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Gravity field recovery from GOCE orbits using the energy conservation approach

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Abstract: The sum of the dissipative energy and energy constant of the GOCE satellite is found by a priori gravity field model at first, and the GOCE dissipative energy is obtained by computing the adjacent epoch difference via the differential method. Then, a gravity field model GOCE-ECP01, which up to the degree and order 80, is recovered by the energy conservation approach from the 103-day precise orbital data of the GOCE satellite collected from November 1, 2009 to January 12, 2010. Finally, the model is compared with existing models EGM96, ITG-CHAMP05S, EIGEN-GRACE2010S, EIGEN-6C and GO_CONS_GCF_2_DIR_R3. The results show that at the same order and degree, the accuracy of model GOCE-EBP01 is higher than those of models EGM96 and ITG-CHAMP05S, but lower than those of models EIGEN-GRACE2010S, EIGEN-6C and GO_CONS_GCF_2_DIR_R3, which is mainly caused by the pole gap.

Key words: energy conservation approach; GOCE satellite; gravity field model; dissipative energy; potential coefficient

1 Introduction

In 2009, the GOCE (Gravity field and steady-state Ocean Circulation Explorer) satellite was launched to determine a geoid with an accuracy of 1 – 2 cm and gravity-field anomalies with an accuracy of 1 mGal at a spatial resolution exceeding 100 km[1]. The GOCE satellite, by combining the high-low satellite-to-satellite tracking (HL-SST) and gravity gradiometry, can effectively recover the medium long wavelength and medium short wavelength information of Earth's gravity field model with a high accuracy. At present, several approaches have been employed to recover the Earth's gravity field model via the tracking data of low earth orbiting satellites (LEOs), such as the Kaula linear perturbation approach, dynamical integral approach, short-arc integral approach, point acceleration approach, average acceleration approach, energy conservation approach and celestial mechanics approach[2-5]. Among the above, the energy conservation approach is simple and effective in the Earth's gravity reversion. Originally proposed by Keefe in 1957, the approach was employed to process the measured low orbiting satellite-to-satellite tracking data by Jekeli in 1999[6]. The approach is about connecting the state vector of force on the satellite with the gravitational potential coefficient to establish an energy conservation equation to recover the gravitational potential coefficient. Since the approach needs no the numerical integral and iterative computations and the original state vector estimation, and its observation equation is linear, many researchers have made in-depth studies on the energy conservation approach, employing it to solve the gravity field models of the CHAMP and GRACE satellites[2-12]. However, the approach is rarely employed to solve the gravity field of GOCE[13].

The paper, after recovering the Earth's gravity field

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model GOCE-ECP01 up to the degree and order 80 based on the 103-day reduced dynamic orbital data of the GOCE satellite by the energy conservation approach, compares the model with other gravity field models. Then, based on the comparison results, it evaluates the accuracy of the recovered model, and analyzes the applicability of applying the energy conservation approach in GOCE Earth gravity field model recovery.

2 Energy conservation equation

The GOCE energy conservation equation in the terrestrial coordinate system reads[11]:

$$T + E_0 = \frac{1}{2}|\dot{\boldsymbol{r}}| - \frac{1}{2}\bar{\omega}^2(r_x^2 + r_y^2) - V_t - U_0 - \Delta C \quad (1)$$

where T is the disturbing potential, E_0 stands for the energy integral constant, V_t is the perturbation potential of the satellite (mainly including the third body perturbation potential, Earth tide perturbation potential, ocean tide perturbation potential, Earth's pole tide and ocean pole tide perturbation potential), U_0 is the normal gravitational potential (only referring to the central gravitational potential of the satellite in this paper), ΔC is the dissipative energy of the satellite, $\boldsymbol{r}(r_x, r_y, r_z)$ and $\dot{\boldsymbol{r}} = (\dot{r}_x, \dot{r}_y, \dot{r}_z)$ are the position and velocity vector of the satellite respectively at an epoch, $\bar{\omega}$ stand for the mean angular velocity of the Earth, and the first and second terms on the right of equation stand for the kinetic energy and rotation energy of the satellite. The computational formula of the disturbing potential adopting the 3D Cartesian coordinate system reads:

$$T = \frac{GM}{R}\sum_{n=2}^{\infty}\sum_{m=0}^{n}(\bar{C}_{nm}\bar{V}_{nm} + \bar{S}_{nm}\bar{W}_{nm}) \quad (2)$$

$\bar{V}_{nm}$ and $\bar{W}_{nm}$ are the Cunningham recursive coefficients. The recursive relation employed in the paper is as follows[11,14]:

$$\begin{cases} \bar{V}_{n,m} = \left(\frac{R}{r}\right)^{n+1}\bar{P}_{n,m}(\sin\varphi)\cos m\lambda \\ \bar{W}_{n,m} = \left(\frac{R}{r}\right)^{n+1}\bar{P}_{n,m}(\sin\varphi)\sin m\lambda \end{cases} \quad (3)$$

Initial value: $\bar{V}_{0,0} = \frac{R}{r}$, $\bar{W}_{0,0} = 0$

$$\begin{cases} \bar{V}_{n,n} = \begin{cases} \sqrt{\frac{2m+1}{m}}\left\{\frac{xR}{r^2}\bar{V}_{n-1,n-1} - \frac{yR}{r^2}\bar{W}_{n-1,n-1}\right\}, & m=1 \\ \sqrt{\frac{2m+1}{2m}}\left\{\frac{xR}{r^2}\bar{V}_{n-1,n-1} - \frac{yR}{r^2}\bar{W}_{n-1,n-1}\right\}, & m>1 \end{cases}, & n=m \\ \bar{W}_{n,n} = \begin{cases} \sqrt{\frac{2m+1}{m}}\left\{\frac{xR}{r^2}\bar{W}_{n-1,n-1} + \frac{yR}{r^2}\bar{V}_{n-1,n-1}\right\}, & m=1 \\ \sqrt{\frac{2m+1}{2m}}\left\{\frac{xR}{r^2}\bar{W}_{n-1,n-1} + \frac{yR}{r^2}K\bar{V}_{n-1,n-1}\right\}, & m>1 \end{cases}, & n=m \end{cases} \quad (4)$$

$$\begin{cases} \bar{V}_{n,n} = \sqrt{\frac{(2n+1)}{(n+m)(n-m)}}\begin{bmatrix} \sqrt{(2n-1)}\frac{zR}{r^2}\bar{V}_{n-1,m} \\ -\sqrt{\frac{(n-m-1)(n+m-1)}{(2n-3)}}\frac{R^2}{r^2}\bar{V}_{n-2,m} \end{bmatrix}, & n>m \\ \bar{W}_{n,n} = \sqrt{\frac{(2n+1)}{(n+m)(n-m)}}\begin{bmatrix} \sqrt{(2n-1)}\frac{zR}{r^2}\bar{W}_{n-1,m} \\ -\sqrt{\frac{(n-m-1)(n+m-1)}{(2n-3)}}\frac{R^2}{r^2}\bar{W}_{n-2,m} \end{bmatrix}, & n>m \end{cases} \quad (5)$$

where, GM stands for gravitational constant times Earth mass, R is the Earth's semi-major axis, r is the geocentric radius vector of the satellite, x, y, z are the positions of the satellite, $\bar{P}_{nm}$ is the normalized Legendre function, n and m are the order and degree respectively, $\bar{C}_{nm}$ and $\bar{S}_{nm}$ are the normalized spherical harmonic coefficients.

Under the hypothesis that the positions of adjacent epochs A and B are (x^A, y^A, z^A) and (x^B, y^B, z^B) respectively, when

$$KE = \frac{1}{2}|\dot{\boldsymbol{r}}| - \frac{1}{2}\bar{\omega}^2(r_x^2 + r_y^2) - V_t - U_0 \quad (6)$$

$$CE = E_0 + \Delta C \quad (7)$$

For epoch A,

$$T^A = KE^A - CE^A \tag{8}$$

and for epoch B,

$$T^B = KE^B - CE^B \tag{9}$$

Compute the difference between the two epochs, namely, equation (8) minus (9).

$$T^A - T^B = KE^A - KE^B - (CE^A - CE^B)$$

where,

$$T^A - T^B = \frac{GM}{R}\sum_{n=2}^{\infty}\sum_{m=0}^{n}((\bar{V}^A_{nm} - \bar{V}^B_{nm})\bar{C}_{nm} + (\bar{W}^A_{nm} - \bar{W}^B_{nm})\bar{S}_{nm}) \tag{11}$$

Plug equation (11) into (10), the observation equation solving the GOCE gravity filed potential coefficients by the energy conservation approach is obtained. Then, the standardized Gauss-Markov model is adopted to formulate an error equation and solves the potential coefficients.

$$V = AX - L \tag{12}$$

3 Recovery of GOCE Earth's gravity field model by using the energy conservation approach

3.1 Data pre-processing

GOCE data is managed and released by the ESA (European Space Agency) and stored in XML (Extensible Markup Language) format, therefore, it is needed to transfer XML data into the data of required format, and at the same time, detect, mark and interpolate the possible data gaps. In this paper, the level 2 orbital data SST_PSO_2, including the kinematic data (SST_PKI_2), reduced-dynamic orbit data (SST_PRD_2), Earth orientation quaternions data (SST_PRM_2) and variance-covariance matrices data (SST_PCV_2) are needed[15]. As the kinematic data contain more data gaps but no the velocity of the satellite, the reduced-dynamic orbit data were employed in this paper. In addition, since both the reduced-dynamic orbit data and the energy conservation equation adopted in this work depended on the celestial coordinate system, there's no need to transfer the coordinate system of the orbital data.

3.2 Computation of the dissipative energy

Since the non-gravitational force in the flight direction of GOCE satellite is compensated by the drag-free control system, and the drag-free compensation is one-dimensional and not fully free of systematic, there will be some remaining signals of the non-gravitational forces which are not compensated[15], therefore, the acceleration of non-gravitational force cannot be obtained and the dissipative energy of the GOCE satellite cannot be computed directly as that of the GRACE satellite, which decides that the processing of the dissipative energy will directly influence the accuracy of the GOCE Earth's gravity field model by the energy conservation approach.

To analyze the dissipative energy of the GOCE satellite, by taking GO_CONS_GCF_2_DIR_R3 Earth's gravity field model as a reference model, the orbit data of November 1, 2009 were employed to imitate the non-spherical gravitational perturbation potential of the satellite as well as to analyze the various potentials energy values of the satellite in the celestial coordinate system (Fig. 1).

In this paper, a gravity field model was employed to find the sum of the energy constant and dissipative energy of the satellite. Since the energy constant is a fixed value, the difference of adjacent epochs actually equals to the difference of the dissipative energy values of adjacent epochs. In order to reflect the degree of difference of the above mentioned sums of the satellite computed based on different models, EIGEN-6C, EIGEN-CG01C, EIGEN-CHAMP05S, EGM2008, EGM96, ITG-Grace2010 and GO_CONS_GCF_2_DIR_R3 were adopted as reference models. Figure 2 shows that the deviations of the computed sums of EGM96, EIGEN-CHAMP05S and ITG-Grace2010s are higher, while the rest are lower than 1 m^2/s^2. To reflect the dissipative energy as true as possible, reference models

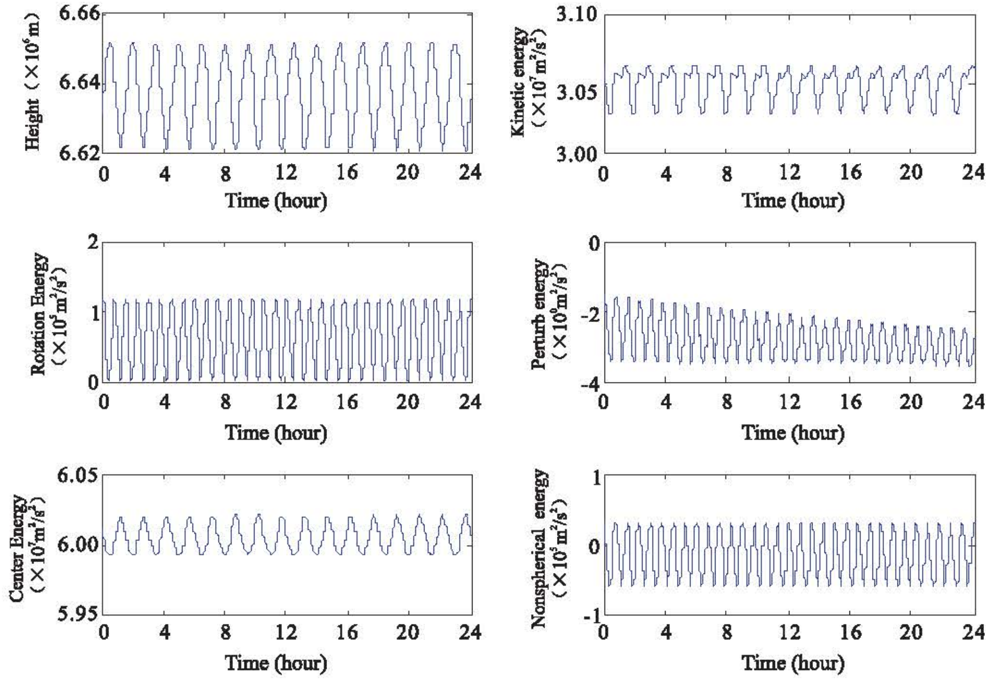


Figure 1 Energy change of GOCE satellite one day

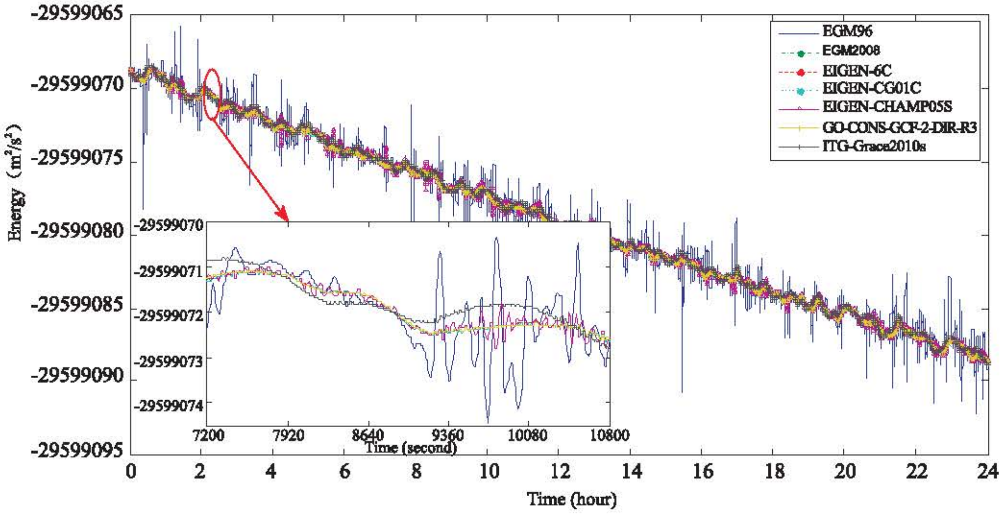


Figure 2 The sum of constant and dissipative energy by different reference gravity field models

with higher accuracies shall be adopted. Therefore, GO_CONS_GCF_2_DIR_R3 was adopted as the reference model to compute the sum of the energy constant and dissipative energy of the satellite.

With adjacent epoch difference, the energy constants were cancelled, leaving only the difference of the dissipative energy of adjacent epochs, thus the first term on the right of equation was obtained (Fig. 3 (a)), and then the second term on the right of equation was obtained via the orbit and velocity of the satellite and force model (Fig. 3 (b)).

3.3 Gravity field model computation

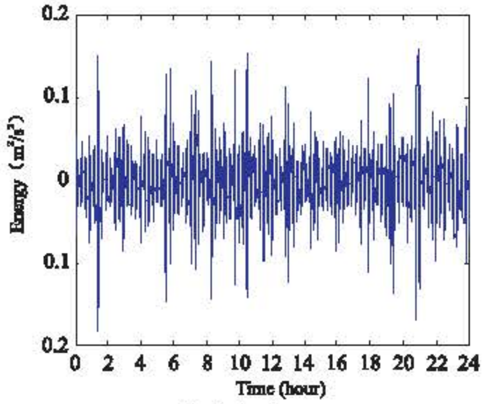
889920 epochs were sampled every 10 s from the 103-day (November 1, 2009 to February 12, 2010) reduced-dynamic orbit data provided by the ESA. Then, equations (10), (11) and (12) were combined to solve equation through direct inversion together with MKL (Math Kernel Library) Mathematical Functions Library. Meanwhile, as Colombo advised[5, 11], the potential coefficients were arranged via the degree-based sequential approach, thus the matrix off diagonal elements of the corresponding coefficient matrix of normal equation caused by the orthogonally of discrete cos and sin sequence functions is disappeared, giving a block diagonal structure to the coefficient matrix. That means even there are errors in the coefficient matrix, it would still have a block diagonal dominant structure (Fig. 4), which is good for the solution of the equations. Through calculation, a gravity field model GOCE-ECP01 up to the degree and order 80 was recovered.

The accuracy of the gravity field model was estimated via the geoid height error and cumulative geoid height error of the potential coefficient differences of different models (Figs. 5 and 6).

$$\sigma_n(\mathrm{Geoid}) = R\sqrt{\sum_{m=0}^{n}[(\bar{C}_{nm}-\bar{C}_{nm}^{\mathrm{ref}})^2+(\bar{S}_{nm}-\bar{S}_{nm}^{\mathrm{ref}})^2]} \tag{13}$$

$$\sigma_N(\mathrm{Geoid}) = R\sqrt{\sum_{n=0}^{N}\sigma_n^2(\mathrm{Geoid})} \tag{14}$$

where $\bar{C}_{nm}^{\mathrm{ref}}$ and $\bar{S}_{nm}^{\mathrm{ref}}$ are the potential coefficients of the reference model, $\bar{C}_{nm}$ and $\bar{S}_{nm}$ are the potential coefficients of the computational model, n and m are the order and degree of the gravity field model. The model EIGEN-5C is selected as a reference model, which is a general reference model in ICGEM (International Centre for Global Earth Models).



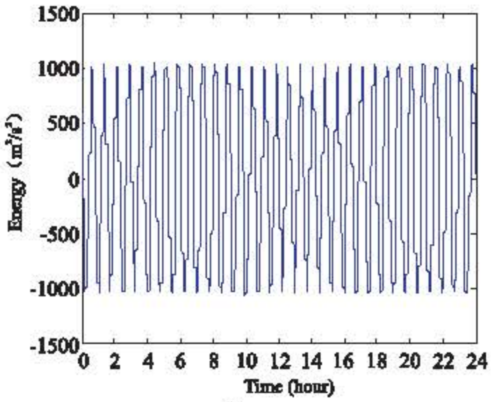


Figure 3 Difference of dissipative energy and KE energy between adjacent epoch

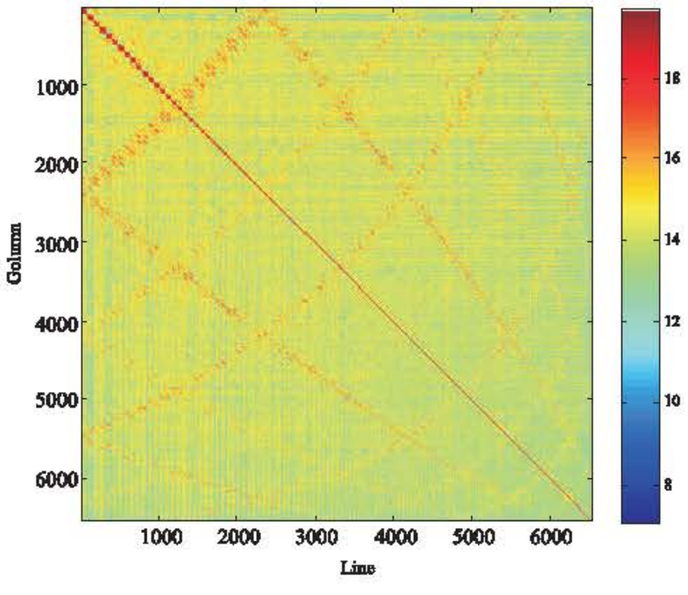


Figure 4 Normal matrix(N = 80. The values of the matix elements are denoted by denary logarithm)

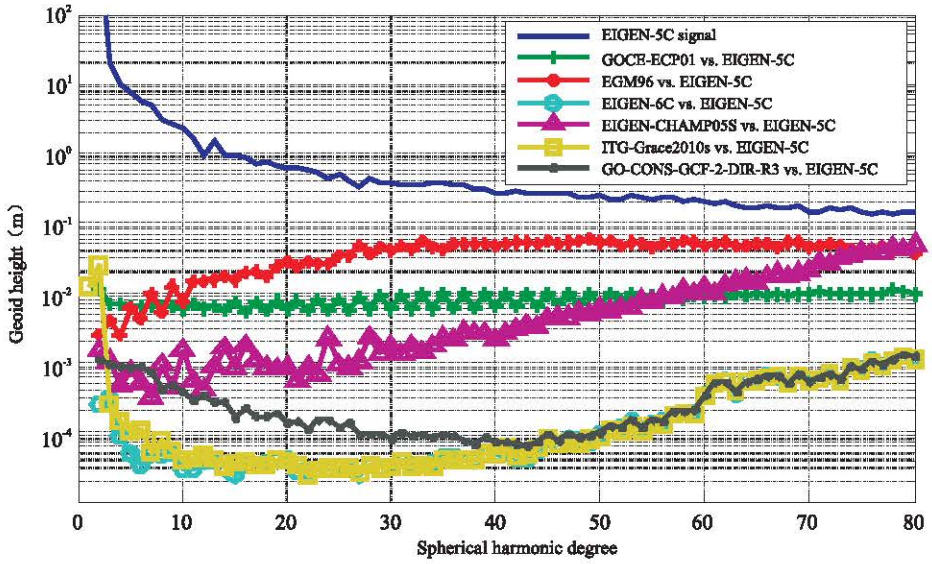


Figure 5 Geoid height between EIGEN-5C and GOCE-ECP01, EGM96, EIGEN-6C, EIGEN-CHAMP05S, GO-DIR-R3, ITG-GRACE2010S (The values of geoid height are denoted by denary logarithm)

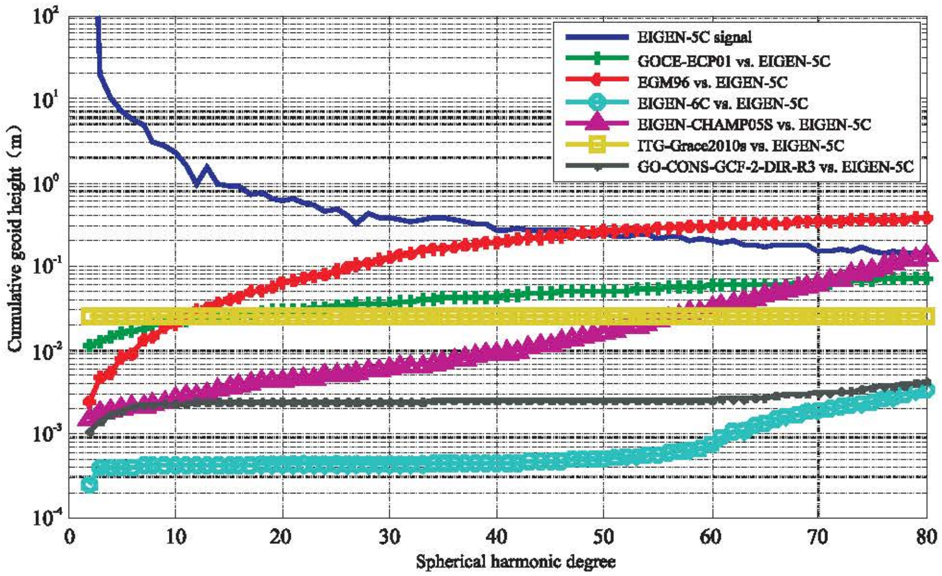


Figure 6 Cumulative geoid height between EIGEN-5C and GOCE-ECP01, EGM96, EIGEN-6C, EIGEN-CHAMP05S, GO-DIR-R3, ITG-GRACE2010S (The values of cumulative geoid height are denoted by denary logarithm)

It can be seen from the figures 5 and 6 that generally speaking, the accuracy of the gravity field model recovered only by GOCE orbit data is lower than that recovered by GRACE, especially at lower degrees and orders, which are mainly caused by the pole gap. Meanwhile, compared with the GOCE Earth's gravity field model GO_CONS_GCF_2_DIR_R3 recovered by the ESA, the model recovered in this paper is lower in accuracy as the result of adopting orbit data rather than gradient data and a smaller data size. However, thanks to the lower orbits and higher orbit accuracy of the GOCE satellite, the accuracy of the gravity field model recovered by GOCE is higher than that of the EGM96 model as a whole, and after the order of 55, its accuracy is higher than that of the CHAMP model.

4 Conclusion

In the paper, a gravity field model GOCE-ECP01, which up to the degree and order 80, was recovered by the energy conservation approach from the 103-day reduced-dynamic orbit data of the GOCE satellite. The overall accuracy of the model was higher than those of the EGM96 and CHAMP Earth's gravity field models, and it showed a geoid height ±1 cm at the degree and order 80.

Since the non-gravitational force in the flight direction of GOCE satellite is compensated by the drag-free control system, the acceleration of the non-gravitational force cannot be obtained; therefore, the dissipative energy of the GOCE satellite cannot be computed directly as those of the GRACE and CHAMP satellites during the process of solving the potential coefficient by the energy conservation approach. The sum of the dissipative energy and energy constant of the GOCE satellite was found by a priori gravity field model, and the GOCE dissipative energy was obtained by computing the adjacent epoch difference, namely, the energy constants were cancelled, leaving only the difference of the dissipative energy values of adjacent epochs, thus indirectly obtained the dissipative energy needed for recovering the GOCE Earth's gravity field model by the energy conservation approach. However, inevitably, the influence of the reference gravity model was brought in at the same time. Therefore, the follow-up study will focus on how to directly computing the dissipative energy of the GOCE satellite.

Since the data gap (pole gap) will appear when the GOCE satellite reaches the poles of the Earth, the overall accuracy of the Earth's gravity field model recovered by GOCE orbit data is lower. Test results show that after the degree and order of 80, the accuracies of directly computed potential coefficients are worse. Therefore, regarding the recovery of a high-accuracy GOCE Earth's gravity field model, how to remove the influence of the pole gap on the accuracy of the potential coefficient is a key subject needing to be researched. In the further research, regularization algorithms will be introduced to improve the computing accuracy of the potential coefficient.

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References

[1] ESA. GOCE Mission Requirements Document, Issue 2, GO-RS-ESA-SY-0001. European Space Age.

[2] Xu Tianhe and Yang Yuanxi. CHAMP gravity field recovery using kinematic orbits. Chinese Journal of Geophysics, 2005, 48(2): 288 – 293. (in Chinese)

[3] Xu Tianhe and Gan Yuehong. CHAMP gravity field recovery based on the improved energy conservation approach. Progress in Geophysics, 2008, 23(1): 63 – 68. (in Chinese)

[4] Wang Zhengtao, Li Jiancheng, Jiang Weiping and Chao Dingbo. Determination of earth gravity field model WHU-GM-05 using GRACE gravity data. Chinese Journal of Geophysics, 2008, 51(5): 1364 – 1371. (in Chinese)

[5] You Wei, Fan Dongming and Guo Jiang. Gravity field recovery by using energy conservation approach. Journal of Geodesy and Geodynamics, 2010, 30(1): 51-55. (in Chinese)

[6] Jekeli C. The determination of gravitational potential differences from satellite-to-satellite tracking. Celestial Mechanics and Dynamical Astronomy, 1999, 75(2): 85 – 101.

[7] Han S C. Efficient global gravity field determination from satellite-to-satellite tracking. Ohio: The Ohio State University, 2003.

[8] Han S C, Jekeli C and Shum C K. Efficient gravity field recovery using in situ disturbing potential observables from CHAMP. Geophysical Research Letters, 2002, 29(16):361-364.

[9] Visser P, Sneeuw N and Gerlach C. Energy integral method for gravity field determination from satellite orbit coordinates. Journal of Geodesy, 2003, 77(3): 207 – 216.

[10] Gerlach Ch, Sneeuw N, Visser P and Svehla D. CHAMP gravity field recovery using the energy balance approach. Advances in Geosciences, 2003, 1: 73 – 80.

[11] Wang Zhengtao. Theory and methodology of Earth gravity field recovery by satellite-to-satellite tracking data. Wuhan: Wuhan University, 2005. (in Chinese)

[12] Song Lei, Huang Teng and Peng Bibo. Recovery of earth gravity field with CHAMP data of orbit and acceleration. Journal of Hohai University (Natural Sciences), 2006, 34(6): 676 – 679. (in Chinese)

[13] Jäggi A, Bock H, Pail R and Goiginger H. Highly-reduced dynamic orbits and their use for global gravity field recovery: A simulation study for GOCE. Studia Geophysica et Geodaetica, 2008, 52(3): 341 – 359.

[14] Yi W Y. The Earth's gravity field from GOCE. München: Technische Universität München, 2011.

[15] EGG-C. GOCE High Level Processing Facility GOCE Level 2 Product Data Hand-book, issue 4.2 revision 2, 23/06/2010, GO-MA-HPF-GS-0110.