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Optimal pump count prediction algorithm for identical pumps working in parallel mode

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Abstract

This paper concentrates on an algorithm for steady running prediction of all identical variable speed pumps (VSP) in group close to the best efficiency point (BEP) given by pump manufacturer. Prediction of starting or stopping pumps based on the pressure and variable demand required is also covered. Additional pumps will be activated when the required pressure or demand cannot be met. Optimal pump working (Q, H) areas, most efficient combination between Q, H and different number of pumps and boundaries between them are calculated and visualized. Results provide simple and easy programmable input to adjust pumping station control systems. The usage of the proposed algorithm is illustrated by a case study based on an existing pumping station.

Keywords: Pumping optimization, Dynamic model, Best efficiency Point, Water distribution systems

1. Introduction

Optimization of pumps schedules and control setting tuning to achieve maximum efficiency is of high current interest. The working condition of an ideal pump to achieve is a situation when the needed demands are met; pressure conditions satisfied and total pumping cost is minimum. Pump (Q, H) performance and efficiency characteristics are usually provided by manufacturers. Different mathematical models of pump stations are used in the implementation of simulation and optimization software and techniques (Zessler and Shamir 1989; Ormsbee et al. 1989; Brdyś and Ulanicki 1994; Yu et al. 1994).

For overall practical estimation of the energy efficiency, a best practice guide was prepared (Energy Efficiency Best Practice Guide, 2009) to help industries improve pumping systems step by step. For pumps working in the oil industry, A. B. Crease (1996) showed that identical pumps work with the highest efficiency (running in parallel) if they operate at the same conditions. Z. Tianyi et al. (2012) also showed that pumps with variable frequencies running in parallel are most efficient when they run at the same speed. Both of these investigations were intended for pumps working outside water distribution systems (in the oil industry and in air-conditioning systems). Analysis of pumps for a water distribution system (WDS) is more complex, as usually a WDS contains water towers and analysis must be made for joint action of pumps and water towers. In this regard, the Tallinn Water WDS differs from other systems because it contains no water towers and the analyses of pumps by A. B. Crease (1996) and Z. Tianyi et al. (2012) may be useful in this case.

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Typically, consumed energy and maintenance cost of a medium-sized industrial pump will account for over 50-95% of a pump's life cycle cost (LCC). Efficiency can be best improved here through the reduction of energy consumed, maintenance recruitments and more closely matching pumping system capacity to actual production requirements. Improved efficiency of pumping systems also contributes to the reduction of greenhouse gas emissions and reduction of environmental impact of operations. In many cases this process includes only limited financial resources but at the same time provides significant savings. Two common issues to be overcome in operation are unnecessary demand on the pumping system and oversized pumps. Focus in this article is on techniques to achieve identical pump group running near the best efficiency point (BEP) and to identify when to start/stop lag pumps based on the flow required. A pump is generally oversized when it is not operated at or with 20% of its BEP (Energy Efficiency Best Practice Guide, 2009).

Nomenclature:

P – electrical power consumed by the station
P_i- electrical power consumed by the ith pump
n – number of pumps at the station
f_i – relative rotational speed of the ith pump
f_i^* – actual rotational speed of the ith pump
f_{i0} – rated rotational speed of the ith pump
ε_0 – binary variable
H,Q – head and water flow required
H_i – pumping head at relative rotational speed of the i^{th} pump
H_{i0} – pumping head at the rated rpm f_{i0}
H_0 – nominal pumping head
η_i – total efficiency of the pump
η_{i0} – rated efficiency of the pump
Q_0 – nominal flow rate
Q_i – flow at relative rotational speed of the i^{th} pump
Q_{i0} – flow at the rated rpm f_{i0}
λ– Lagrange multiplier
ρ – density of water
g – gravitational acceleration
N_i – effective power of pump at relative rotational speed of the i^{th} pump
N_{i0} – effective power of pump at the rated rpm f_{i0}

2. Identification of the optimization task

The task is to optimize the work of a pumping station equipped with identical centrifugal pumps running in parallel mode. It is necessary to estimate the parameters derived from the network characteristics, the working mode where electrical power consumption is minimal. Let us assume the following: the number of pumps in the pumping station is n, all the pumps can be switched on and off, and their revolutions per minute (rpm) are adjustable. In such a case a complex optimization task should be resolved with the objective function which minimizes energy consumption

$$P = \sum_{i=1}^{n} \varepsilon_i P_i\left(Q_i, f_i\right) \quad (1)$$

where

$$f_i = \frac{f_i^*}{f_{i0}} \quad (2)$$

ε_i= 1 if the ith pump is operational, and $\varepsilon_i = 0$ if the pump is not operational. It is necessary to find such values of ε_i and f_i where the value of the objective function (1) is minimal, provided that the following conditions have been met.

1) Condition derived from the network characteristic is

$$H = H(Q), \tag{3}$$

where

$$Q = \sum_{i=1}^{n} \varepsilon_i Q_i \tag{4}$$

2) Conditions derived from the pump pressure characteristics are described as

$$H_i = f_i^2 H_{i0}\left(Q_{i0}\right) = f_i^2 H_{i0}\left(\frac{Q_i}{f_i}\right) \tag{5}$$

Usually it is assumed that

$$H_{i0} = a_{i1} + a_{i2} Q_{i0} + a_{i3} Q_{i0}^2 \tag{6}$$

in which case

$$H_i = a_{i1} f_i^2 + a_{i2} f_i Q_i + a_{i3} Q_i^2 \tag{7}$$

3) Conditions derived from the power characteristic are described as

$$N_i = f_i^3 N_{i0}\left(\frac{Q_i}{f_i}\right) \tag{8}$$

if

$$N_{i0} = b_{i1} Q_{i0} + b_{i2} Q_{i0}^2 + b_{i3} Q_{i0}^3 \tag{9}$$

then

$$N_i = b_{i1} f_i^2 Q_i + b_{i2} f_i Q_i^2 + b_{i3} Q_i^3 \tag{10}$$

4) Relationship between the effective power of the pump and the electrical power consumed is

$$P_i = \frac{N_i}{\eta_i} \tag{11}$$

5) Restrictions of the pump's rotational speed

$$f_{i\min} \le f_i \le f_{i\max} \tag{12}$$

The reason for the latter is the need to avoid motor overloads.

The optimization task will be solved below in two stages. First, it is necessary to optimize the pumping station operations with a constant number of pumps. Then, the optimal number of pumps will be established.

3. Optimization of pumping station operations with a constant number of pumps

Mathematically, the problem set leads to solving of a non-linear planning task. It involves finding non-negative values f_i, which satisfy the equalities (3), (4), (11) and inequality (12) and provide the minimum value of the objective function (1) at these conditions.

Let us assume that there are n identical pumps at the pumping station. In such case

$$\varepsilon_i = 1 \qquad i = 1,2,...n$$

and the objective function (1) takes the form

$$P = \sum_{i=1}^{n} P_i(Q_i, f_i) \tag{13}$$

If the pumps are identical, and their pressure characteristic is

$$H_0 = H_0(Q_0) \tag{14}$$

then

$$H_i = f_i^2 H_0(Q_0)\,,\ N_i = f_i^3 N_0(Q_0) \tag{15}$$

With the given pumping head H, the total efficiency of the pump η_i can be expressed as

$$\eta_i = f(Q_i) = f\left(Q_0 \frac{n_i}{n_0}\right) \tag{16}$$

Therefore, in the case of identical pumps, in accordance with the equality (11)

$$P_i = \frac{\rho g Q_i H}{\eta_i} \tag{17}$$

and

$$Q_i = f_i Q_0 \tag{18}$$

Taking the equality (4), we will now find that

$$Q = \sum_{i=1}^{n} f_i Q_0 \tag{19}$$

Let us find the objective function (13) minimum with the additional condition of (19).
To solve the task the Lagrange function is used, which, using equalities (17) and (19), can be written in the following form:

$$L = \sum_{i=1}^{n} \frac{\rho g Q_0 f_i H_0}{f_0 \eta_i} + \lambda\left(Q_0 - \sum_{i=1}^{n} f_i Q_0\right). \tag{20}$$

Equating the partial derivatives of the Lagrange function to zero

$$\frac{\partial L}{\partial f_i} = 0 \qquad i = 1,2,\ldots n \tag{21}$$

the equations for establishing the f_i-s

$$\frac{Q_0 H_0 - f_0 \dfrac{\partial \eta_i}{\partial f_i}}{f_0 \eta_i^2} - \lambda Q_0 = 0 \qquad i = 1,2,\ldots n \tag{22}$$

$$Q - Q_0 \sum_{i=1}^{n} f_i = 0 \tag{23}$$

Ecuationts (22) and (23) form a system of n+1 nonlinear equations that can be solved numerically.
First n equations (22) will all be the same. It is easy to see that the system has a solution if we take $f_i = f$ for each i.
Equation (23) will have the form of

$$Q - Q_0 n f = 0 \tag{24}$$

and can be used for finding f

$$f = \frac{Q}{n Q_0} \tag{25}$$

From here we can calculate our Lagrange multiplier. However, this solution may not be unique. Therefore, we tried to use real pump characteristics (Fig. 1) to find the efficiency of three pumps working with different frequencies at H_0 equal to 27 m. Let us suppose that a WDS needs 1200 l/s and use different flows for two pumps. The third pump flow is calculated according to Eq. 23 as a difference between 1200 l/s and the sum of two pumps. The efficiency of each pump is estimated from Fig. 1. Fig. 2 presents the results of calculations. The figure shows that the surface has one minimum only and it is just at point 400,400,400.

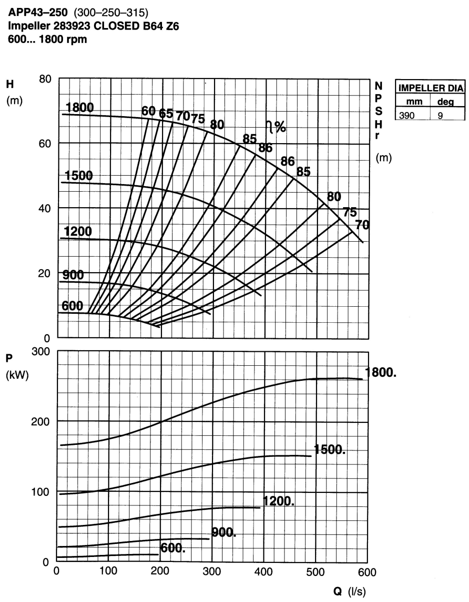

Fig.1. Pressure and efficiency characteristics of Ahlstrom APP 43–250 centrifugal pump

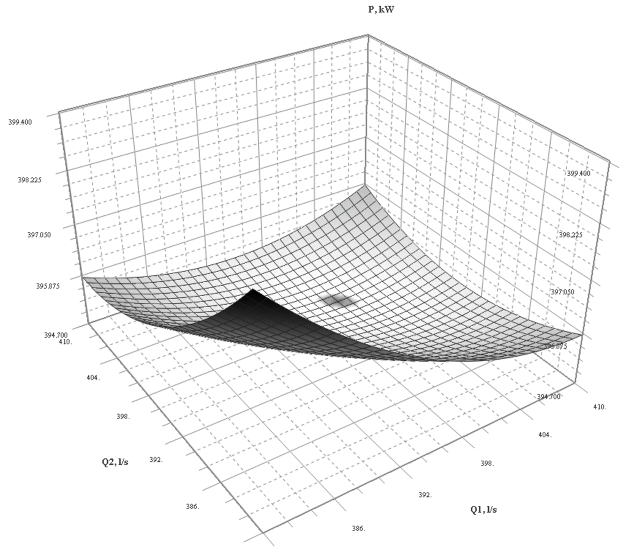

Fig. 2. Summarized power consumption of pumps at different flows

Therefore, with the constant number of pumps in the pumping station, the optimal solution is to keep the pumps running with one and the same rotational speed.

4. Establishing the optimal number of pumps

Assume that the pumping station is equipped with identical pumps, which run with identical rpm. The optimal number of running pumps for the given pumping head H and the flow rate Q is calculated as follows:

Let the efficiency of the pump be

$$\eta = \eta(Q, H) \quad (26)$$

in which case the efficiency of each n identical pump is

$$\eta_n = \eta\left(\frac{Q}{n}, H\right) \quad (27)$$

Accordingly, the overall electrical power consumed by the pumping station pumps can be described in the following form:

$$P = \frac{\rho g Q H}{\eta\left(\frac{Q}{n}, H\right)} \quad (28)$$

In order for the electrical power to be minimal, the efficiency η_n must be maximal. Consequently, it is necessary to find η_n for all n-s, and to establish at which n the η_n will obtain its highest value. The respective n will give us the optimal number of pumps. Let us assume that the number of pumps is a constantly changing value x, which is found from the condition of the maximum of the function $\eta\left(\frac{Q}{x}, H\right)$

$$\frac{d\eta\left(\frac{Q}{x}, H\right)}{dx} = 0 \quad (29)$$

Let approximate the efficiency of the pump by polynomial interpolation as

$$\eta(Q,H) = c_0 + c_1Q + c_2Q^2 + c_3H + c_4QH + c_5H^2 + c_6Q^2H + c_7QH^2 \tag{30}$$

where c_0 – c_7 are coefficients found with the non-linear least squares method from Fig. 1.

in which case the efficiency of n pumps is

$$\eta_n = c_0 + c_1\frac{Q}{n} + c_2\frac{Q^2}{n^2} + c_3H + c_4\frac{Q}{n}H + c_5H^2 + c_6\frac{Q^2}{n^2}H + c_7\frac{Q}{n}H^2 \tag{31}$$

Replacing n by x and taking into account (29) we will now find that the optimal number of pumps must be an integer around the value

$$x = -\frac{(2c_2 + 2c_6H)Q}{(c_1 + c_4H + c_7H^2)} \tag{32}$$

Based on the optimal number of pumps, the (Q, H) area can be divided into subareas. Boundaries separating subareas can be found using the following equalities:

$$\eta\left(\frac{Q}{n-1}, H\right) = \eta\left(\frac{Q}{n}, H\right). \tag{33}$$

Using the efficiency expression (31) with the condition (33), the boundaries for Q, when it is necessary to increase pump number from n-1 to n can be calculated

$$Q_{n-1,n} = -\frac{(c_1 + c_4H + c_7H^2)n(n-1)}{(c_2 + c_6H)(2n-1)} . \tag{34}$$

The approximation (31) has been checked and it showed quite good results. It may be possible to obtain better results with the cubic approximation of efficiency (Ulanicki et al. 2008) but in this paper only approximation (31) is used.

5. Case Study

This case study is targeted to optimizing the work of a pumping station equipped with four identical Ahlstrom APP 43–250 (300-250-315) centrifugal pumps with a closed impeller, running in parallel. The pressure and efficiency characteristics are presented in Fig. 1.

Using the least squares method, the efficiency of the pump is calculated based on Fig. 1.

$$\eta_n = 65.85 + 0.316\frac{Q}{n} - 0.00097\frac{Q^2}{n^2} - 2.224H + 0.0075\frac{Q}{n}H + 0.023H^2 + 0.0000085\frac{Q^2}{n^2}H - 0.000115\frac{Q}{n}H^2 \tag{35}$$

Let us now use the efficiency expression (35) to establish the optimal number of pumps. With the formula (32) we obtain

$$x = -\frac{Q(-0{,}00193 + 0{,}000017H)}{0{,}3166 + 0{,}0075H - 0{,}000115H^2} \tag{36}$$

If x is an integer, $x = n$, then it ensures from the equality (36) that the pumps are running in the optimal modes. The most efficient combination between Q and H with different n can be found from (37).

$$Q = -\frac{n(0{,}3166 + 0{,}0075H - 0{,}000115H^2)}{-0{,}00193 + 0{,}000017H} \tag{37}$$

According to equalities (34) and (35), the boundaries of the (Q, H) areas with *n*-1 and n number of pumps can be calculated as

$$Q_{n-1,n} = -\frac{(0.3167 + 0.0075H - 0.000115H^2)n(n-1)}{(-0.00097 + 0.0000085H)(2n-1)} \tag{38}$$

Figure 3 presents the results of calculations based on (37) and (38).

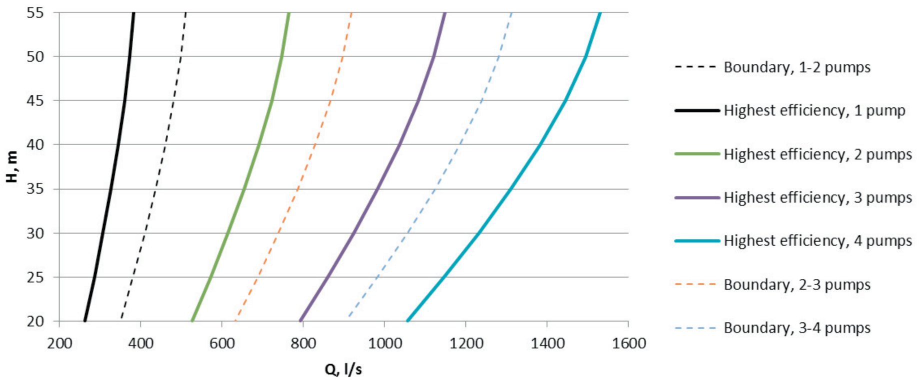

Fig. 3. Optimal pump count (Q, H) areas and boundaries

6. Conclusions

To achieve the highest efficiency at the Tallinn Water pumping station, VSP pumps in parallel should be run at the same rotational speed. Approximation of pump characteristics by mathematical equations allows us to create analytical formulas for selection of an efficient number of pumps when Q and H change. The mathematical algorithm used in this article helps to generate easily understandable optimal pump count (Q, H) areas and boundaries graphs for pumping station operational crew. Optimal number of pumps for adjusting pumping station controls to achieve maximal efficiency can be calculated automatically and be used as input to adjust pumping station controls.

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