



# Trapping penguins with entangled B mesons



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## ABSTRACT

The first direct observation of time-reversal (T) violation in the  $B\bar{B}$  system has been reported by the BaBar Collaboration, employing the method of Bañuls and Bernabéu. Given this, we generalize their analysis of the time-dependent T-violating asymmetry ( $A_T$ ) to consider different choices of CP tags for which the dominant amplitudes have the same weak phase. As one application, we find that it is possible to measure departures from the universality of  $\sin(2\beta)$  directly. If  $\sin(2\beta)$  is universal, as in the Standard Model, the method permits the direct determination of penguin effects in these channels. Our method, although no longer a strict test of T, can yield tests of the  $\sin(2\beta)$  universality, or, alternatively, of penguin effects, of much improved precision even with existing data sets.

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## 1. Introduction

A goal of  $B$ -physics is to study the nature of CP violation and to discern, ultimately, whether sources of CP violation exist beyond that of the Standard Model (SM). This means the weak phases associated with various decays are measured to test whether they fit the SM pattern or not. Thus far such searches have proven nil, noting, e.g., Ref. [1] and its update in Ref. [2], and it is of interest to carry these tests to higher precision. For example, in the SM the CP asymmetries associated with the quark decays  $b \rightarrow c\bar{c}s$ ,  $b \rightarrow c\bar{c}d$ , and  $b \rightarrow s\bar{s}s$  measure  $\sin(2\beta)$ , up to penguin contributions and new physics in the decay amplitudes [3].<sup>1</sup> Measurements of the time-dependent asymmetry in the penguin mode  $B \rightarrow \phi K_S$  ( $b \rightarrow s\bar{s}s$ ) and others are statistics limited, and follow-up studies are planned at Belle-II [4]. A compilation of existing measurements can be found in Ref. [5]. Improved tests of weak-phase universality, notably that of  $\sin(2\beta)$ , using the usual measurement of time-dependent CP asymmetries will require experiments at new facilities. In this paper, we propose a more accessible way to sharpen these tests by determining effective weak-phase differences through a single asymmetry measurement; thus an improved test can come from existing data sets.

The BaBar Collaboration has observed direct T violation [6] by exploiting the quantum entanglement of the  $B\bar{B}$  mesons produced

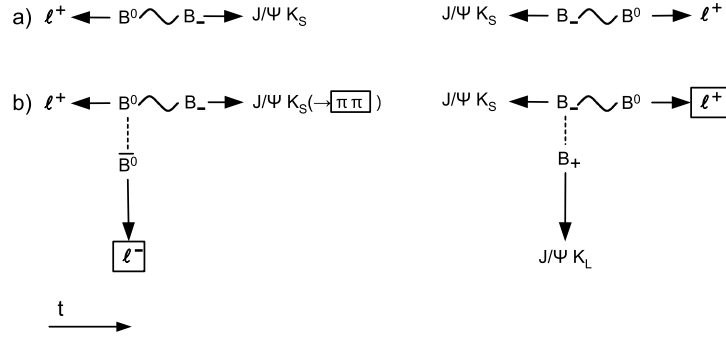
in  $\Upsilon(4S)$  decays, as long familiar from other contexts [7–10]. That is, because the  $\Upsilon(4S)$  state has definite flavor and CP, the flavor- or CP-state of a  $B$  meson can be determined, or “tagged,” at a time  $t$  by measuring the decay of the other  $B$  meson at that instant. In a seminal paper, Bañuls and Bernabéu showed that by selecting suitable combinations of flavor and CP tags of the  $B$ -mesons in the entangled pair, CP, T, and CPT asymmetries [11] can all be constructed. Consequently, BaBar uses the final states  $J/\Psi K_L$  (CP = +) and  $J/\Psi K_S$  (CP = −) as CP tags and the sign of the charged lepton in  $\ell^\pm X$  decay as a flavor tag. Thus by employing either flavor or CP tagging they are able to form a time-dependent asymmetry  $A_T$ , such as  $A_T = (\Gamma(B^0 \rightarrow B_+) - \Gamma(B_+ \rightarrow B_0))/(\Gamma(B^0 \rightarrow B_+) + \Gamma(B_+ \rightarrow B_0))$ , where  $B_\pm$  denotes a state with CP = ± [6,11–13]. Thus if the rates of  $B^0 \rightarrow B_+$  and  $B_+ \rightarrow B^0$  are not the same, i.e., not in “detailed balance,” then time-reversal symmetry is broken. BaBar measures the T-violating parameters  $\Delta S_T^+ = -1.37 \pm 0.14_{\text{stat}} \pm 0.06_{\text{sys}}$  and  $\Delta S_T^- = 1.17 \pm 0.18_{\text{stat}} \pm 0.11_{\text{sys}}$ , so that both measurements exceed discovery significance, and reports observing T violation with an effective significance of  $14\sigma$  [6]. Previously a failure of detailed balance was reported in  $K^0 \leftrightarrow \bar{K}^0$  transitions by CPLEAR [14], but the concomitant claim of direct T violation of  $\langle A_T^{\text{exp}} \rangle = (6.6 \pm 1.3_{\text{stat}} \pm 1.0_{\text{sys}}) \times 10^{-3}$  is only of  $4\sigma$  significance if statistical and systematic errors are combined in quadrature. Moreover, the interpretation of the experiment as a test of T has been criticized [15,16]. In the case of the concept [11,12] employed by the BaBar experiment [6], the use of entanglement with distinct kinds of tags allows the reservations [15,16] levied against the CPLEAR experiment to be set to rest [2,17,18].

Nevertheless, there has been discussion of the conditions under which a measured non-zero value of  $A_T$  proves that time-reversal

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<sup>1</sup> Recall  $\beta \equiv \arg[-V_{cd}^* V_{cb}^*/(V_{td}^* V_{tb}^*)]$ , where  $V_{ij}$  is an element of the Cabibbo–Kobayashi–Maskawa (CKM) matrix. In this paper we use “penguin contributions” to connote all wrong phase contributions to the decay amplitude.

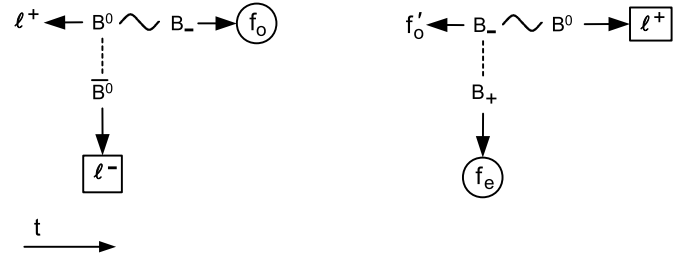


**Fig. 1.** The transition  $B^0 \rightarrow B_-$  and the construction of its time-conjugate  $B_- \rightarrow B^0$ . a) Idealized: the initial detection of  $\ell^-$  projects the other  $B$  into the orthogonal flavor state, realizing  $B^0 \rightarrow B_-$  upon subsequent detection of  $J/\psi K_S$ , whereas the initial detection of  $J/\psi K_L$  projects the other  $B$  into the CP = - state. In this latter case subsequent detection of  $\ell^+ X$  realizes  $B_- \rightarrow B^0$ , the time-reversed process associated with  $B^0 \rightarrow B_-$ . The initial-state projections can be thought of as inverse decays of  $\ell^+$  and  $J/\psi K_S$ , respectively [19]. b) Expanded to include the particles that are detected (boxes) to tag the initial and final states of the  $B$ -meson. The second process is not the time conjugate of the first once direct CP violation in the tagging decay is included. The CP state of the  $B$ -meson prepared through inverse decay is not identical to that of the  $B$  which decays to  $J/\psi K_S(\pi^+\pi^-)$ . Note at the  $B$ -factories that  $K_L$  is reconstructed through its interactions with the detector [22].

symmetry is broken. Generally, the existence of penguins complicates the interpretation of these measurements as tests of T (or of CPT), though in the specific final states studied by BaBar [6]  $A_T$  is a true test of T irrespective of penguin effects in the  $B$ -meson decay [19]. Direct CP violation in the CP tag, however, which is possible if  $K_{S,L}$  are reconstructed through their hadronic decays, also causes the interpretation of  $A_T$  as a test of T to fail – this has also been noted by Ref. [20] in an analogous study of  $K\bar{K}$  transitions and in Ref. [21]. In this paper we break the interpretation of  $A_T$  as a test of T purposefully through the choice of different CP tags, and the resulting variations in the effective T violation can be used to probe the existence of different small effects. In particular, we show that with specially chosen “generalized” CP tags the dominant amplitudes cancel in observables associated with  $A_T$ , thus yielding a direct test of weak phase universality, or, alternatively, a measurement of differences of penguin pollution in the SM. These differences have been difficult to quantify [5], and our procedure gives direct access to them. To explicate this, we shall start by revisiting the interpretation of  $A_T$ .

## 2. Interpreting $A_T$

The combination of Einstein–Podolsky–Rosen (EPR) entanglement in the  $B\bar{B}$  system from  $\Upsilon(4s)$  decay with the possibility of *both* lepton and CP tagging (using  $J/\psi K_{S,L}$ ) allows a near-perfect experimental realization of a process and its time-reversal conjugate, making the measurement of  $A_T$  a true test of time-reversal symmetry. The first tag at  $t_0$ , of CP (or flavor), sets the initial state of the remaining particle. Following the formalism of the recent analysis of BaBar’s measured  $A_T$  by Applebaum et al. [19], the state assignment of the remaining  $B$ -meson can be thought of as an inverse decay at  $t_0$  from the opposite CP (or flavor) tag. Fig. 1 visualizes this result. The inverse decay is realized through EPR entanglement and the decay of another particle, and Applebaum et al. state the conditions under which a nonzero  $A_T$  reveals T violation, though, as we will show, the conditions turn out to be necessary but not sufficient. That is, they note that (i) the absence of CPT violation in strangeness changing decays and (ii) the absence of wrong sign decays or the absence of direct CP violation in semileptonic decays if wrong sign decays occur are required to interpret  $A_T$  as a test of T invariance [19]. (A complementary discussion of the conditions under which  $A_T$  serves as a test of T can be found in Ref. [21].) Fig. 1a illustrates the ideal case in which the detection of one state projects the other  $B$ -meson into the state orthogonal to it, thus realizing the exchange of initial and final states needed to construct the time-conjugate process.



**Fig. 2.** The  $B^0 \rightarrow B_-$  transition and its time conjugate using general CP tags  $f_0$  and  $f'_e$ , which are odd and even, respectively, under CP. In this case the interpretation of  $A_T$  as a test of T can be broken at the tag level. Thus detecting  $f_e$  at  $t_0$  is tantamount to the inverse decay  $f'_0 \rightarrow B_-$ , where  $f_0$  and  $f'_0$  are distinct states. Here circles are used to indicate that the CP tag may be reconstructed rather than directly detected.

There is one more effect to consider in interpreting  $A_T$  as a test of T, and it can arise if the CP tagging state is itself reconstructed through its decay to hadrons. That is, direct CP violation in the decay of CP tag to hadronic final states breaks the ability to construct the time-reversed process. (This is distinct from the complications due to  $\epsilon_K$ , noted in Ref. [19].) Fig. 1b illustrates this, though the details are provided in the following section. Ideally,  $K_S$  and  $K_L$  can be reconstructed unambiguously, but direct CP violation in the reconstruction of the  $K_S$  from  $K_S \rightarrow \pi\pi$  decay prevents this. In the formalism of [19], it appears as if it were a CPT-violating effect. Of course, CPT is not actually broken, but, rather, the relationships between the T and CP asymmetries expected under an assumption of CPT invariance will not hold because of direct CP violation in the kaon decay. The effect of direct CP violation in  $K_S \rightarrow \pi\pi$  is numerically very small [23]. Nevertheless it can limit the sensitivity of CPT tests that follow from comparing T and CP asymmetries,  $A_T$  and  $A_{CP}$ . (We note that the best limits on the real part of the CPT-violating parameter  $z$  in the  $B$  system come from studies of  $b \rightarrow c\bar{c}s$  decay [24,25].) The new method we propose exploits the potential failure of  $A_T$  as a test of T by selecting CP tags of common dominant weak phase (in the SM) but differing penguin pollution, e.g., to yield new observables – this is illustrated in Fig. 2. These new observables probe small effects that have not previously been directly measured. In these cases as well we find  $|A_T| \neq |A_{CP}|$  without CPT violation. We now turn to the details.

## 3. Details

The time-dependent decay rate for  $B\bar{B}$  mesons produced in  $\Upsilon(4S)$  decay, in which one  $B$  decays to final state  $f_1$  at time  $t_1$

and the other decays to final state  $f_2$  at a later time  $t_2$  has been analyzed in the presence of CPT violation, wrong-sign semileptonic decays, and wrong strangeness decays [19]. In what follows we assume all of these refinements to be completely negligible. Moreover, we neglect CP violation in  $B\bar{B}$  mixing and set the width difference of the  $B$ -meson weak eigenstates to zero, i.e.,  $\Gamma_H - \Gamma_L = 0$ . The decay rate to  $f_1$  and then  $f_2$  is denoted as  $\Gamma_{(f_1)\perp, f_2}$  and is thus given by

$$\Gamma_{(f_1)\perp, f_2} = \mathcal{N}_1 \mathcal{N}_2 e^{-\Gamma(t_1+t_2)} [1 + C_{(1)\perp, 2} \cos(\Delta m_B t) + S_{(1)\perp, 2} \sin(\Delta m_B t)], \quad (1)$$

with  $\Gamma \equiv (\Gamma_H + \Gamma_L)/2$ ,  $\Delta m_B \equiv m_H - m_L$ ,  $t = t_2 - t_1 \geq 0$ ,  $S_{(1)\perp, 2} \equiv C_1 S_2 - C_2 S_1$ , and  $C_{(1)\perp, 2} \equiv -[C_2 C_1 + S_2 S_1]$  [19]. Moreover,  $C_f \equiv (1 - |\lambda_f|^2)/(1 + |\lambda_f|^2)$  and  $S_f \equiv 2\Im(\lambda_f)/(1 + |\lambda_f|^2)$ , where  $\lambda_f \equiv (q/p)(\bar{A}_f/A_f)$ , noting  $A_f \equiv A(B^0 \rightarrow f)$ ,  $\bar{A}_f \equiv A(\bar{B}^0 \rightarrow f)$ ,  $\mathcal{N}_f \equiv A_f^2 + \bar{A}_f^2$ , and  $q$  and  $p$  are the usual  $B\bar{B}$  mixing parameters [23]. Since we neglect wrong-sign semileptonic decay,  $C_{\ell^+X} = -C_{\ell^-X} = 1$ . Defining normalized rates as per  $\Gamma'_{(f_1)\perp, f_2} \equiv \Gamma_{(f_1)\perp, f_2}/(\mathcal{N}_{f_1} \mathcal{N}_{f_2})$  we have, in the case of the asymmetry illustrated in Fig. 1,

$$A_T = \frac{\Gamma'_{(\ell^-X)\perp, J/\psi K_S} - \Gamma'_{(J/\psi K_L)\perp, \ell^+X}}{\Gamma'_{(\ell^-X)\perp, J/\psi K_S} + \Gamma'_{(J/\psi K_L)\perp, \ell^+X}}. \quad (2)$$

Note that normalizing each rate is important to a meaningful experimental asymmetry because the  $J/\psi K_S$  (or, more generally,  $c\bar{c}K_S$ ) and  $J/\psi K_L$  final states have different reconstruction efficiencies [13]. BaBar constructs four different asymmetries, based on four distinct subpopulations of events, namely, those for  $\Gamma_{(\ell^+X)\perp, c\bar{c}K_S}$  ( $\bar{B}^0 \rightarrow B_-$ ),  $\Gamma_{(c\bar{c}K_S)\perp, \ell^+X}$  ( $B_+ \rightarrow B^0$ ),  $\Gamma_{(\ell^+X)\perp, J/\psi K_L}$  ( $\bar{B}^0 \rightarrow B_+$ ),  $\Gamma_{(J/\psi K_L)\perp, \ell^+X}$  ( $B_- \rightarrow B^0$ ), and their T conjugates, respectively, and finds the measurements of the individual asymmetries to be compatible [6]. We note that the normalization factors  $\mathcal{N}_f$  for general CP tags will differ; nevertheless, meaningful experimental asymmetries can be constructed through the use of normalized decay rates as already implemented in BaBar's  $A_T$  analysis [6].

In what follows we generalize the choice of CP final states, so that  $J/\psi K_S \rightarrow f_o$  and  $J/\psi K_L \rightarrow f_e$ , where ‘‘o’’ (‘‘e’’) denotes a CP-odd (even) final state. We define

$$\begin{aligned} A_{CP}^{e+} &\equiv \frac{\Gamma'_{(\ell^-X)\perp, f_e} - \Gamma'_{(\ell^+X)\perp, f_e}}{\Gamma'_{(\ell^-X)\perp, f_e} + \Gamma'_{(\ell^+X)\perp, f_e}} \\ &= C_e \cos(\Delta m_B t) - S_e \sin(\Delta m_B t), \quad (3) \\ A_{CP}^{e-} &\equiv \frac{\Gamma'_{(f_e)\perp, \ell^-X} - \Gamma'_{(f_e)\perp, \ell^+X}}{\Gamma'_{(f_e)\perp, \ell^-X} + \Gamma'_{(f_e)\perp, \ell^+X}} \\ &= C_e \cos(\Delta m_B t) + S_e \sin(\Delta m_B t), \quad (4) \end{aligned}$$

where  $A_{CP}^{e+} \rightarrow A_{CP}^{o+}$  and  $A_{CP}^{e-} \rightarrow A_{CP}^{o-}$  follow by replacing  $f_e \rightarrow f_o$ . Note that  $A_{CP}^{f+}$  and  $A_{CP}^{f-}$  employ distinct data samples. Moreover,

$$\begin{aligned} A_T^{o+} &\equiv \frac{\Gamma'_{(f_o)\perp, \ell^-X} - \Gamma'_{(\ell^+X)\perp, f_o}}{\Gamma'_{(f_o)\perp, \ell^-X} + \Gamma'_{(\ell^+X)\perp, f_o}} \\ &= \frac{(C_e + C_o) \cos(\Delta m_B t) + (S_o - S_e) \sin(\Delta m_B t)}{2 + (C_o - C_e) \cos(\Delta m_B t) + (S_o + S_e) \sin(\Delta m_B t)}, \quad (5) \\ A_T^{o-} &\equiv \frac{\Gamma'_{(\ell^-X)\perp, f_o} - \Gamma'_{(f_o)\perp, \ell^+X}}{\Gamma'_{(\ell^-X)\perp, f_o} + \Gamma'_{(f_o)\perp, \ell^+X}} \\ &= \frac{(C_e + C_o) \cos(\Delta m_B t) - (S_o - S_e) \sin(\Delta m_B t)}{2 + (C_o - C_e) \cos(\Delta m_B t) - (S_o + S_e) \sin(\Delta m_B t)}, \quad (6) \end{aligned}$$

and

$$\begin{aligned} A_T^{e+} &\equiv \frac{\Gamma'_{(f_e)\perp, \ell^-X} - \Gamma'_{(\ell^+X)\perp, f_o}}{\Gamma'_{(f_e)\perp, \ell^-X} + \Gamma'_{(\ell^+X)\perp, f_o}} \\ &= \frac{(C_e + C_o) \cos(\Delta m_B t) - (S_o - S_e) \sin(\Delta m_B t)}{2 - (C_o - C_e) \cos(\Delta m_B t) + (S_o + S_e) \sin(\Delta m_B t)}, \quad (7) \\ A_T^{e-} &\equiv \frac{\Gamma'_{(\ell^-X)\perp, f_e} - \Gamma'_{(f_o)\perp, \ell^+X}}{\Gamma'_{(\ell^-X)\perp, f_e} + \Gamma'_{(f_o)\perp, \ell^+X}} \\ &= \frac{(C_e + C_o) \cos(\Delta m_B t) + (S_o - S_e) \sin(\Delta m_B t)}{2 - (C_o - C_e) \cos(\Delta m_B t) - (S_o + S_e) \sin(\Delta m_B t)}. \quad (8) \end{aligned}$$

Each time-dependent asymmetry has four parameters made distinguishable by the various time-dependent functions, and they can be measured experimentally. Indeed the individual asymmetries can be simultaneously fit for  $S_o + S_e$ ,  $S_o - S_e$ ,  $C_o + C_e$ , and  $C_o - C_e$ . Note that if  $C_o = C_e$  and  $S_o = -S_e$ ,  $A_{CP}^{e+} = A_{CP}^{o-} = A_T^{o+} = A_T^{e-}$  and  $A_{CP}^{e-} = A_{CP}^{o+} = A_T^{e+} = A_T^{o-}$ . Neglecting CP violation in kaon decay, we note that  $\lambda_{J/\psi K_S} = -\lambda_{J/\psi K_L}$ . The  $K_S$  is reconstructed through its decays to  $\pi\pi$  ( $2\pi$ ), whereas the  $K_L$ , at BaBar and Belle, is not determined from its decay to  $\pi^0\pi^0\pi^0$ , though this can be done at DAPHNE [20]. We calculate  $\lambda_{2\pi}$

$$\lambda_{2\pi} = \frac{q \langle \bar{K}^0 | \bar{B}^0 \rangle}{p \langle K^0 | B^0 \rangle} \frac{1 + \epsilon_K}{1 - \epsilon_K} \frac{1 + \eta_{2\pi}}{1 - \eta_{2\pi}}, \quad (9)$$

where  $\eta_{2\pi} \equiv (2\pi | K_L \rangle) / (2\pi | K_S \rangle)$  and  $\epsilon_K$  captures CP violation in  $K\bar{K}$  mixing. Since  $\eta_{2\pi} \neq 0$  [23], we find  $C_{2\pi} \neq C_{K_L}$  and  $S_{2\pi} \neq -S_{K_L}$ , yielding  $|A_{CP}| \neq |A_T|$  (in all cases) without CPT violation. Though we concur with Ref. [19] that neither direct CP violation in  $B$  meson decay nor CP violation in  $K\bar{K}$  mixing can generate this effect, we see explicitly that the effect of direct CP violation in  $K$  decay can be included through a nonzero  $\theta_f$ , a nominally CPT-violating parameter, in the formalism of Ref. [19]. We note the criteria of Applebaum et al. [19], enumerated in the previous section, should be supplemented with the neglect of direct CP violation in kaon decay, if the kaon is reconstructed through its hadronic decays, in order to interpret  $A_T$  as a test of T.

Thus far we have discussed the CP final states  $f_o = J/\psi K_S$  and  $f_e = J/\psi K_L$ , though other choices are possible. If we choose CP final states that share a dominant weak phase with each other and with  $J/\psi K_{S,L}$ , we have  $f_{o'} = \phi K_S, \eta' K_L, \rho^0 K_S, \omega K_S, \pi^0 K_L$  and  $f_{e'} = \phi K_L, \eta' K_S, \rho^0 K_L, \omega K_L, \pi^0 K_S$ , respectively, with the prime notation henceforth representing a CP tag other than  $J/\psi K_{S,L}$ . These are the two-body ‘‘sin(2 $\beta$ )’’ modes commonly studied<sup>2</sup> to test its universality [23,28]. Not only can we use these modes to form the  $A_T$  asymmetries we have discussed thus far [29], such as the comparison of  $\bar{B}^0 \rightarrow B_{o'}$  with  $B_{o'} \rightarrow \bar{B}^0$ , we can form two more for each one: e.g., we can compare  $\bar{B}^0 \rightarrow B_{o'}$  to  $B_o \rightarrow \bar{B}^0$ , as well as  $\bar{B}^0 \rightarrow B_o$  to  $B_{o'} \rightarrow \bar{B}^0$ . Turning to Eq. (8), we see that the parameters associated with the  $\sin(\Delta m_B t)$  terms in these comparisons are, e.g.,  $S_{o'} - S_e$  and  $S_{o'} + S_e$ . In  $S_{o'} + S_e$  the dominant weak phase contributions (in the SM) cancel, and the small terms, namely, the penguin contributions, as well as possible contributions from new physics, are determined directly. In the analogous comparison of  $\bar{B}^0 \rightarrow B_{e'}$  with  $B_{e'} \rightarrow \bar{B}^0$  decay, the dominant weak phases cancel in  $S_o + S_{e'}$ . Note that the possibility of a direct measurement of a quantity in which the dominant weak phases can cancel is special to the  $A_T$  construction.

<sup>2</sup> Three-body decays, such as  $K_S K_S K_S$  or  $K^+ K^- K_S$ , have also been studied, though determining the CP content of the  $K^+ K^- K_S$  Dalitz plot requires an angular moment analysis [26,27].

In order to demonstrate this, we first define the parameter  $\lambda_f$  on which  $S_{o,e}$  depend. There is a factor of  $\exp(-i2\beta)$  from  $B\bar{B}$  mixing, and, in general, the decay amplitude can be written as a linear combination of 2 weak phases (we select “up” and “charm”):  $A_f = a_f^c e^{-i\theta_c} + a_f^u e^{-i\theta_u}$ , in which “ $a_f^c$ ”, “ $a_f^u$ ” contain the magnitudes of the amplitude associated with each phase, including diagrammatic tree and penguin contributions. The associated weak phases are  $\theta_c = 0$  and  $\theta_u \equiv \gamma$ . The dominant weak phase is determined by the quark-flavor content of the final state. Our focus is on the  $\sin(2\beta)$  modes, for which  $a_f^c$  is the dominant amplitude. Defining

$$\lambda_f = -\eta_{CP}^f e^{-2i\beta} \frac{1 + d_f e^{-i\gamma}}{1 + d_f e^{i\gamma}}, \quad (10)$$

$$d_f \equiv \left| \frac{V_{ub}^* V_{us}}{V_{cb}^* V_{cs}} \right| \frac{a_f^u}{a_f^c}, \quad (11)$$

where  $\text{CP}|f\rangle = \eta_{CP}^f |f\rangle$ , a simple calculation gives us: [30]

$$S_f = -\eta_{CP}^f \frac{\sin(2\beta) + 2\Re(d_f) \sin(2\beta + \gamma) + |d_f|^2 \sin(2\beta + 2\gamma)}{1 + |d_f|^2 + 2\Re(d_f) \cos(\gamma)},$$

$$C_f = \frac{-2\Im(d_f) \sin(\gamma)}{1 + |d_f|^2 + 2\Re(d_f) \cos(\gamma)}. \quad (12)$$

As long familiar, a difficulty arises in attempting to separate the dominant term from any small effects. Setting the smaller, wrong phase contribution to zero, we recover the simplified expressions  $C_f = 0$ ,  $S_f = -\eta_{CP}^f \sin(2\beta)$  for all  $f$ . It is convenient to define  $\delta S_f$  such that  $S_f = -\eta_{CP}^f (\sin(2\beta) + \delta S_f)$ .<sup>3</sup>

Several theoretical studies have been made of the deviations of  $S_f$ , measured through  $A_{CP}^{f+}$ , from  $\sin(2\beta)$ , through computation of the amplitudes in the SM [30–36], as well as through approaches using SU(3)-flavor-based assumptions [37,38,43]. A particular effort has been placed on determining the size of the small penguin pollution in the golden  $J/\psi K_{S,L}$  modes, for which ancillary data and flavor-based relations can be used [39–43]. Experimentally one can form

$$\delta S_f = -\eta_{CP}^f S_f - \sin(2\beta) \quad (13)$$

using the determination of  $\sin(2\beta)$  in  $B \rightarrow c\bar{c} K_S$  and  $J/\psi K_L$  final states [6,44,45], though the error in  $\delta S_f$  is dominantly that in  $S_f$ . We now compare this procedure to our  $A_T$  method with generalized CP tags. In this new case, assuming  $\sin(2\beta)$  universality, the  $\sin(2\beta)$  term in  $S_f$  cancels, yielding

$$(S_e + S_o) = \delta S_o - \delta S_e \quad (14)$$

and providing a direct measurement of the difference of deviations from  $\sin(2\beta)$  for the chosen CP tags. If we use a golden mode for which  $\delta S_{e(o)} \approx 0$ , such as  $J/\psi K_{S,L}$ , to define  $\sin(2\beta)$ , then  $S_e + S_o \approx \pm \sin(2\beta_{o(e)}) \mp \sin(2\beta) \pm \delta S_{o(e)}$ , where the upper sign is associated with  $o$ . Thus we test the deviation of  $S_f$  from  $\sin(2\beta)$  through a single asymmetry measurement, whereas a “double” difference appears in Eq. (13). Of course  $\sin(2\beta)$  in  $B \rightarrow c\bar{c} K_S$ ,  $J/\psi K_L$  decays is very well known ( $0.677 \pm 0.020$  [5]), so that it is more pertinent to note that the asymmetry  $A_T$  can directly employ these highly precise decay samples as well [6,44,45].

<sup>3</sup> We use “ $\delta S_f$ ” in place of the “ $\Delta S_f$ ” used in Refs. [30–36] in order to avoid confusion with the quantities  $\Delta S_T^\pm$  of Refs. [6,13,19] that we have already introduced.

An asymmetry  $A_T$  generally requires the comparison of the rates ( $(\ell^\pm X)_\perp$ ,  $f_{o(e)}$ ) and ( $(f_{e'(e')})_\perp$ ,  $\ell^\pm X$ ), or of their time conjugates, while  $A_{CP}$  only requires the comparison of the ( $(\ell^\pm X)_\perp$ ,  $f_{o'(e')}$ ) rates. Thus in the case of  $\eta' K_S$ , e.g., the determination of  $S_{e'}$  via  $A_{CP}$  employs two subsamples of limited statistics, whereas the determination of  $S_{e'} + S_o$  via  $A_T$  is formed from the comparison of a limited statistics sample with the plentiful statistics of  $c\bar{c} K_S$ . Consequently, we expect improved access to  $\delta S_{e'}$ , for any of the CP-even modes that probe  $\sin(2\beta)$ , and analogous improvements to the determination of  $\delta S_{o'}$  for any of the CP-odd modes. Current experimental results for  $S_f$  have limited precision in many of the  $\sin(2\beta)$  modes previously listed as CP-tag candidates (e.g.,  $-\eta_{CP}^f S_{\pi^0 K_S} = 0.57 \pm 0.17$ ;  $-\eta_{CP}^f S_{\omega K_S} = 0.45 \pm 0.24$  [5]). Our method will be of greatest impact for these more poorly known modes. Comparing these results against predicted values of  $\delta S_{o'(e')}$  in the SM should then yield sharper tests of new physics. Such sharpened determinations should also improve the ability to extract the true value of  $\sin(2\beta)$  from fits to the experimental results in a theoretical framework including leading SU(3) flavor-breaking effects [38], again leading to improved tests of new physics. We note that diverse sources of the latter have been proposed [3,33,46–48].

Our method requires the construction of normalized subsample rates as in Eq. (2); normalized subsample rates have already been employed in BaBar’s  $A_T$  analysis [6]. The efficacy of this procedure can be roughly assessed through the comparison of BaBar’s claimed significance for the observation of T and CP violation through the measurement of  $A_T$  and  $A_{CP}$ , respectively. In this exact case BaBar measures T violation at  $14\sigma$  and CP violation at  $17\sigma$  [6], so that they are not very different, particularly when one notes that the  $A_T$  measurement employs a  $J/\psi K_L$  subsample as well. Consequently, for various  $f_{o'(e')}$  we can expect a sharper determination of  $\delta S_{o'(e')}$  through the measurement of  $A_T$  than possible through study of  $A_{CP}$  alone.

The method we have proposed can be generalized to other sorts of decay modes, such as those that probe  $\sin(2\alpha)$  [49]. The basic idea is that the CP-tagging modes are chosen so that their dominant decay amplitudes (in the SM) share the same weak phase. In the cases we have considered in this paper, the CP-even and odd tags are chosen with a common dominant weak phase of  $\sin(2\beta)$ . In so doing,  $A_T$  is no longer a true test of T, but we introduce new observables that permit a direct measurement of small departures from weak-phase universality. If the dominant weak phase is universal, then these observables measure the penguin pollution in these decays. We emphasize that although the phrase “penguin trapping” has previously been used to refer to the specific reconstruction of the penguin amplitude using flavor-based assumptions and empirical data [50], we use it here to refer to a method by which a more precise empirical assessment can be made of observables in which penguin effects can appear.

#### 4. Summary

We have described how a broader measurement program of the time-dependent asymmetry  $A_T$  with generalized CP tags, possible at a B factory, can be used to measure small departures from weak-phase universality. Generally an analysis of  $A_T$  provides four parameters composed of linear combinations of  $S_{o(e)}$  and  $C_{o(e)}$ ; under the use of generalized CP tags the asymmetry  $A_T$  no longer serves as a genuine T test – and  $|A_T| \neq |A_{CP}|$  can appear without CPT violation. However, the new observables the  $A_T$  construction offers allow the direct measurement of the penguin effects with improved statistical control, information that can be used to test the universality of  $\sin(2\beta)$ . New results of greater precision can be obtained from existing B-factory data using this method, and we

believe it can also greatly enable precision studies of CP violation anticipated with the Belle II detector at KEK.

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