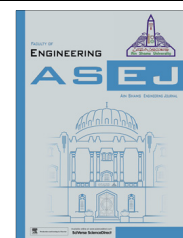




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## ELECTRICAL ENGINEERING

# Impact of DC link control strategies on the power-flow convergence of integrated AC–DC systems



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**Abstract** For the power-flow solution of integrated AC–DC systems, five quantities are required to be solved per converter, against three independent equations available. These three equations consist of two basic converter equations and one DC network equation, corresponding to each converter. Thus, for solution, two additional equations are required. These two equations are derived from the control specifications adopted for the DC link. Depending on the application, several combinations of valid control specifications are possible. A set of valid control specifications constitutes a control strategy. It is observed that the control strategy adopted for the DC link strongly affects the power-flow convergence of integrated AC–DC systems. This paper investigates how different control strategies affect the power flow convergence of integrated AC–DC systems. Sequential method is used to solve the DC variables in the Newton Raphson (NR) power flow model. Seven typical control strategies have been taken into consideration. This is validated by numerous case studies carried out with multiple DC links incorporated in the IEEE 118-bus and 300-bus test systems.

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## 1. Introduction

With ever-increasing load demands, system stability issues and different operating frequencies may render AC transmission

infeasible. In this respect, HVDC transmission allows power transmission between asynchronous AC transmission systems, and can increase the system stability by preventing cascading failures due to phase instability from propagating from one part of a wider power transmission grid to another. For lengths exceeding about 500 km, HVDC transmission is proving to be more economical than AC [1–3].

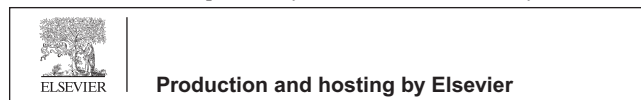
For planning, operation and control of power systems with HVDC links, power-flow solution of power systems incorporated with HVDC links is required [4–6]. For power flow solution of hybrid AC–DC systems, corresponding to each converter, five quantities are required to be solved. These are the DC voltage, the DC current, the control angle, the converter transformer tap ratio and the converter power factor.

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**List of symbols**

|  |  |                       |   |
|--|--|-----------------------|---|
| $S_{base}$                                 | base MVA   | $V_i \angle \theta_i$ | AC bus voltage magnitude (rms) and phase angle at $i$ th bus  |
| $V_{ac\ base}, I_{ac\ base}, Z_{ac\ base}$ | AC base voltage, AC base current and AC base impedance, respectively               | $V_j \angle \theta_j$ | AC bus voltage magnitude (rms) and phase angle at $j$ th bus  |
| $V_{dc\ base}, I_{dc\ base}, Z_{dc\ base}$ | DC base voltage, DC base current and DC base impedance, respectively               | $Y_{ik}, \phi_{ik}$   | magnitude and phase angle of the element in the $i$ th row and $k$ th column of the bus admittance matrix |
| $n_b$                                      | number of bridges  | $P_{Di}, Q_{Di}$      | active and reactive power demands at bus ' $i$ ', respectively  |
| $X_c$                                      | commutating reactance  | $P_{Dj}, Q_{Dj}$      | active and reactive power demands at bus ' $j$ ', respectively  |
| $V_d, I_d$                                 | DC voltage and current, respectively   | $V_{doR}, V_{doI}$    | no load direct voltages at the rectifier and inverter sides, respectively                                 |
| $\phi_R, \phi_I$                           | power factor angles at the rectifier and inverter ends, respectively               | $P_{dR}, Q_{dR}$      | active and reactive powers at the rectifier side, respectively  |
| $\alpha_R, \gamma_I$                       | firing angle of the rectifier and extinction angle of the inverter, respectively   | $P_{dI}, Q_{dI}$      | active and reactive powers at the inverter side, respectively   |
| $V_{dR}, V_{dI}$                           | DC voltages at the rectifier and inverter sides, respectively                      |                       |   |
| $a_R, a_I$                                 | converter transformer tap ratios on the rectifier and inverter sides, respectively |                       |   |
| $R_d$                                      | resistance of DC link  |                       |   |

On the other hand, only three independent equations comprising two basic converter equations and one DC network equation exist per converter. Thus for solution, two additional equations are required. These two equations are obtained from the control specifications adopted for the DC link. Thus, mathematically, the control specifications are used to bridge the gap between the number of independent equations and the number of unknowns. Control specifications usually include specified values of converter transformer tap ratio, converter control angle, DC voltage, DC current or power. Several combinations of valid control specifications are feasible, depending on the application. Each set of valid control specifications is known as a control strategy. Although a large number of control strategies are feasible, only some are adopted in practice.

The earliest algorithms for power flow were based on the Gauss–Siedel method, which exhibited poor convergence characteristics. Subsequently, the Newton–Raphson (NR) method was developed, which had better convergence characteristics. Gradually, it was adopted as the de facto standard in the industry.

For power flow solution of integrated AC–DC systems using the NR method, two different algorithms have generally been reported in the literature. These are known as the unified and the sequential method, respectively. Some excellent research works on the unified and the sequential power flow methods are presented in [4–15], respectively. Unlike the unified method, the sequential method is easier to implement and poses lesser computational burden due to the smaller size of the Jacobian matrix. Consequently, in this work, only the sequential AC–DC power-flow algorithm has been considered.

In the sequential AC–DC power-flow algorithm, the AC and DC systems are solved separately in each iteration and are coupled by injecting an equivalent amount of real and reactive power at the terminal AC buses. It is observed that each control strategy affects the sequential power flow convergence in a uniquely different manner. It was reported by [11] that for standard control strategies e.g. constant DC voltage or current or power, the convergence rate can be improved by decoupling the DC and AC systems and solving them independently. On the other hand, for non-standard ones such as constant tap

ratios and constant terminal voltage, the convergence may suffer. The mechanism by which this occurs has not been very clear and has not been exclusively addressed in the literature. This motivated the authors to investigate how different control strategies affect the power flow convergence. Numerous case studies are carried out by adopting seven different control strategies on HVDC links incorporated in the IEEE 118-bus and 300-bus test systems [16] for validation.

This paper is organized as follows: In Section 2, the mathematical modeling of the integrated AC–DC system is presented. Section 3 details some of the typical DC link control strategies adopted in practice. In Section 4, the power flow equations of the integrated AC–DC system are presented, with the DC link acting as an equivalent load on the converter AC buses. Section 5 details the case studies carried out by incorporating DC links in the IEEE 118 and 300 bus test systems. The conclusions are presented in Section 6.

## 2. System modelling

Fig. 1 shows a typical AC–DC power system network in which a HVDC link is connected in the branch “ $i$ – $j$ ” between any two buses “ $i$ ” and “ $j$ ” of the network. The two converters representing the rectifier and the inverter are connected to the AC system at buses “ $i$ ” and “ $j$ ” respectively, through their respective converter transformers. The HVDC link is accounted for as equivalent amount of real and reactive power injections  $P_{dR}$  and  $Q_{dR}$ ,  $P_{dI}$  and  $Q_{dI}$  at the converters’ AC terminal buses “ $i$ ” and “ $j$ ”, respectively. Although these power injections are not shown in Fig. 1, they are included in the analysis by appropriate modifications of the power flow equations, as detailed later in Section 4. Fig. 2 shows the equivalent circuit for the network shown in Fig. 1.

Prior to the selection of variables and formulation of the equations, several basic assumptions are required which are generally accepted in the analysis of steady state DC converter operation [1–3]. These are as follows:

- The AC voltages at the terminal bus bars are balanced and sinusoidal.



**Table 3** First study of IEEE 118-bus system.

| HVDC link          |                   | $P_{base}$ (pu) | HVDC link specification |                    | Power flow solution |                   |              |                |                |            |                |              |         |
|--------------------|-------------------|-----------------|-------------------------|--------------------|---------------------|-------------------|--------------|----------------|----------------|------------|----------------|--------------|---------|
| From bus no        | To bus no         |                 | Spec. values            |                    | ACSV                |                   | DCSV         |                |                |            |                |              |         |
|                    |                   |                 |                         | Control strategy 1 |                     | AC terminal buses |              | HVDC variables |                |            |                |              |         |
| 11                 | 13                | 0.4081          | Control strategy 1      | $P_{dR}$ (pu)      | 0.5                 | $V_{11}$          | 0.9803       | $V_{dR}$       | 1.005          |            |                |              |         |
|                    |                   |                 |                         | $V_{dI}$ (pu)      | 1.0                 | $\theta_{11}$     | 10.739°      | $I_d$          | 0.4975         |            |                |              |         |
|                    |                   |                 | $\alpha_R$ (deg.)       | 5°                 | $V_{13}$            | 0.9598            | $a_R$        | 1.08           |                |            |                |              |         |
|                    |                   |                 | $\gamma_I$ (deg.)       | 18°                | $\theta_{13}$       | 11.349°           | $a_I$        | 1.15           |                |            |                |              |         |
|                    |                   |                 | Control strategy 2      | $\cos\phi_R$       | 0.94                | AC terminal buses | $V_{11}$     | 0.9801         | $\cos\phi_I$   | 0.90       | NI             | 6            |         |
|                    |                   |                 |                         | $\cos\phi_I$       | 0.90                |                   |              |                | HVDC variables |            | $V_{dR}$       | 1.005        |         |
|                    |                   |                 | $P_{dR}$ (pu)           | 0.5                | $V_{11}$            | 0.9801            | $I_d$        | 0.4975         | $\alpha_R$     | 9.1566°    | $\gamma_I$     | 17.969°      |         |
|                    |                   |                 | $V_{dI}$ (pu)           | 1                  | $\theta_{11}$       | 10.741°           | $a_R$        | 1.09           | $\cos\phi_R$   | 0.94       | $\cos\phi_I$   | 0.90         |         |
|                    |                   |                 | $a_R$                   | 1.09               | $V_{13}$            | 0.9596            | $\cos\phi_R$ | 0.94           | NI             | 13         | HVDC variables |              |         |
|                    |                   |                 | $a_I$                   | 1.15               | $\theta_{13}$       | 11.351°           | $\cos\phi_I$ | 0.90           | NI             | 13         | HVDC variables |              |         |
|                    |                   |                 | Control strategy 3      | AC terminal buses  |                     | $V_{11}$          |              | 0.9803         |                | $V_{dR}$   | 1.005          | $P_{dR}$     | 0.5025  |
|                    |                   |                 |                         | $I_d$ (pu)         | 0.5                 | $V_{11}$          | 0.9803       | $a_R$          | 1.08           | $a_I$      | 1.15           | $\cos\phi_R$ | 0.94    |
|                    |                   |                 | $V_{dI}$ (pu)           | 1                  | $\theta_{11}$       | 10.737°           | $\cos\phi_I$ | 0.90           | NI             | 6          | HVDC variables |              |         |
|                    |                   |                 | $\alpha_R$ (deg.)       | 5°                 | $V_{13}$            | 0.9597            | $V_{dR}$     | 1.005          | $I_d$          | 0.4975     | $a_R$          | 1.08         |         |
|                    |                   |                 | $\gamma_I$ (deg.)       | 18°                | $\theta_{13}$       | 11.358°           | $a_I$        | 1.13           | $\cos\phi_R$   | 0.94       | $\cos\phi_I$   | 0.90         |         |
|                    |                   |                 | Control strategy 4      | AC terminal buses  |                     | $V_{11}$          |              | 0.9843         |                | $V_{dR}$   | 1.005          | $I_d$        | 0.4975  |
|                    |                   |                 |                         | $P_{dR}$ (pu)      | 0.5                 | $V_{11}$          | 0.9843       | $a_R$          | 1.08           | $\gamma_I$ | 17.868°        | $\cos\phi_R$ | 0.94    |
|                    |                   |                 | $V_{dI}$ (pu)           | 1                  | $\theta_{11}$       | 10.727°           | $\cos\phi_I$ | 0.90           | NI             | 6          | HVDC variables |              |         |
|                    |                   |                 | $\alpha_R$ (deg.)       | 5                  | $V_{13}$            | 0.9767            | $V_{dR}$     | 1.005          | $I_d$          | 0.4975     | $\alpha_R$     | 10.678°      |         |
|                    |                   |                 | $a_I$                   | 1.13               | $\theta_{13}$       | 11.099°           | $a_I$        | 1.13           | $\cos\phi_R$   | 0.93       | $\cos\phi_I$   | 0.90         |         |
|                    |                   |                 | Control strategy 5      | AC terminal buses  |                     | $V_{11}$          |              | 0.9847         |                | $V_{dR}$   | 1.005          | $I_d$        | 0.4975  |
|                    |                   |                 |                         | $P_{dR}$ (pu)      | 0.5                 | $V_{11}$          | 0.9847       | $a_R$          | 1.09           | $\alpha_R$ | 10.707°        | $a_I$        | 1.13    |
|                    |                   |                 | $V_{dI}$ (pu)           | 1                  | $\theta_{11}$       | 10.702°           | $\cos\phi_R$ | 0.93           | $\cos\phi_I$   | 0.90       | NI             | 6            |         |
|                    |                   |                 | $a_R$                   | 1.09               | $V_{13}$            | 0.977             | $V_{dR}$     | 1.005          | $I_d$          | 0.4975     | $\alpha_R$     | 10.678°      |         |
|                    |                   |                 | $\gamma_I$ (deg.)       | 18°                | $\theta_{13}$       | 11.069°           | $a_I$        | 1.13           | $\cos\phi_R$   | 0.93       | $\cos\phi_I$   | 0.90         |         |
|                    |                   |                 | Control strategy 6      | AC terminal buses  |                     | $V_{11}$          |              | 0.9809         |                | $V_{dR}$   | 0.9076         | $I_d$        | 0.5509  |
|                    |                   |                 |                         | $P_{dR}$ (pu)      | 0.5                 | $V_{11}$          | 0.9809       | $a_R$          | 1              | $V_{dI}$   | 0.902          | $\alpha_R$   | 10.707° |
|                    |                   |                 | $a_R$                   | 1                  | $\theta_{11}$       | 10.755°           | $\cos\phi_R$ | 0.92           | $\cos\phi_I$   | 0.89       | NI             | 9            |         |
| $a_I$              | 1.05              | $V_{13}$        | 0.9624                  | $\cos\phi_R$       | 0.92                | $\cos\phi_I$      | 0.89         | HVDC variables |                |            |                |              |         |
| $\gamma_I$ (deg.)  | 18°               | $\theta_{13}$   | 11.335°                 | $V_{dI}$           | 0.9817              | $\gamma_I$        | 15.4291°     | $a_R$          | 1.09           |            |                |              |         |
| Control strategy 7 | AC terminal buses |                 | $V_{11}$                |                    | 0.9817              |                   | $V_{dI}$     | 0.9965         | $P_{dR}$       | 0.35       |                |              |         |
|                    | $V_{dR}$ (pu)     | 1               | $V_{11}$                | 0.9817             | $\theta_{11}$       | 10.8211°          | $a_R$        | 1.09           | $\cos\phi_R$   | 0.9277     |                |              |         |
| $I_d$ (p.u.)       | 0.35              | $\theta_{11}$   | 10.8211°                | $\cos\phi_I$       | 0.9313              | $\cos\phi_I$      | 0.9313       | NI             | 11             |            |                |              |         |
| $a_I$              | 1.11              | $V_{13}$        | 0.964                   | $\cos\phi_R$       | 0.9277              | $\cos\phi_I$      | 0.9313       | NI             | 11             |            |                |              |         |
| $\alpha_R$ (deg.)  | 6°                | $\theta_{13}$   | 10.7882°                | NI                 | 11                  |                   |              |                |                |            |                |              |         |

side transformer. The tap ratio on the rectifier side transformer ' $a_R$ ' along with the inverter side extinction angle can be calculated subsequent to the AC load flow.

### 3.5. Control strategy 5

In this control strategy, the extinction angle of the inverter is specified along with the tap ratio of the rectifier side. On the other

hand, the firing angle of the rectifier and the inverter side transformer tap ratio is computed subsequent to the AC load flow.

3.6. Control strategy 6

In this control strategy, the firing angle and the dc voltage of the rectifier side is computed given the tap ratios of the converter transformers along with the extinction angle of the inverter. The equivalent reactive power injections on both the rectifier and inverter sides are updated in each iteration, rendering this control strategy slightly harder to implement.

3.7. Control strategy 7

In this control strategy, the extinction angle of the inverter and DC voltage of the Inverter are computed while the transformer

tap ratio at the inverter side and the DC voltage of the rectifier are specified. The equivalent reactive power injection of the inverter is updated in each iteration, rendering this control strategy slightly harder to implement.

4. AC-DC power-flow equations

As discussed in Section 2, the effect of the DC link is included in the power flow equations by injecting equivalent amount of real and reactive powers at the terminal AC buses connected to the converters. This results in appropriate modifications of the mismatch equations at the converter terminal AC buses, as given below.

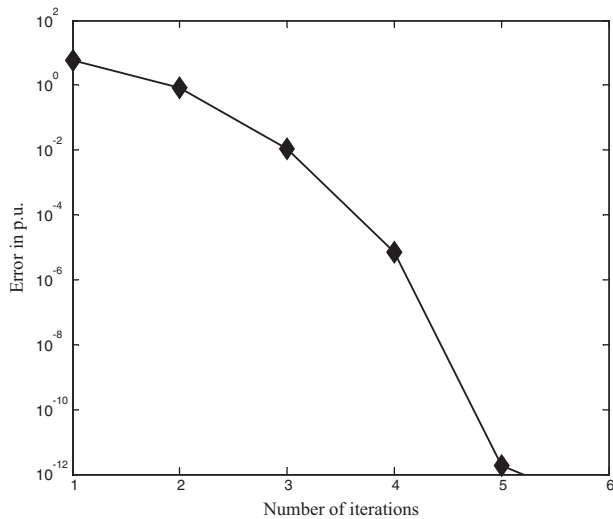


Figure 3 Convergence characteristics of base case power flow in IEEE-118 bus test system.

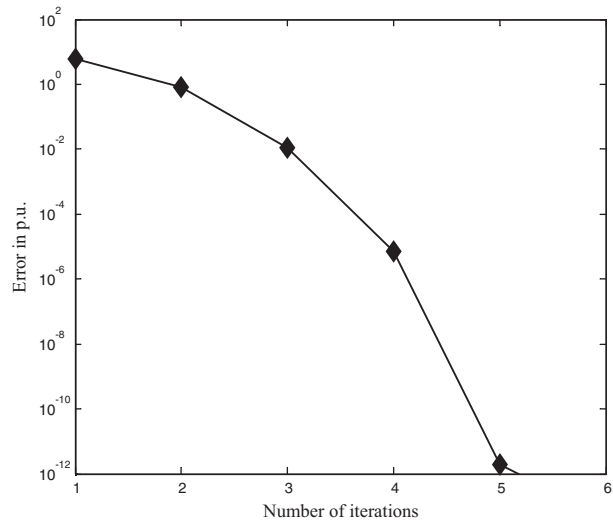


Figure 4 Convergence characteristics for the case study of Table 3 with control strategy 1.

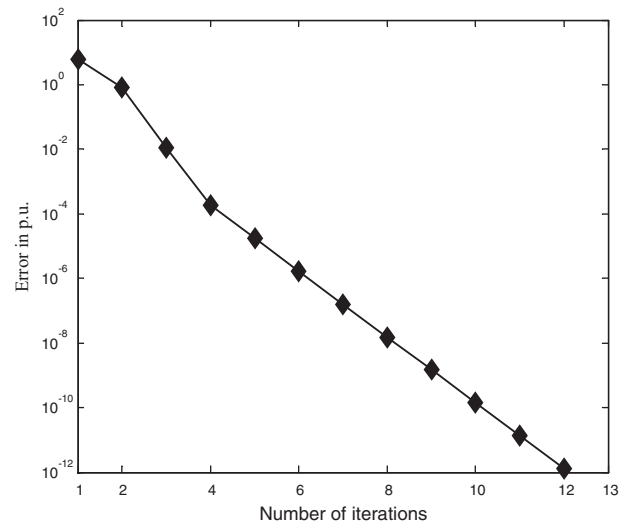


Figure 5 Convergence characteristics for the case study of Table 3 with control strategy 2.

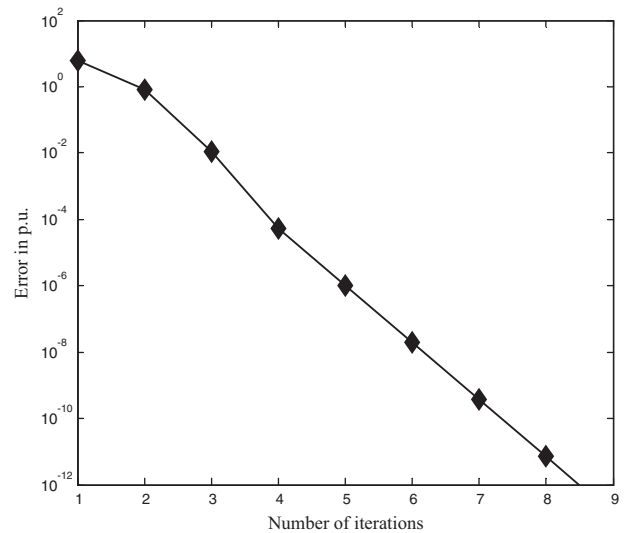


Figure 6 Convergence characteristics for the case study of Table 3 with control strategy 6.

For any AC bus ‘*i*’, which is not connected to any DC link, the mismatches in the active and reactive power injections are given respectively, by

$$\Delta P_i = P_i^{sp} - \sum_{k=1}^n V_i V_k Y_{ik} \cos(\theta_i - \theta_k - \phi_{ik}) \quad (8)$$

$$\Delta Q_i = Q_i^{sp} - \sum_{k=1}^n V_i V_k Y_{ik} \sin(\theta_i - \theta_k - \phi_{ik}) \quad (9)$$

If however, a DC link exists between arbitrary AC buses ‘*i*’ and ‘*j*’ with buses ‘*i*’ and ‘*j*’ connected to the rectifier and inverter respectively, the effect of the DC link can be incorporated in the AC power flow as equivalent active and reactive power injections ‘ $P_{dR}$ ’ and ‘ $Q_{dR}$ ’ at the rectifier bus ‘*i*’ and ‘ $P_{dI}$ ’ and ‘ $Q_{dI}$ ’ at the inverter bus ‘*j*’, respectively. Therefore, the mismatches in the active and reactive power injections can be written as

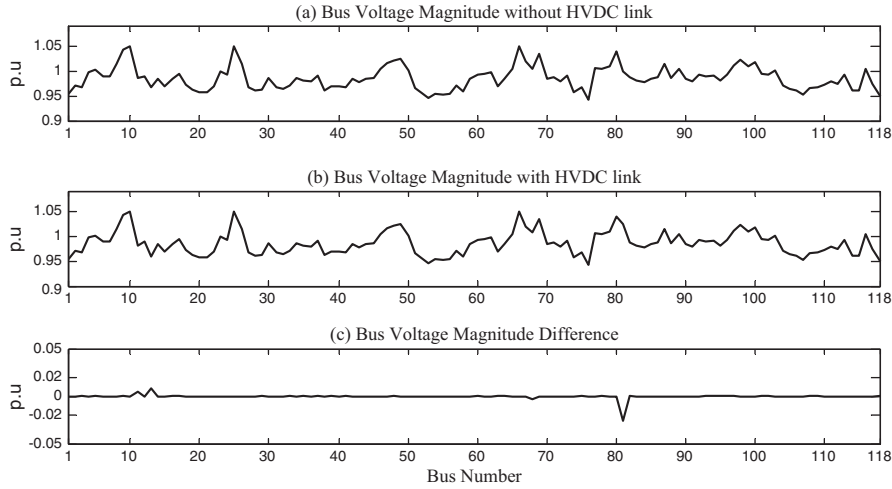


Figure 7 Bus voltage profile for the case study of Table 3 with control strategy 2.

Table 4 Variation of tap setting and reactive power consumption of rectifier with firing angle.

| HVDC links    |              | HVDC link specification   | Power flow solutions      |  |  |
|---------------|--------------|---|---------------------------|--|--|
| Rectifier bus | Inverter bus | Control strategy-5  | AC terminal buses         | HVDC variables   |  |
| 11            | 13           | $\gamma_I = 18^\circ$ ;<br>$V_{dI} = 1$<br>(p.u.);<br>$I_d = 0.5$<br>(p.u.) | $\alpha_R$<br>(deg.)<br>5 | $V_{11} = 0.9803 \angle 10.737$ ;<br>$V_{13} = 0.9597 \angle 13.358$   | $V_{dR} = 1.005$ ; $P_{dR} = 0.5025$ ; $a_R = 1.08$ ; $a_I = 1.15$ ; $Q_{dR} = 0.167$ ; $Q_{dI} = 0.2339$ ;<br>$\cos\phi_I = 0.9$ ; $\cos\phi_R = 0.94$ ; NI = 6;          |
|               |              |   | 6                         | $V_{11} = 0.9798 \angle 11.1165$ ;<br>$V_{13} = 0.9593 \angle 11.7535$ | $V_{dR} = 1.005$ ; $P_{dR} = 0.5025$ ; $a_R = 1.08$ ; $a_I = 1.15$ ; $Q_{dR} = 0.1698$ ; $Q_{dI} = 0.2339$ ;<br>$\cos\phi_I = 0.9058$ ; $\cos\phi_R = 0.9474$ ; NI = 6;    |
|               |              |   | 7                         | $V_{11} = 0.9797 \angle 11.117$ ;<br>$V_{13} = 0.9593 \angle 11.7539$  | $V_{dR} = 1.005$ ; $P_{dR} = 0.5025$ ; $a_R = 1.08$ ; $a_I = 1.15$ ; $Q_{dR} = 0.173$ ; $Q_{dI} = 0.2339$ ;<br>$\cos\phi_I = 0.9058$ ; $\cos\phi_R = 0.9455$ ; NI = 6;     |
|               |              |   | 8                         | $V_{11} = 0.9797 \angle 11.1175$ ;<br>$V_{13} = 0.9592 \angle 11.7543$ | $V_{dR} = 1.005$ ; $P_{dR} = 0.5025$ ; $a_R = 1.09$ ; $a_I = 1.15$ ; $Q_{dR} = 0.1768$ ; $Q_{dI} = 0.2339$ ;<br>$\cos\phi_I = 0.9058$ ; $\cos\phi_R = 0.9433$ ; NI = 6;    |
|               |              |   | 9                         | $V_{11} = 0.9796 \angle 11.1181$ ;<br>$V_{13} = 0.9592 \angle 11.7548$ | $V_{dR} = 1.005$ ;<br>$P_{dR} = 0.5025$ ; $a_R = 1.09$ ; $a_I = 1.15$ ; $Q_{dR} = 0.1809$ ; $Q_{dI} = 0.2339$ ; $\cos\phi_I = 0.9058$ ;<br>$\cos\phi_R = 0.9409$ ; NI = 6; |
|               |              |   | 10                        | $V_{11} = 0.9796 \angle 11.1188$ ;<br>$V_{13} = 0.9591 \angle 11.7553$ | $V_{dR} = 1.005$ ; $P_{dR} = 0.5025$ ; $a_R = 1.09$ ; $a_I = 1.15$ ; $Q_{dR} = 0.1855$ ; $Q_{dI} = 0.2339$ ;<br>$\cos\phi_I = 0.9058$ ; $\cos\phi_R = 0.9381$ ; NI = 6;    |
|               |              |   | 11                        | $V_{11} = 0.9795 \angle 11.1195$ ;<br>$V_{13} = 0.9591 \angle 11.7559$ | $V_{dR} = 1.005$ ; $P_{dR} = 0.5025$ ; $a_R = 1.1$ ; $a_I = 1.15$ ; $Q_{dR} = 0.1904$ ; $Q_{dI} = 0.2339$ ;<br>$\cos\phi_I = 0.9058$ ; $\cos\phi_R = 0.9351$ ; NI = 6;     |
|               |              |   | 12                        | $V_{11} = 0.9794 \angle 11.1202$ ;<br>$V_{13} = 0.959 \angle 11.765$   | $V_{dR} = 1.005$ ; $P_{dR} = 0.5025$ ; $a_R = 1.01$ ; $a_I = 1.15$ ; $Q_{dR} = 0.1958$ ; $Q_{dI} = 0.2339$ ;<br>$\cos\phi_I = 0.9058$ ; $\cos\phi_R = 0.9318$ ; NI = 6;    |
|               |              |   | 13                        | $V_{11} = 0.9794 \angle 11.121$ ;<br>$V_{13} = 0.959 \angle 11.7571$   | $V_{dR} = 1.005$ ; $P_{dR} = 0.5025$ ; $a_R = 1.1$ ; $a_I = 1.15$ ; $Q_{dR} = 0.2014$ ; $Q_{dI} = 0.2339$ ;<br>$\cos\phi_I = 0.9058$ ; $\cos\phi_R = 0.9282$ ; NI = 6;     |
|               |              |   | 14                        | $V_{11} = 0.9793 \angle 11.1218$ ;<br>$V_{13} = 0.9591 \angle 11.755$  | $V_{dR} = 1.005$ ; $P_{dR} = 0.5025$ ; $a_R = 1.11$ ; $a_I = 1.15$ ; $Q_{dR} = 0.2075$ ; $Q_{dI} = 0.2339$ ;<br>$\cos\phi_I = 0.9058$ ; $\cos\phi_R = 0.9243$ ; NI = 6;    |
|               |              |   | 15                        | $V_{11} = 0.9792 \angle 11.1227$ ;<br>$V_{13} = 0.9589 \angle 11.7585$ | $V_{dR} = 1.005$ ; $P_{dR} = 0.5025$ ; $a_R = 1.12$ ; $a_I = 1.15$ ; $Q_{dR} = 0.2138$ ; $Q_{dI} = 0.2339$ ;<br>$\cos\phi_I = 0.9058$ ; $\cos\phi_R = 0.9201$ ; NI = 6;    |

$$\Delta P_i = P_i^{sp} - \sum_{k=1}^n V_i V_k Y_{ik} \cos(\theta_i - \theta_k - \phi_{ik}) - P_{dR} \quad (10)$$

$$\Delta Q_i = Q_i^{sp} - \sum_{k=1}^n V_i V_k Y_{ik} \sin(\theta_i - \theta_k - \phi_{ik}) - Q_{dR} \quad (11)$$

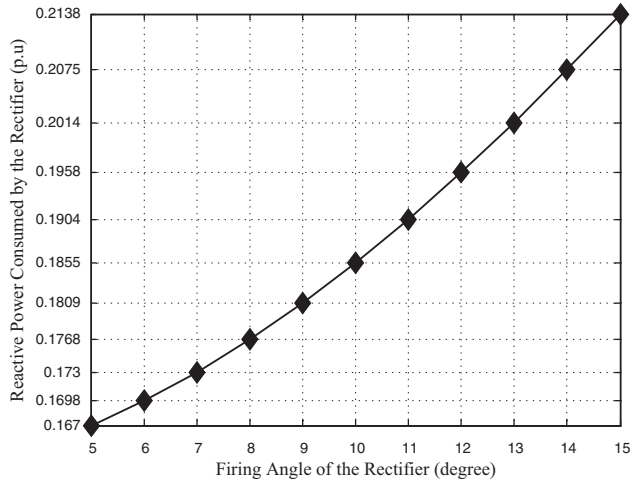
$$\Delta P_j = P_j^{sp} - \sum_{k=1}^n V_j V_k Y_{jk} \cos(\theta_j - \theta_k - \phi_{jk}) + P_{dI} \quad (12)$$

$$\Delta Q_j = Q_j^{sp} - \sum_{k=1}^n V_j V_k Y_{jk} \sin(\theta_j - \theta_k - \phi_{jk}) - Q_{dI} \quad (13)$$

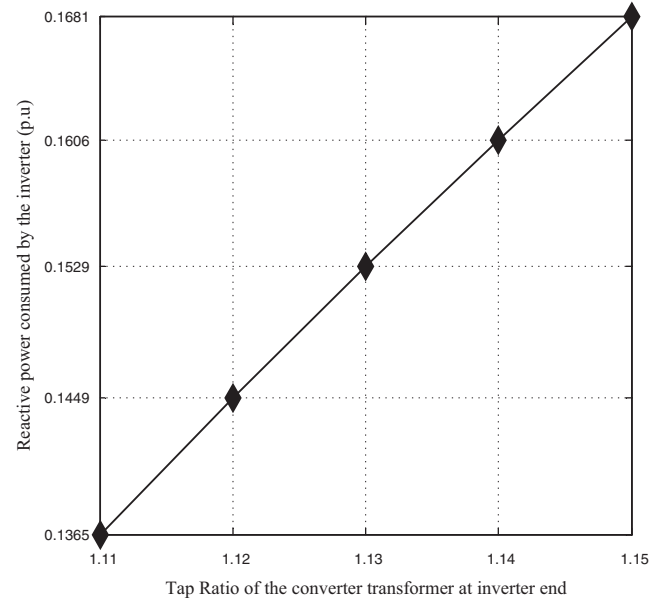
where  $P_{dR} = V_{dR} I_d$ ,  $Q_{dR} = P_{dR} \tan \phi_R$ ,  $P_{dI} = V_{dI} I_d$ , and  $Q_{dI} = P_{dI} \tan \phi_I$ .

In the above equations, the equivalent active power injections 'P<sub>dR</sub>' and 'P<sub>dI</sub>' are usually specified or can be very easily computed by manipulation of the specified variables. However, for the equivalent reactive power injections Q<sub>dR</sub> and Q<sub>dI</sub>, the case is different, depending on the control strategy

adopted for the DC link. For control strategies 1, 3, 4 and 5,  $\phi_R$  and  $\phi_I$  (and hence Q<sub>dR</sub> and Q<sub>dI</sub>) can be computed by manipulation of the specified variables. However, for control strategies 2 and 6,  $\phi_R$  and  $\phi_I$  (and hence Q<sub>dR</sub> and Q<sub>dI</sub>) are dependent on both the specified variables as well as the AC state variables, which are updated every iteration. This affects the convergence pattern. The steps involved in the computation of the active and reactive power injections pertaining to control strategies 1 and 2 only are detailed in Table B.1 of Appendix B. From Table B.1, it is observed that unlike control strategy-1, the reactive power injection in control strategy-2 is updated every iteration, adversely affecting the convergence. Although the steps involved in the computation of the power



**Figure 8** Variation of reactive power consumed by the rectifier with its firing angle.

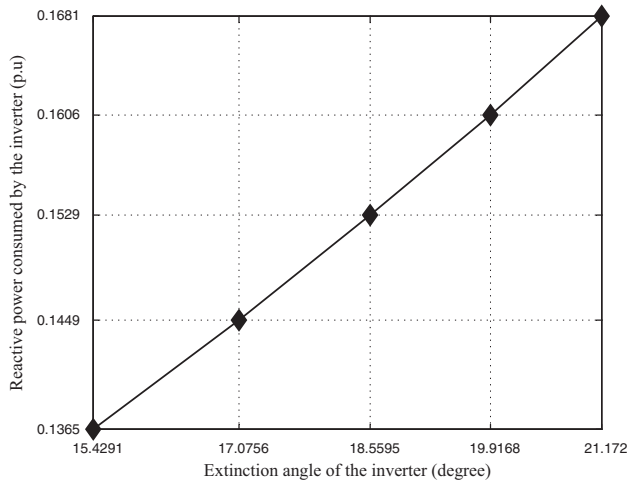


**Figure 9** Variation of the reactive power consumed by the inverter with its tap setting.

**Table 5** Variation of extinction angle and reactive power consumption of inverter with its transformer tap setting.

| HVDC links    |                | HVDC link specification   |       | Power flow solutions   |  |
|---------------|----------------|---|-------|--|--|
| Rectifier bus | Inverter buses | Control strategy-5  |       | AC terminal buses  | HVDC variables   |
| 11            | 13             | $\alpha_R = 6^\circ$ ; $V_{dR} = 1$ (p.u.); $I_d = 0.35$ (p.u.) | $a_I$ |  |  |
|               |                |   | 1.11  | $V_{11} = 0.9817 \angle 10.8211$ ;<br>$V_{13} = 0.964 \angle 10.7882$  | $V_{dI} = 0.9965$ ; $P_{dR} = 0.35$ ; $a_R = 1.1$ ; $Q_{dR} = 0.1408$ ; $Q_{dI} = 0.1365$ ;<br>$\gamma_I = 15.4291^\circ$ ; $\cos \phi_I = 0.9313$ ; $\cos \phi_R = 0.9277$ ; NI = 11;       |
|               |                |   | 1.12  | $V_{11} = 0.9816 \angle 10.8215$ ;<br>$V_{13} = 0.9635 \angle 10.7972$ | $V_{dI} = 0.9965$ ; $P_{dR} = 0.35$ ; $a_R = 1.1$ ; $Q_{dR} = 0.1408$ ; $Q_{dI} = 0.1783$ ;<br>$\gamma_I = 17.0756^\circ$ ; $\cos \phi_I = 0.9235$ ; $\cos \phi_R = 0.9277$ ; NI = 11;       |
|               |                |   | 1.13  | $V_{11} = 0.9816 \angle 10.8218$ ;<br>$V_{13} = 0.9629 \angle 10.8058$ | $V_{dI} = 0.9965$ ; $P_{dR} = 0.35$ ; $a_R = 1.0982$ ; $Q_{dR} = 0.1408$ ;<br>$Q_{dI} = 0.1529$ ; $\gamma_I = 18.5595^\circ$ ; $\cos \phi_I = 0.9158$ ; $\cos \phi_R = 0.9277$ ;<br>NI = 11; |
|               |                |   | 1.14  | $V_{11} = 0.9815 \angle 10.822$ ;<br>$V_{13} = 0.9624 \angle 10.8141$  | $V_{dI} = 0.9965$ ; $P_{dR} = 0.35$ ; $a_R = 1.0983$ ; $Q_{dR} = 0.1408$ ;<br>$Q_{dI} = 0.1606$ ; $\gamma_I = 19.9168^\circ$ ; $\cos \phi_I = 0.9083$ ; $\cos \phi_R = 0.9277$ ;<br>NI = 11; |
|               |                |   | 1.15  | $V_{11} = 0.9814 \angle 10.8223$ ;<br>$V_{13} = 0.9619 \angle 10.822$  | $V_{dI} = 0.9965$ ; $P_{dR} = 0.35$ ; $a_R = 1.1$ ; $Q_{dR} = 0.1408$ ; $Q_{dI} = 0.1681$ ;<br>$\gamma_I = 21.172^\circ$ ; $\cos \phi_I = 0.9009$ ; $\cos \phi_R = 0.9277$ ; NI = 11;        |





**Figure 10** Variation of the reactive power consumed by the inverter with its extinction angle.

injections pertaining to control strategies 3, 4, 5 and 6 are not shown due to limitation of space, they can be done in a similar manner.

It is important to note the conventions of the signs of the equivalent real and reactive power injections representing the DC link. It is assumed that the rectifier consumes both real and reactive power from the AC grid while the inverter supplies real power and consumes reactive power from it.

## 5. Case studies and results

To analyze how AC–DC power flow convergence is affected by the control strategy adopted for the HVDC link, several case studies were carried out with multiple HVDC links incorporated in the IEEE 118-bus and IEEE 300-bus test systems [16]. All the simulations were carried out in MATLAB. Although several control strategies are feasible, due to a shortage of space, only case studies pertaining to seven typical control strategies are reported in this paper. Two comprehensive case studies on the IEEE 118-bus test system and three on the IEEE 300-bus test systems are reported. Each comprehensive case study comprises seven separate power flow studies, each pertaining to the application of a particular control strategy to a DC link. For all the case studies, the commutating reactance and the DC link resistance were chosen as 0.1 p.u. and 0.01 p.u. respectively. The number of bridges ‘ $n_b$ ’ for all the converters [11] was taken to be equal to 2. To minimize the reactive power requirement at the rectifier and inverter terminals and the overall system losses, the values of the rectifier firing angles and the inverter extinction angles are kept within 5–7° and 15–22° respectively, for all the case studies. In a similar manner, the tap ratios of the converter transformers are set to keep the above control angles within the above limits. A convergence tolerance of  $10^{-12}$  p.u. was uniformly adopted for all the case studies. In each of the case studies, ‘NI’ refers to the number of iterations taken by the algorithm to converge to the specified tolerance ( $10^{-12}$  p.u.). In general, ‘NI’ is representative of the degree of convergence of a power flow study.

### 5.1. Case I: First study of IEEE-118 bus system

In this study, a single HVDC link is incorporated in the transmission line between buses 11 and 13. The base case (without any HVDC link) active power flow in this line is found to be 40.81 MW. Subsequently, applying Control Strategy 1 to the HVDC link, its active power-flow is set to a value of 50 MW. The rectifier firing angle and the inverter extinction angle were set to 5° and 18° respectively. On account of the relationship between the inverter side DC voltage, the AC bus voltage magnitude, the converter transformer tap ratio and the power factor at the inverter end {Eq. (4)}, the inverter side DC voltage is set to a value of 1.0 p.u. These are detailed in columns 1–5 of Table 3. The power-flow solution corresponding to these specifications are also shown in columns 6–9 of Table 3. The state variables pertaining to the AC and DC systems are denoted as ACSV and DCSV respectively. It requires six iterations to converge. In a similar manner, the HVDC link specifications corresponding to the six other control strategies and their power-flow solution are shown in Table 3.

From Table 3, it is observed that almost similar convergence pattern is exhibited for all the control strategies except Control Strategy 2, 6 and 7 where the number of iterations taken to converge is more. This is reiterated from the convergence characteristics shown in Figs. 3–6, corresponding to the base case (without any HVDC link) and three typical control strategies 1, 2 and 6, respectively. In Figs. 3–6, ‘error’ refers to the maximum absolute power mismatch (in p.u.). From Figs. 3–6, it is observed that the power flow convergence with control strategy 1 is almost similar to that of the base case as the power injections (at the terminal buses connected to the rectifier and the inverter) can be computed a priori and remain constant. It is also observed from Figs. 3–6 that the convergence characteristics with control strategies 2 and 6 are adversely affected as compared to that with control strategy 1. This is due to the fact that with control strategies 2 and 6, the equivalent reactive power injections are updated in every iteration, as already explained in Appendix B. The convergence patterns for control strategies 3–5 and control strategy 7 although similar to control strategy 1 and control strategy 2 or 6 respectively, are not shown due to limitations of space.

The bus voltage profile for the power-flow solution pertaining to the case with control strategy 2 is shown in Fig. 7. It is observed that bus voltage profile hardly changes except the AC terminal buses connected to the rectifier and the inverter. However, the bus voltage profiles of the other case studies of Table 3 are not shown due to limitations of space.

It may be noted from Table 3 that for control strategy 3, the firing angle of the rectifier and the extinction angle of the inverter are maintained at constant values of 5° and 18°, respectively. This is done in order to minimize the reactive power consumptions of the converters. A separate case study was carried out to elaborate this. The rectifier firing angle was gradually increased from 5° to 15° while maintaining the DC current and voltage at the inverter end at 0.5 p.u. and 1 p.u., respectively. The specified quantities and the power flow solutions are shown in columns 3–4 and 5–6 of Table 4 respectively. The variation of the reactive power consumed by the rectifier with the firing angle is shown in Fig. 8. It is observed that the reactive power consumed by the rectifier ‘ $Q_{dR}$ ’ increases

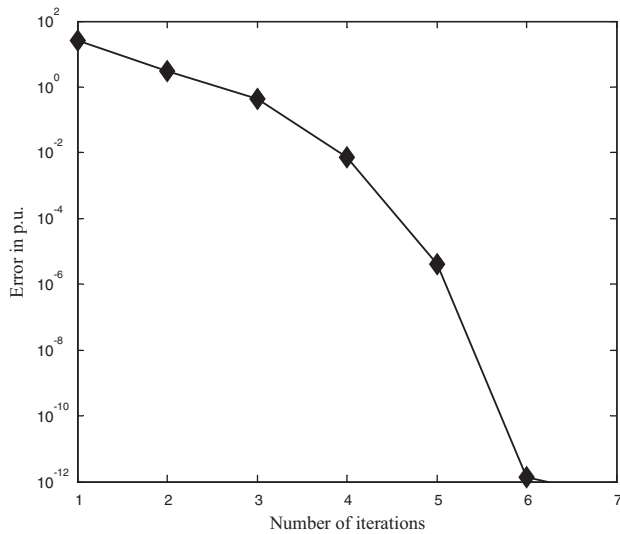


**Table 6** Second study of IEEE 118-bus system.

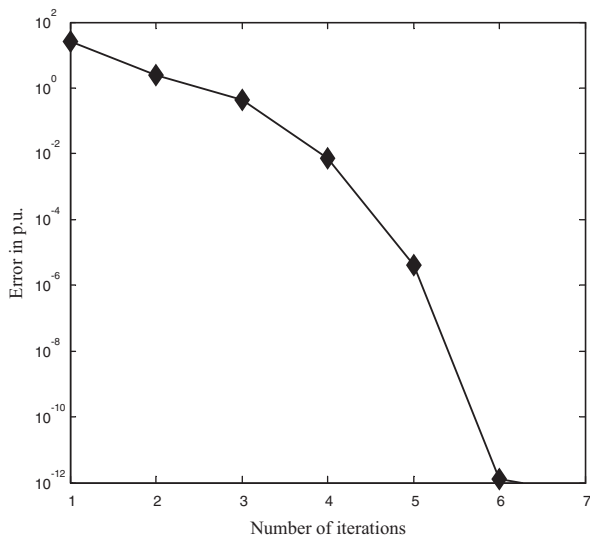
| HVDC link          |           | $P_{base}$ (pu)    | HVDC link specification |                   | Power flow solution |                |                   |         |                |         |
|--------------------|-----------|--------------------|-------------------------|-------------------|---------------------|----------------|-------------------|---------|----------------|---------|
| From bus no        | To bus no |                    | Spec. values            |                   | ACSV                |                | DCSV              |         |                |         |
|                    |           | Control strategy 1 |                         | AC terminal buses |                     | HVDC variables |                   |         |                |         |
| 6                  | 7         | 0.3386             | $P_{dR}$ (pu)           | 0.5               | $V_6$               | 0.99           | $V_{dR}$          | 1.005   |                |         |
|                    |           |                    | $V_{dI}$ (pu)           | 1.0               | $\theta_6$          | 11.037°        | $I_d$             | 0.4975  |                |         |
|                    |           |                    | $\alpha_R$ (deg.)       | 5°                | $V_7$               | 0.9878         | $a_R$             | 1.07    |                |         |
|                    |           |                    | $\gamma_I$ (deg.)       | 18°               | $\theta_7$          | 11.150°        | $a_I$             | 1.12    |                |         |
|                    |           |                    | Control strategy 2      |                   | AC terminal buses   |                | AC terminal buses |         | HVDC variables |         |
|                    |           |                    | $P_{dR}$ (pu)           | 0.5               | $V_6$               | 1.0            | $V_{dR}$          | 1.005   | $I_d$          | 0.4975  |
|                    |           |                    | $V_{dI}$ (pu)           | 1                 | $\theta_6$          | 10.926°        | $\alpha_R$        | 12.419° | $\gamma_I$     | 17.929° |
|                    |           |                    | $a_R$                   | 1.08              | $V_7$               | 0.994          | $\cos\phi_R$      | 0.93    | $\cos\phi_I$   | 0.90    |
|                    |           |                    | $a_I$                   | 1.1               | $\theta_7$          | 11.079°        | NI                | 8       | HVDC variables |         |
|                    |           |                    | Control strategy 3      |                   | AC terminal buses   |                | AC terminal buses |         | HVDC variables |         |
|                    |           |                    | $I_d$ (pu)              | 0.5               | $V_6$               | 0.99           | $V_{dR}$          | 1.005   | $P_{dR}$       | 0.5025  |
|                    |           |                    | $V_{dI}$ (pu)           | 1                 | $\theta_6$          | 11.035°        | $a_R$             | 1.07    | $a_I$          | 1.11    |
|                    |           |                    | $\alpha_R$ (deg.)       | 5°                | $V_7$               | 0.9878         | $\cos\phi_R$      | 0.94    | $\cos\phi_I$   | 0.90    |
|                    |           |                    | $\gamma_I$ (deg.)       | 18°               | $\theta_7$          | 11.151°        | NI                | 6       | HVDC variables |         |
|                    |           |                    | Control strategy 4      |                   | AC terminal buses   |                | AC terminal buses |         | HVDC variables |         |
|                    |           |                    | $P_{dR}$ (pu)           | 0.5               | $V_6$               | 1.00           | $V_{dR}$          | 1.005   | $I_d$          | 0.4975  |
|                    |           |                    | $V_{dI}$ (pu)           | 1                 | $\theta_6$          | 10.92°         | $a_R$             | 1.06    | $\gamma_I$     | 12.862° |
|                    |           |                    | $\alpha_R$ (deg.)       | 5                 | $V_7$               | 0.997          | $\cos\phi_R$      | 0.94    | $\cos\phi_I$   | 0.92    |
|                    |           |                    | $a_I$                   | 1.08              | $\theta_7$          | 11.038°        | NI                | 6       | HVDC variables |         |
|                    |           |                    | Control strategy 5      |                   | AC terminal buses   |                | AC terminal buses |         | HVDC variables |         |
|                    |           |                    | $P_{dR}$ (pu)           | 0.5               | $V_6$               | 0.99           | $V_{dR}$          | 1.005   | $I_d$          | 0.4975  |
|                    |           |                    | $V_{dI}$ (pu)           | 1                 | $\theta_6$          | 11.036°        | $\alpha_R$        | 12.200° | $a_I$          | 1.11    |
|                    |           |                    | $a_R$                   | 1.09              | $V_7$               | 0.9908         | $\cos\phi_R$      | 0.93    | $\cos\phi_I$   | 0.90    |
|                    |           |                    | $\gamma_I$ (deg.)       | 18°               | $\theta_7$          | 11.109°        | NI                | 6       | HVDC variables |         |
|                    |           |                    | Control strategy 6      |                   | AC terminal buses   |                | AC terminal buses |         | HVDC variables |         |
|                    |           |                    | $P_{dR}$ (pu)           | 0.5               | $V_6$               | 1.00           | $V_{dR}$          | 0.9946  | $I_d$          | 0.5027  |
|                    |           |                    | $a_R$                   | 1.05              | $\theta_6$          | 10.92°         | $V_{dI}$          | 0.9896  | $\alpha_R$     | 5.6583° |
|                    |           |                    | $a_I$                   | 1.1               | $V_7$               | 0.994          | $\cos\phi_R$      | 0.94    | $\cos\phi_I$   | 0.90    |
| $\gamma_I$ (deg.)  | 18°       | $\theta_7$         | 11.079°                 | NI                | 6                   | HVDC variables |                   |         |                |         |
| Control strategy 7 |           | AC terminal buses  |                         | AC terminal buses |                     | HVDC variables |                   |         |                |         |
| $V_{dR}$ (p.u.)    | 1         | $V_6$              | 1                       | $V_{dI}$          | 0.995               | $\gamma_I$     | 17.1234°          |         |                |         |
| $I_d$ (p.u.)       | 0.5       | $\theta_6$         | 10.8952°                | $a_R$             | 1.09                | $P_{dR}$       | 0.5               |         |                |         |
| $a_I$              | 1.1       | $V_7$              | 0.9941                  | $\cos\phi_R$      | 0.9144              | $\cos\phi_I$   | 0.9099            |         |                |         |
| $\alpha_R$ (deg.)  | 6°        | $\theta_7$         | 11.0473°                | NI                | 8                   | HVDC variables |                   |         |                |         |

**Table 7** First study of IEEE 300-bus system.

| HVDC link         |            | $P_{base}$ (pu) | HVDC link specification |                    | Power flow solution |                    |              |                |                    |              |
|-------------------|------------|-----------------|-------------------------|--------------------|---------------------|--------------------|--------------|----------------|--------------------|--------------|
| From bus no       | To bus no  |                 | Spec. values            |                    | ACSV                |                    | DCSV         |                |                    |              |
|                   |            |                 |                         | Control strategy 1 |                     | AC terminal buses  |              | HVDC variables |                    |              |
| 270               | 292        | 0.3652          | $P_{dR}$ (pu)           | 0.4                | $V_{270}$           | 1.008              | $V_{dR}$     | 1.004          | $I_d$              | 0.3984       |
|                   |            |                 |                         |                    |                     |                    |              |                |                    |              |
|                   |            |                 | $\alpha_R$ (deg.)       | $5^\circ$          | $V_{292}$           | 1.00               | $a_I$        | 1.09           |                    |              |
|                   |            |                 |                         |                    |                     |                    |              |                | $\gamma_I$ (deg.)  | $18^\circ$   |
|                   |            |                 | Control strategy 2      | $\cos\phi_I$       | 0.91                | NI                 | 7            | HVDC variables |                    |              |
|                   |            |                 |                         |                    |                     |                    |              |                | $P_{dR}$ (pu)      | 0.4          |
|                   |            |                 | $V_{dI}$ (pu)           | 1                  | $\theta_{270}$      | $-11.41^\circ$     | $I_d$        | 0.3984         |                    |              |
|                   |            |                 |                         |                    |                     |                    |              |                | $a_R$              | 1.06         |
|                   |            |                 | $a_I$                   | 1.08               | $\theta_{292}$      | $-10.68^\circ$     | $\gamma_I$   | 15.673°        |                    |              |
|                   |            |                 |                         |                    |                     |                    |              |                | Control strategy 3 | $\cos\phi_R$ |
|                   |            |                 | $I_d$ (pu)              | 0.4                | $V_{270}$           | 1.008              | $V_{dR}$     | 1.004          |                    |              |
|                   |            |                 |                         |                    |                     |                    |              |                | $V_{dI}$ (pu)      | 1            |
|                   |            |                 | $\alpha_R$ (deg.)       | $5^\circ$          | $V_{292}$           | 1.00               | $a_R$        | 1.09           |                    |              |
|                   |            |                 |                         |                    |                     |                    |              |                | $\gamma_I$ (deg.)  | $18^\circ$   |
|                   |            |                 | Control strategy 4      | $\cos\phi_I$       | 0.91                | NI                 | 7            | HVDC variables |                    |              |
|                   |            |                 |                         |                    |                     |                    |              |                | $P_{dR}$ (pu)      | 0.4          |
|                   |            |                 | $V_{dI}$ (pu)           | 1                  | $\theta_{270}$      | $-11.41^\circ$     | $I_d$        | 0.3984         |                    |              |
|                   |            |                 |                         |                    |                     |                    |              |                | $\alpha_R$ (deg.)  | 5            |
|                   |            |                 | $a_I$                   | 1.1                | $\theta_{292}$      | $-10.69^\circ$     | $\gamma_I$   | 19.036°        |                    |              |
|                   |            |                 |                         |                    |                     |                    |              |                | Control strategy 5 | $\cos\phi_R$ |
|                   |            |                 | $P_{dR}$ (pu)           | 0.4                | $V_{270}$           | 1.0031             | $V_{dR}$     | 1.004          |                    |              |
|                   |            |                 |                         |                    |                     |                    |              |                | $V_{dI}$ (pu)      | 1            |
|                   |            |                 | $a_R$                   | 1.05               | $V_{292}$           | 1.00               | $\alpha_R$   | 7.6894°        |                    |              |
|                   |            |                 |                         |                    |                     |                    |              |                | $\gamma_I$ (deg.)  | $18^\circ$   |
|                   |            |                 | Control strategy 6      | $\cos\phi_I$       | 0.90                | NI                 | 7            | HVDC variables |                    |              |
|                   |            |                 |                         |                    |                     |                    |              |                | $P_{dR}$ (pu)      | 0.4          |
|                   |            |                 | $a_R$                   | 1.08               | $\theta_{270}$      | $-11.41^\circ$     | $I_d$        | 0.3958         |                    |              |
|                   |            |                 |                         |                    |                     |                    |              |                | $a_I$              | 1.1          |
| $\gamma_I$ (deg.) | $18^\circ$ | $\theta_{292}$  | $-10.68^\circ$          | $\alpha_R$         | 13.551°             |                    |              |                |                    |              |
|                   |            |                 |                         |                    |                     | Control strategy 7 | $\cos\phi_R$ | 0.93           | NI                 | 10           |
| $V_{dR}$ (p.u.)   | 1          | $V_{270}$       | 0.99                    | $V_{dI}$           | 0.996               |                    |              |                |                    |              |
|                   |            |                 |                         |                    |                     | $I_d$ (p.u.)       | 0.4          | $\theta_{270}$ | 18.582°            | $\gamma_I$   |
| $a_I$             | 1.09       | $V_{292}$       | 1                       | $a_R$              | 1.08                |                    |              |                |                    |              |
|                   |            |                 |                         |                    |                     | $\alpha_R$ (deg.)  | $6^\circ$    | $\theta_{292}$ | 19.310°            | $P_{dR}$     |
|                   |            |                 |                         | $\cos\phi_R$       | 0.9232              |                    |              |                |                    |              |
|                   |            |                 |                         |                    |                     |                    |              |                |                    | $\cos\phi_I$ |
|                   |            |                 |                         | NI                 | 8                   |                    |              |                |                    |              |



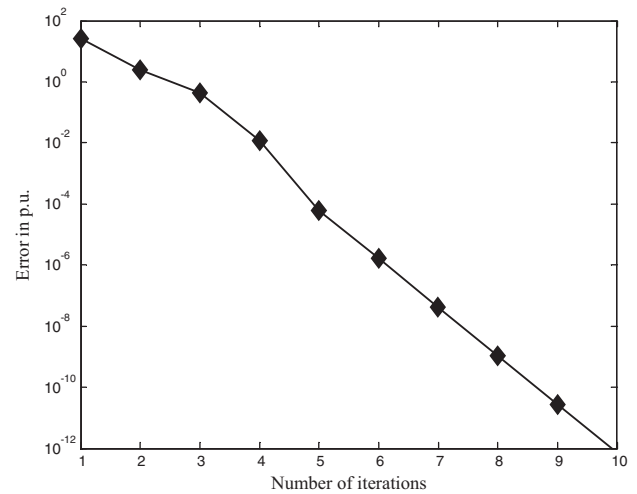
**Figure 11** Convergence characteristics of base case power flow in IEEE-300 bus test system.



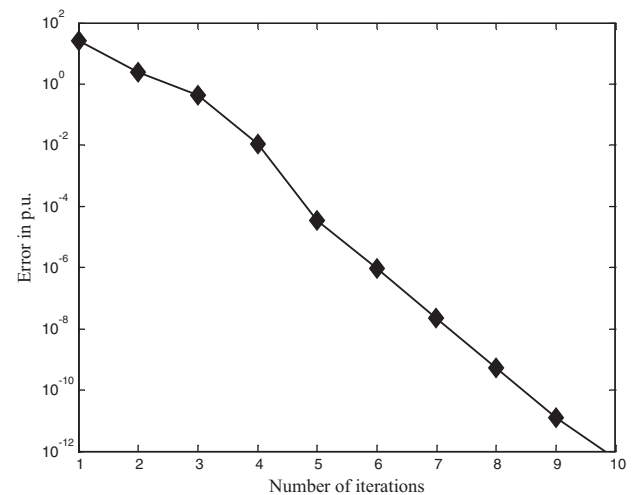
**Figure 12** Convergence characteristics for the case study of Table 5 with control strategy 1.

as the firing angle of the rectifier ' $\alpha_R$ ' increased. The tap setting of the rectifier transformer is kept within its specified limits to minimize the reactive power consumed by the rectifier.

In a similar manner, corresponding to control strategy 7 in Table 3, the tap setting of the inverter transformer is gradually increased from 1.11 to 1.15 while maintaining the DC voltage of the rectifier and the DC current to 1 p.u. and 0.35 p.u., respectively. The firing angle of the rectifier is also maintained to a constant value  $6^\circ$ . The specified quantities and the power flow solutions are shown in columns 3–4 and 5–6 of Table 5 respectively. The variation of the reactive power consumed by the inverter with the transformer tap ratio and extinction angle of the inverter is shown in Figs. 9 and 10 respectively. It is observed that the reactive power consumed by the inverter ' $Q_{di}$ ' increases as the extinction angle of the inverter ' $\gamma_I$ '



**Figure 13** Convergence characteristics for the case study of Table 5 with control strategy 2.

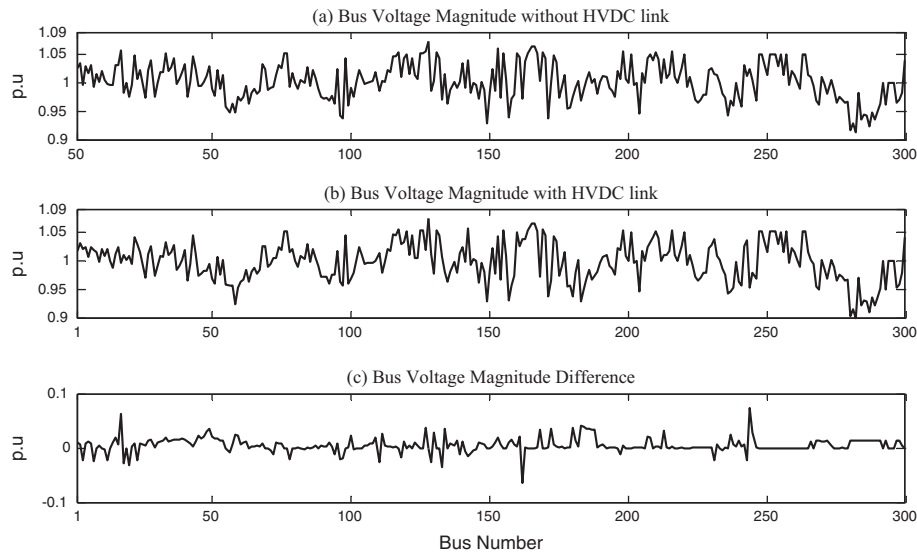


**Figure 14** Convergence characteristics for the case study of Table 5 with control strategy 6.

increased. Therefore, the tap setting of the inverter is adjusted to minimize the extinction angle of the inverter and the reactive power consumed by the inverter.

## 5.2. Case II: Second study with the IEEE 118-bus system

In this study, a single HVDC link was incorporated in the transmission line between buses 6 and 7. The base case active power flow in this line was found to be 33.86 MW. Subsequently, applying Control Strategy 1 to the HVDC link, the active power flow is set to a value of 50 MW. The rectifier firing angle and the inverter extinction angle were again set to  $5^\circ$  and  $18^\circ$  respectively. The inverter side DC voltage is set to a value 1 p.u. These are detailed in columns 1–5 of Table 6. The power-flow solution corresponding to these specifications are also shown in columns 6–9 of Table 6. It requires six iterations to converge. In a similar manner, the HVDC link specifications corresponding to the five other control strategies and their



**Figure 15** Bus voltage profile for the case study of Table 5 with control strategy 2.

power-flow solutions are shown in Table 6. It can be observed that NI is more for control strategy 2, as is expected. It is also observed that in Table 6, the convergence pattern corresponding to control strategies 2 and 6 are slightly better than those in Table 3. This is because the convergence pattern is also dependent on the location of the DC link i.e. the AC system buses between which the link is incorporated. This is reiterated from the subsequent case studies with the IEEE 300 bus system.

### 5.3. Case III: First study of IEEE 300-bus system

In this study, a HVDC link is first incorporated in the transmission line between buses 270 and 292. The base case power flow in this line is 36.52 MW. The power-flow with the HVDC link is set to 40 MW. For all the seven control strategies, the different HVDC link specifications along with the corresponding power-flow solutions are shown in Table 7.

From Tables 6 and 7, it can be observed that with the HVDC link incorporated, the IEEE 300-bus system takes more number of iterations to converge than the IEEE 118-bus system. Also, from Table 7, it is observed that similar convergence patterns is exhibited for all the control strategies except control strategies 2, 6 and 7, where the number of iterations taken to converge is more. This is reinforced from the convergence characteristics shown in Figs. 11–14, corresponding to the base case (without any HVDC link) and three typical control strategies 1, 2 and 6, respectively. From Fig. 11–14, it is observed that the power flow convergence with control strategy 1 is as good as that of the base case while those with control strategies 2 and 6 are adversely affected. This is expected, as the reactive power injections are updated every iteration with control strategies 2, 6 and 7 unlike in control strategy 1. The convergence characteristics for control strategies 3–5 and control strategy 7, although similar to control strategy 1 and control strategy 2 or 6, are not shown due to limitations of space.

The bus voltage profile for the power-flow solution pertaining to the case with control strategy 2 is shown in Fig. 15. It is observed that bus voltage profile hardly changes except the AC terminal buses connected to the rectifier and the inverter.

However, due to lack of space, the bus voltage profiles of the other case studies of Table 7 could not be accommodated.

### 5.4. Case IV: Second study of IEEE 300-bus system

In this study, a HVDC link is first incorporated in the transmission line between buses 1 and 3. The base case power flow in this line is 24.04 MW. The power-flow with the HVDC link is set to 40 MW. For all the seven control strategies, the different HVDC link specifications along with the corresponding power-flow solutions are shown in Table 8.

From Table 8, it can be observed that almost identical convergence characteristics are exhibited for all the control strategies. In addition, for control strategy 2, 6 and 7, the number of iterations taken to converge is more, as expected. Also, in comparison with Table 7, the convergence pattern with control strategies 2, 6 and 7 are slightly better. This is because the convergence pattern also depends on the location of the DC link.

### 5.5. Case V: Third study of IEEE 300-bus system

In this study, a HVDC link is first incorporated in the transmission line between buses 199 and 197. The base case power flow in this line is 32.13 MW. The power-flow with the HVDC link is set to 40 MW. For all the seven control strategies, the different HVDC link specifications along with the corresponding power-flow solutions are shown in Table 9. From Table 9, it can be observed that the model exhibits almost similar convergence characteristics for all the control strategies except control strategies 2, 6 and 7. For control strategy 2, 6 and 7, the NI taken to converge is more, as is expected from the explanations given in Appendix B.

From the case studies, it is also observed that in addition to the control strategy adopted, the location of the DC link i.e. the system buses between which the DC link is incorporated, also affects the power flow convergence pattern, although to a lesser extent.

**Table 8** Second study of IEEE 300-bus system.

| HVDC link          |           | $P_{base}$ (pu)    | HVDC link specification |                   | Power flow solution |                |                   |         |                |         |
|--------------------|-----------|--------------------|-------------------------|-------------------|---------------------|----------------|-------------------|---------|----------------|---------|
| From bus no        | To bus no |                    | Spec. values            |                   | ACSV                |                | DCSV              |         |                |         |
|                    |           | Control strategy 1 |                         | AC terminal buses |                     | HVDC variables |                   |         |                |         |
| 3                  | 1         | 0.2404             | $P_{dR}$ (pu)           | 0.4               | $V_3$               | 1.0187         | $V_{dR}$          | 1.004   |                |         |
|                    |           |                    | $V_{dI}$ (pu)           | 1.0               | $\theta_3$          | 6.6208°        | $I_d$             | 0.3984  |                |         |
|                    |           |                    | $\alpha_R$ (deg.)       | 5°                | $V_1$               | 1.0147         | $a_R$             | 1.03    |                |         |
|                    |           |                    | $\gamma_I$ (deg.)       | 18°               | $\theta_1$          | 6.4066°        | $a_I$             | 1.08    |                |         |
|                    |           |                    | Control strategy 2      |                   | AC terminal buses   |                | AC terminal buses |         | HVDC variables |         |
|                    |           |                    | $P_{dR}$ (pu)           | 0.4               | $V_3$               | 1.0186         | $V_{dR}$          | 1.004   | $I_d$          | 0.3984  |
|                    |           |                    | $V_{dI}$ (pu)           | 1                 | $\theta_3$          | 6.6218°        | $\alpha_R$        | 9.8257° | $\gamma_I$     | 19.893° |
|                    |           |                    | $a_R$                   | 1.04              | $V_1$               | 1.0145         | $\cos\phi_R$      | 0.94    | $\cos\phi_I$   | 0.90    |
|                    |           |                    | $a_I$                   | 1.09              | $\theta_1$          | 6.408°         | NI                | 9       | HVDC variables |         |
|                    |           |                    | Control strategy 3      |                   | AC terminal buses   |                | AC terminal buses |         | HVDC variables |         |
|                    |           |                    | $I_d$ (pu)              | 0.4               | $V_3$               | 1.0187         | $V_{dR}$          | 1.004   | $P_{dR}$       | 0.4016  |
|                    |           |                    | $V_{dI}$ (pu)           | 1                 | $\theta_3$          | 6.6206°        | $a_R$             | 1.03    | $a_I$          | 1.08    |
|                    |           |                    | $\alpha_R$ (deg.)       | 5°                | $V_1$               | 1.0147         | $\cos\phi_R$      | 0.95    | $\cos\phi_I$   | 0.91    |
|                    |           |                    | $\gamma_I$ (deg.)       | 18°               | $\theta_1$          | 6.4083°        | NI                | 7       | HVDC variables |         |
|                    |           |                    | Control strategy 4      |                   | AC terminal buses   |                | AC terminal buses |         | HVDC variables |         |
|                    |           |                    | $P_{dR}$ (pu)           | 0.4               | $V_3$               | 1.017          | $V_{dR}$          | 1.004   | $I_d$          | 0.3984  |
|                    |           |                    | $V_{dI}$ (pu)           | 1                 | $\theta_3$          | 6.641°         | $a_R$             | 1.03    | $\gamma_I$     | 21.472° |
|                    |           |                    | $\alpha_R$ (deg.)       | 5                 | $V_1$               | 1.0158         | $\cos\phi_R$      | 0.95    | $\cos\phi_I$   | 0.89    |
|                    |           |                    | $a_I$                   | 1.1               | $\theta_1$          | 6.4176°        | NI                | 7       | HVDC variables |         |
|                    |           |                    | Control strategy 5      |                   | AC terminal buses   |                | AC terminal buses |         | HVDC variables |         |
|                    |           |                    | $P_{dR}$ (pu)           | 0.4               | $V_3$               | 1.0194         | $V_{dR}$          | 1.004   | $I_d$          | 0.3984  |
|                    |           |                    | $V_{dI}$ (pu)           | 1                 | $\theta_3$          | 6.611°         | $\alpha_R$        | 6.2125° | $a_I$          | 1.08    |
|                    |           |                    | $a_R$                   | 1.03              | $V_1$               | 1.0167         | $\cos\phi_R$      | 0.95    | $\cos\phi_I$   | 0.91    |
|                    |           |                    | $\gamma_I$ (deg.)       | 18°               | $\theta_1$          | 6.3904°        | NI                | 7       | HVDC variables |         |
|                    |           |                    | Control strategy 6      |                   | AC terminal buses   |                | AC terminal buses |         | HVDC variables |         |
|                    |           |                    | $P_{dR}$ (pu)           | 0.4               | $V_3$               | 1.0186         | $V_{dR}$          | 1.0264  | $I_d$          | 0.3897  |
|                    |           |                    | $a_R$                   | 1.08              | $\theta_3$          | 6.6228°        | $V_{dI}$          | 1.0225  | $\alpha_R$     | 14.424° |
|                    |           |                    | $a_I$                   | 1.1               | $V_1$               | 1.0146         | $\cos\phi_R$      | 0.93    | $\cos\phi_I$   | 0.91    |
| $\gamma_I$ (deg.)  | 18°       | $\theta_1$         | 6.4085°                 | NI                | 8                   | HVDC variables |                   |         |                |         |
| Control strategy 7 |           | AC terminal buses  |                         | AC terminal buses |                     | HVDC variables |                   |         |                |         |
| $V_{dR}$ (p.u.)    | 1         | $V_3$              | 1.0185                  | $V_{dI}$          | 0.996               | $\gamma_I$     | 17.3967°          |         |                |         |
| $I_d$ (p.u.)       | 0.4       | $\theta_3$         | 36.623°                 | $a_R$             | 1.06                | $P_{dR}$       | 0.4               |         |                |         |
| $a_I$              | 1.07      | $V_1$              | 1.0146                  | $\cos\phi_R$      | 0.9232              | $\cos\phi_I$   | 0.9172            |         |                |         |
| $\alpha_R$ (deg.)  | 6°        | $\theta_1$         | 36.4084°                | NI                | 9                   | HVDC variables |                   |         |                |         |

**Table 9** Third study of IEEE 300-bus system.

| HVDC link          |            | $P_{base}$ (pu)   | HVDC link specification |                    | Power flow solution |                   |                |                |  |
|--------------------|------------|-------------------|-------------------------|--------------------|---------------------|-------------------|----------------|----------------|--|
| From bus no        | To bus no  |                   | Spec. values            |                    | ACSV                |                   | DCSV           |                |  |
|                    |            |                   |                         | Control strategy 1 |                     | AC terminal buses |                | HVDC variables |  |
| 199                | 197        | 0.3213            | $P_{dR}$ (pu)           | 0.4                | $V_{199}$           | 1.008             | $V_{dR}$       | 0.9044         |  |
|                    |            |                   | $V_{dI}$ (pu)           | 0.9                | $\theta_{199}$      | $-22.19^\circ$    | $I_d$          | 0.4423         |  |
|                    |            |                   | $\alpha_R$ (deg.)       | $10^0$             | $V_{197}$           | 1.0159            | $a_R$          | 0.96           |  |
|                    |            |                   | $\gamma_I$ (deg.)       | $22^\circ$         | $\theta_{197}$      | $-22.63^\circ$    | $a_I$          | 1.00           |  |
|                    |            |                   | Control strategy 2      |                    | AC terminal buses   |                   | $\cos\phi_R$   | 0.93           |  |
|                    |            |                   | $P_{dR}$ (pu)           | 0.4                | $V_{199}$           | 1.000             | $\cos\phi_I$   | 0.88           |  |
|                    |            |                   | $V_{dI}$ (pu)           | 0.9                | $\theta_{199}$      | $-22.15^\circ$    | NI             | 7              |  |
|                    |            |                   | $a_R$                   | 0.96               | $V_{197}$           | 1.0124            | HVDC variables |                |  |
|                    |            |                   | $a_I$                   | 1                  | $\theta_{197}$      | $-22.64^\circ$    | $V_{dR}$       | 0.9044         |  |
|                    |            |                   | Control strategy 3      |                    | AC terminal buses   |                   | $I_d$          | 0.4423         |  |
|                    |            |                   | $I_d$ (pu)              | 0.4                | $V_{199}$           | 1.000             | $\alpha_R$     | $8.819^\circ$  |  |
|                    |            |                   | $V_{dI}$ (pu)           | 0.9                | $\theta_{199}$      | $-22.65^\circ$    | $\gamma_I$     | $21.44^\circ$  |  |
|                    |            |                   | $\alpha_R$ (deg.)       | $10^\circ$         | $V_{197}$           | 1.0162            | $\cos\phi_R$   | 0.94           |  |
|                    |            |                   | $\gamma_I$ (deg.)       | $22^\circ$         | $\theta_{197}$      | $-22.65^\circ$    | $\cos\phi_I$   | 0.88           |  |
|                    |            |                   | Control strategy 4      |                    | AC terminal buses   |                   | NI             | 9              |  |
|                    |            |                   | $P_{dR}$ (pu)           | 0.4                | $V_{199}$           | 1.000             | HVDC variables |                |  |
|                    |            |                   | $V_{dI}$ (pu)           | 0.9                | $\theta_{199}$      | $-22.15^\circ$    | $V_{dR}$       | 0.9040         |  |
|                    |            |                   | $\alpha_R$ (deg.)       | $10^0$             | $V_{197}$           | 1.0157            | $P_{dR}$       | 0.3616         |  |
|                    |            |                   | $a_I$                   | 1.0                | $\theta_{197}$      | $-22.15^\circ$    | $a_R$          | 0.95           |  |
|                    |            |                   | Control strategy 5      |                    | AC terminal buses   |                   | $a_I$          | 1.00           |  |
|                    |            |                   | $P_{dR}$ (pu)           | 0.4                | $V_{199}$           | 1.008             | $\cos\phi_R$   | 0.94           |  |
|                    |            |                   | $V_{dI}$ (pu)           | 0.9                | $\theta_{199}$      | $-22.18^\circ$    | $\cos\phi_I$   | 0.88           |  |
|                    |            |                   | $a_R$                   | 0.95               | $V_{197}$           | 1.0193            | NI             | 7              |  |
|                    |            |                   | $\gamma_I$ (deg.)       | $22^\circ$         | $\theta_{197}$      | $-22.63^\circ$    | HVDC variables |                |  |
|                    |            |                   | Control strategy 6      |                    | AC terminal buses   |                   | $V_{dR}$       | 0.8985         |  |
|                    |            |                   | $P_{dR}$ (pu)           | 0.4                | $V_{199}$           | 1.000             | $I_d$          | 0.4452         |  |
|                    |            |                   | $a_R$                   | 0.95               | $\theta_{199}$      | $-22.15^\circ$    | $V_{dI}$       | 0.8941         |  |
|                    |            |                   | $a_I$                   | 1                  | $V_{197}$           | 1.00123           | $\alpha_R$     | $6.947^\circ$  |  |
| $\gamma_I$ (deg.)  | $22^\circ$ | $\theta_{197}$    | $-22.64^\circ$          | $\cos\phi_R$       | 0.94                |                   |                |                |  |
| Control strategy 7 |            | AC terminal buses |                         | $\cos\phi_I$       | 0.88                |                   |                |                |  |
| $V_{dR}$ (p.u.)    | 1          | $V_{199}$         | 1                       | NI                 | 9                   |                   |                |                |  |
| $I_d$ (p.u.)       | 0.4        | $\theta_{199}$    | $7.8464^\circ$          | HVDC variables     |                     |                   |                |                |  |
| $a_I$              | 1.07       | $V_{197}$         | 1.0129                  | $V_{dI}$           | 0.996               |                   |                |                |  |
| $\alpha_R$ (deg.)  | $6^\circ$  | $\theta_{197}$    | $7.3457^\circ$          | $\gamma_I$         | $17^\circ$          |                   |                |                |  |
|                    |            |                   |                         | $a_R$              | 1.08                |                   |                |                |  |
|                    |            |                   |                         | $P_{dR}$           | 0.4                 |                   |                |                |  |
|                    |            |                   |                         | $\cos\phi_R$       | 0.9232              |                   |                |                |  |
|                    |            |                   |                         | $\cos\phi_I$       | 0.9189              |                   |                |                |  |
|                    |            |                   |                         | NI                 | 10                  |                   |                |                |  |



**Table A.1** Different base values for DC system.

$$V_{dc \text{ base}} = kV_{ac \text{ base}}; \text{ where } k = \frac{3\sqrt{2}}{\pi} n_b$$

$$I_{dc \text{ base}} = \frac{\sqrt{3}}{k} I_{ac \text{ base}}$$

$$Z_{dc \text{ base}} = k^2 Z_{ac \text{ base}}$$

$$R_{dc \text{ base}} = \frac{3}{\pi} n_b X_c \text{ base}$$

## 6. Conclusions

For the power-flow solution of integrated AC–DC systems, the DC link control specifications bridge the gap between the number of independent equations and the number of unknown quantities. Depending on the application, several combinations of valid control specifications are possible. Each combination of a set of valid control specifications comprises a control strategy. It is observed that the power-flow convergence of integrated AC–DC systems is strongly affected by the control strategy adopted. For a majority of the possible control strategies, the equivalent real and reactive power injections at the concerned buses are independent of the NR iterative loop. However, for others, the equivalent reactive power injections need to be computed every NR iteration. This affects the convergence of the algorithm. This is validated by power flow convergence characteristics with different control strategies and multiple power flow case studies with application of seven different control strategies on DC links incorporated in the IEEE-118 and 300 bus test systems.

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## Appendix A. See Table A.1.

$$V_{ac \text{ base}} = V(\text{line to line rms voltage})$$

$$I_{ac \text{ base}} = \frac{S_{base}}{\sqrt{3}V_{ac \text{ base}}} \quad (\text{A.1})$$

$$Z_{ac \text{ base}} = \frac{V_{ac \text{ base}}}{\sqrt{3}I_{ac \text{ base}}} \quad (\text{A.2})$$

**Appendix B.** As already explained in Section 4, for solving the AC power flow, the equivalent active and reactive power injections at the AC terminal buses connected to the converters need to be computed. The steps involved in the computation of the active and reactive power injections for control strategies 1 and 2 is shown in Table B.1 below.

From Table B.1, it can be observed that the active power injections can be computed prior to the AC power flow. It is also observed that in Control Strategy-1, the quantities  $\phi_R$  and  $\phi_I$  and hence the reactive power injections  $Q_{dR}$  and  $Q_{dI}$  can be computed prior to the AC power flow by manipulation of the specified variables. On the other hand, in Control Strategy-2, they are also dependent on the AC power flow variables and thus, need to be updated every iteration.

Although the steps pertaining to the other control strategies could not be detailed due to a shortage of space, they can also be done in a similar manner.

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**Table B.1** Steps to compute active and reactive power injections in control strategies 1 and 2.

| Control Strategy-1   |   | Control Strategy-2  |   |
|--|---|---|---|
| Specified quantities   | Unknown quantities                              | Specified quantities  | Unknown quantities  |
| $\alpha_R, P_{dR}, \gamma_I, V_{dI}$   | $a_R, a_I, V_{dR}, \phi_R, \phi_I, I_d, P_{dI}$ | $a_R, P_{dR}, a_I, V_{dI}$  | $\alpha_R, \gamma_I, V_{dR}, \phi_R, \phi_I, I_d, P_{dI}$ |
| Step 1: Compute $V_{dR} = \frac{V_{dI} + \sqrt{V_{dI}^2 + 4R_{dc}P_{dR}}}{2}$  |   | Step 1: Compute $V_{dR} = \frac{V_{dI} + \sqrt{V_{dI}^2 + 4R_{dc}P_{dR}}}{2}$   |   |
| Step 2: Compute $I_d = \frac{P_{dR}}{V_{dR}}$  |   | Step 2: Compute $I_d = \frac{P_{dR}}{V_{dR}}$   |   |
| Step 3: Compute $P_{dI} = V_{dI}I_d$   |   | Step 3: Compute $P_{dI} = V_{dI}I_d$  |   |
| Step 4: Compute $\cos \phi_R = \frac{V_{dR}\cos \alpha_R}{V_{dR} + X_c I_d}$   |   | Step 4: Compute $\cos \phi_R = \frac{V_{dR}}{a_R V_i}$ ; <b>Note 1:</b> $V_i$ is an AC power flow variable and is updated every iteration. Hence, $\cos \phi_R$ changes in every iteration  |   |
| Step 5: Compute $Q_{dR} = P_{dR}\tan \phi_R$   |   | Step 5: Compute $Q_{dR} = P_{dR}\tan \phi_R$  |   |
| Step 6: Compute $\cos \phi_I = \frac{V_{dI}\cos \gamma_I}{V_{dI} + X_c I_d}$   |   | Step 6: Compute $\cos \phi_I = \frac{V_{dI}}{a_I V_j}$ ; <b>Note 2:</b> $V_j$ is also an AC power flow variable and is updated every iteration, along with $\cos \phi_I$  |   |
| Step 7: Compute $Q_{dI} = P_{dI}\tan \phi_I$ ; <b>Note:</b> $P_{dR}$ is specified. $P_{dI}, Q_{dR}$ and $Q_{dI}$ can be computed prior to the AC power flow and hence, are independent of the iterative loop |   | Step 7: Compute $Q_{dI} = P_{dI}\tan \phi_I$ ; <b>Note 3:</b> $P_{dR}$ is specified. $P_{dI}$ can be computed prior to the AC power flow. However, $Q_{dR}$ and $Q_{dI}$ depend upon $\phi_R$ and $\phi_I$ respectively, and need to be updated every iteration |   |

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