}$$

Using these figures as a benchmark for $2^3P_0 c\bar{c} \to K\bar{K}^*$ indicates that multigluon annihilation of charmonium χ'_{c1} is roughly 1000 times larger than the wavefunction suppressed decay of the molecular $\hat{\chi}$. Thus finding the X in $K\bar{K}^*$ at the keV level may be an indication that it is a charmonium and not a molecular state.

The same computation applies to the $\pi\rho$ decay mode with the exception that the strange quark mass is replaced with the up or down constituent mass, the kinematics are slightly different, and both the charged and neutral $D\bar{D}^*$ components of the $\hat{\chi}$ wavefunction contribute to $\pi^+\rho^- +$ c.c. and $\pi^0\rho^0$ decays. Taking $\beta_\pi = \beta_\rho = 0.4$ GeV yields

$$\Gamma(\hat{\chi}_{c1} \to \pi^+\rho^- + \text{c.c.}) = \Gamma\left(\hat{\chi}_{c1} \to \pi^0\rho^0\right) = 40\ \text{eV}. \tag{13}$$

One expects that the analogous charmonium rates are several orders of magnitude greater than this so that discovering the X in the $\pi\rho$ channel could be an indication that it is a charmonium state.

4. Conclusions

Radiative decays of the $X(3872)$ offer a promising method to distinguish the charmonium and molecular assignments for this state. Unfortunately the predicted rate for charmonium 2^3P_1 to $\gamma J/\psi$ is very sensitive to model details so that a comparison to the molecular rate is less significant that desired. Nevertheless, one expects $\Gamma(\hat{\chi}_{c1} \to \gamma J/\psi) \lesssim \Gamma(\chi'_{c1} \to \gamma J/\psi)$. Furthermore, determining this width for the genuine charmonium state will provide a demanding test of our understanding of heavy quark systems.

Alternatively, the charmonium width for $\gamma\psi'$ is relatively stable and is much larger than the molecular rate, which can only proceed via light quark annihilation, and is therefore wavefunction suppressed. Thus collecting sufficient X events to verify that the $\gamma\psi'$ final state is not seen at the 100 keV level will provide compelling evidence of the molecular nature of the X.

It has been argued that existence of $2\pi J/\psi$ and $3\pi J/\psi$ decay modes of the X are a strong indication of the isospin symmetry violating, and hence molecular, nature of the X. In a similar fashion, the suppressed hidden flavour changing decay $X \to K\bar{K}^*$ strongly favours the charged kaon final state over neutral kaons. Alternatively, one expects equal production of charged and neutral kaons from charmonium decay. An explicit computation shows that the molecular rate is very small, and probably several orders of magnitude smaller than the analogous charmonium decay. Thus a direct measurement of this rate or of the ratio of charged to neutral kaons may be a useful test of the nature of the $X(3872)$.

Acknowledgements

I thank Ted Barnes, Eric Braaten, Olga Lakhina, Philip Page, Walter Toki, and the participants of the BaBar Collaboration workshop on charmonium for discussions. This work was supported by the DOE under contract DE-FG02-00ER41135.

References

[1] S.K. Choi, et al., Belle Collaboration, Phys. Rev. Lett. 91 (2003) 262001.
[2] D. Acosta, et al., CDF II Collaboration, hep-ex/0312021.
[3] V.M. Abazov, et al., D0 Collaboration, hep-ex/0405004.
[4] B. Aubert, et al., BaBar Collaboration, hep-ex/0402025.
[5] T. Barnes, S. Godfrey, E.S. Swanson, in preparation.
[6] T. Barnes, S. Godfrey, Phys. Rev. D 69 (2004) 054008; See also, E.J. Eichten, K. Lane, C. Quigg, Phys. Rev. D 69 (2004) 094019.
[7] F.E. Close, P.R. Page, Phys. Lett. B 578 (2004) 119; F.E. Close, S. Godfrey, Phys. Lett. B 574 (2003) 210.
[8] N.A. Törnqvist, hep-ph/0308277; E. Braaten, M. Kusunoki, Phys. Rev. D 69 (2004) 074005; C.Y. Wong, Phys. Rev. C 69 (2004) 055202; S. Pakvasa, M. Suzuki, Phys. Lett. B 579 (2004) 67.
[9] Z.H. Mei, X.Q. Luo, Int. J. Mod. Phys. A 18 (2003) 5713.
[10] E.S. Swanson, Phys. Lett. B 588 (2004) 189.
[11] S.K. Choi, Belle Collaboration, hep-ex/0405014.
[12] C.Z. Yuan, X.H. Mo, P. Wang, Phys. Lett. B 579 (2004) 74.
[13] P. Zweber, CLEO Collaboration, Talk presented at APS April Meeting, 2004.