Study of free convection in a square cavity with magnetic field and semi circular heat source of different orientations

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Abstract

Free convection flow in the presence of magnetic field in a square cavity with semi circular heat source was studied in this paper. The lower wall of cavity was heated from below and upper wall of cavity was cold whereas side walls of the cavity were thermally insulated. The governing differential equations are solved by finite element method. The effects of magnetic field were studied on flow and temperature field and heat transfer performance at a wide range of parameter Hartmann (0 ≤ Ha ≤ 100) number. The study is performed for different orientation of heated solid and Hartmann numbers at Pr=0.71 and (10 ≤ Ra ≤ 10000).

Keywords: Free convection; semi circular heat source; orientation; finite element method.

1. Introduction

Free convection in cavity has received considerable attention from researchers. Most of the cavities commonly used in industries are circular, square, rectangular, trapezoidal and triangular etc. Square cavities have received a considerable attention for its application in various fields. Taghikhani and Chavoshi [1] numerically investigated two dimensional magneto-hydrodynamics (MHD) free convection with internal heating in a square cavity. They observed that the effect of the magnetic field is to reduce the convective heat transfer inside the cavity. Bakhshan

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and Ashoori [2] investigated numerically analysis of a fluid behavior in a rectangular enclosure under the effect of magnetic field. They observed that Nusselt number rises with increasing Grashof and Prandtl numbers and decreasing Hartmann and orientation of magnetic field. Öztöp and Al-Salem [3] numerically investigated effects of joule heating on MHD natural convection in non-isothermally heated enclosure. They observed that positive stream functions are decreased with increasing of Hartmann number and thermal boundary layer becomes larger. Parvin and Nasrin [4] investigated numerically analysis of the flow and heat transfer characteristics for MHD free convection in an enclosure with a heated obstacle. They found that buoyancy-induced vortex in the streamlines is increased and thermal layer near the heated surface becomes thick with increasing Rayleigh number. Natural convection in a square cavity localized heating from below was investigated by Santosh et al. [5]. They observed that heat transfer increases when the heater source is placed towards the cold wall. In the light of the above literatures, the present study addresses the effects of MHD free convection on heat flow within a square cavity.

**Nomenclature**

- $B_0$: Applied magnetic field
- $C_p$: Specific heat at constant pressure (J kg$^{-1}$ K$^{-1}$)
- $g$: Acceleration due to gravity (m s$^{-1}$)
- $h$: Local heat transfer coefficient (W m$^{-2}$ K$^{-1}$)
- $Ha$: Hartmann number
- $k$: Thermal conductivity (W m$^{-1}$ K$^{-1}$)
- $L$: Length of the enclosure (m)
- $Nu$: Nusselt number
- $p$: Pressure of the fluid
- $P$: Dimensionless pressure of the fluid
- $rot$: Rotation angle
- $Pr$: Prandtl number
- $Ra$: Rayleigh number
- $T$: Dimensional temperature (K)
- $T_c$: Cold temperature of fluid (K)
- $T_h$: Heat temperature of fluid (K)
- $u, v$: Dimensional velocity components (m s$^{-1}$)
- $U, V$: Dimensionless velocities
- $x, y$: Dimensional coordinates (m)
- $X, Y$: Dimensionless coordinates

**Greek Symbols**

- $\alpha$: Fluid thermal diffusivity (m$^2$ s$^{-1}$)
- $\beta$: Thermal expansion coefficient (K$^{-1}$)
- $\nu$: Kinematic viscosity (m$^2$ s$^{-1}$)
- $\theta$: Dimensionless temperature
- $\rho$: Density (kg m$^{-3}$)
- $\mu$: Dynamic viscosity (N s m$^{-2}$)
- $\sigma$: Electrical conductivity of the fluid

**2. Model and mathematical formulation**

A schematic diagram of the system considered in present study is shown in Fig. 1. The system consists of a square cavity with sides of length $L$ and a semi-circular solid block is located at the centre of the enclosure. The left and right walls are considered to be adiabatic. The top wall is kept at a constant temperature $T_c$. The bottom wall and semi-circular obstacle are assumed to be uniform temperature be $T_h$. Here $T_c$ is less than $T_h$. The uniform magnetic field $B_0$ is also applied to the fluid in the direction parallel to $y$. Based on the model, two dimensional, laminar, incompressible steady equations are written by considering a uniform applied magnetic field. The gravitational force ($g$) acts in the vertically downward direction. We assumed that Boussinesq approximation is valid and radiation mode of the heat transfer and Joule heating are neglected. Thus, using the coordinate system shown in Fig. 1, the governing equations can be written in dimensional form

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0
\]

\[
\rho \left( u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = -\frac{\partial p}{\partial x} + \mu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right)
\]
We introduce the following dimensionless variables

\[ X = \frac{x}{L}, \quad Y = \frac{y}{L}, \quad U = \frac{uL}{\alpha}, \quad V = \frac{vL}{\alpha}, \quad P = \frac{pL^2}{\rho\alpha^2}, \quad \theta = \frac{T - T_c}{T_h - T_c}, \quad \text{Pr} = \frac{\nu}{\alpha}, \]

\[ Gr = \frac{g\beta L^3(T_h - T_c)}{\nu^2}, \quad Ha^2 = \frac{\sigma B_0^2 L^2}{\mu}, \quad Ra = \frac{g\beta L^3(T_h - T_c)\text{Pr}}{\nu^2}, \quad \sigma = \frac{\rho^2\alpha}{L}, \quad \alpha = \frac{k}{\rho C_p} \]

Thus, Eqs. (1)-(4) become

\[
\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0
\]

\[
U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = -\frac{\partial P}{\partial X} + \text{Pr} \left( \frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} \right)
\]

\[
U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} = -\frac{\partial P}{\partial Y} + \text{Pr} \left( \frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2} \right) + \frac{Ra}{\text{Pr}} \theta - Ha^2 \text{Pr} V
\]

\[
U \frac{\partial \theta}{\partial X} + V \frac{\partial \theta}{\partial Y} = \frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2}
\]

The boundary conditions are: at top wall and semi circular heated boundaries \( U = V = 0, \theta = 1 \)

At bottom wall \( U = V = 0, \theta = 0 \)

At the other walls \( U = V = 0, \frac{\partial \theta}{\partial N} = 0 \)

At the inside and on the wall of enclosure fluid pressure \( P = 0 \)

3. Numerical implementation

The dimensional governing equations of this physical problem are transferred to the non-dimensional form. The finite element method is one of the numerical methods that have expected popularity due to its capability for solving complex structural problems. This is due to the fact that the governing differential equations for general flow problems consist of several coupled equations which are intrinsically nonlinear. It will be assumed that the fluid inside the cavity is viscous and incompressible. To handle the cavity situation, the finite element method (FEM) will be considered. The discretized energy and momentum equations subjected to the boundary conditions simultaneously will be solved using the finite element method (FEM). The numerical technique based on the Galerkin weighted residual method of finite element formulation is used in this work. The solution method has been chosen to solve the governing equations, which are linearized implicitly with respect to the equation’s dependent variable. The advantage of using this method is that the global system matrix is decomposed into smaller sub-matrices and then solved in a sequential manner using a nonlinear parametric solver. This approach will result in
significantly fast convergence assurance. A complete explanation of this procedure can be found in finite element method (FEM) [6].

3.1. Code validation

Validation of the code was done by comparing streamlines and isotherms with results shown in Fig. 2(a) and Fig. 2(b) by Santosh et al. [5] while Pr=0.71, Ra=10^5. As seen from this figure the obtained results show good agreement.

![Fig. 2(a). Obtained results by Santosh [5]](image)

![Fig. 2(b). Obtained results by Present work](image)

4. Results and discussion

A numerical analysis has been performed in this work to investigate the effects of magnetic field in a square cavity with a semi circular heated obstacle of different orientation parameter (rot). The ranges rot and Ha for this investigation vary from 0° to 90° and 0 to 100 respectively whereas other parameters are fixed at Ra = 10^4 and Pr = 0.71. In this section the influence of the Hartmann number (Ha = 0, 50,100) on the flow and temperature fields for Ra=10^4 and three values of the orientation parameter (rot=0°, 60°,90°) are presented. Fig. 3 illustrates the streamlines and isotherms for rot=0°, where the buoyancy effects dominate the flow field in the cavity and the heat transfer is mainly due to natural convection. The results show that the buoyancy induced vortices in the cavity in the absence of the magnetic field (Ha = 0). Four vortices are evident inside of the cavity for Ha = 0. When the Hartmann number increases, both of them are vanished due to the effect of the magnetic field. The isotherms distribution is also affected by the effect of the magnetic field. Less bend of the isotherms lines is observed as the Hartmann number increases. Fig. 4 shows the streamlines and isotherms for rot=60°. In the absence of the magnetic field, one vortex is
appearing clearly right side of the heated body due to buoyancy force. As the Hartmann number increases size of the vortex larger and move to the top side and another vortex created on the left top corner. Bending of the isotherms lines are a lesser amount of with increasing Hartmann number. Fig. 5 shows the streamlines and isotherms for rot=90°. In the absence of the magnetic field, elliptic size only one vortex is generated right side of the heated body due to buoyancy force. As the Hartmann number increases shape of the vortex seems to be almost circular and move to the top side and another vortex created on the left top corner. Effect of the magnetic field is also significant for isotherms lines. The average Nusselt number is plotted as a function of Hartmann number and Rayleigh number respectively as shown in Fig. 6 and Fig. 7 for five different orientation (rot = 0°, 15°, 30°, 45°, 90°) while Pr=0.71. The maximum heat transfer rate is obtained for the lowest Ha. This is because the magnetic field retards the flow.

Fig. 3. Streamlines (on the top) and isotherms (on the bottom) for rotation 0° with Pr=0.71 and Ra=10000.

Fig. 4. Streamlines (on the top) and isotherms (on the bottom) for rotation 60° with Pr=0.71 and Ra=10000.
5. Conclusion

The effects of magnetic field parameter $Ha$ due to semi circular heat source, Prandtl number $Pr$ and Rayleigh number $Ra$ on flow and temperature field have been studied in detail. From the present investigation the following conclusions may be drawn: if Hartman number increases the Nusselt number, representing heat transfer from the cavity decreases and if Rayleigh number increases the Nusselt number increases.

References