Neutron star structure in an in-medium modified chiral soliton model

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ABSTRACT

We study the internal structure of a static and spherically symmetric neutron star in the framework of an in-medium modified chiral soliton model. The Equations of State describing an infinite and asymmetric nuclear matter are obtained introducing the density dependent functions into the low energy free space Lagrangian of the model starting from the phenomenology of pionic atoms. The parametrizations of density dependent functions are related to the properties of isospin asymmetric nuclear systems at saturation density of symmetric nuclear matter $\rho_0 \simeq 0.16$ fm^{-3}. Our results, corresponding to the compressibility of symmetric nuclear matter in the range 250 MeV $\leq K_0 \leq$ 270 MeV and the slop parameter value of symmetry energy in the range 30 MeV $\leq L_S \leq$ 50 MeV, are consistent with the results from other approaches and with the experimental indications. Using the modified Equations of State, near the saturation density of symmetric nuclear matter ρ_0, the extrapolations to the high density and highly isospin asymmetric regions have been performed. The calculations showed that the properties of $\sim 1.4 M_\odot$ and $\sim 2 M_\odot$ neutron stars can be well reproduced in the framework of present approach.

1. Introduction

Analysis of the neutron star structure is an interesting topic of the modern astrophysics. In particular, in the nuclear astrophysics the structure studies of neutron stars are related to the Equations of State (EOS) of nuclear matter which describes the pressure density versus energy density relation for a broad range of density values (for example, see recent review [1] and references therein). The peculiarities of EOS are well know at the densities below the saturation density of symmetric nuclear matter ρ_0 while at the high density regions they are still remaining not clear. The high density behavior of EOS are poorly understood because of the difficulty of direct experimental accessibility in laboratories and because of the absence of *ab initio* theoretical calculations. Therefore, from the experimental point of view, the neutron star studies may serve as a laboratory for understanding the behavior of EOS at high densities. From the theoretical point of view, instead of *ab initio* calculations one can start from the phenomenological framework taking into account the well known properties of EOS at the low density region and extrapolate to the high density regions trying to describe the properties of matter under the extreme (high density and high temperature) conditions.

In this context and if one able to formulate further in-medium modifications, a chiral soliton model of Skyrme, describing the sin-

gle nucleon properties in free space [2,3], or its variations including the explicit vector mesonic degrees of freedom [4] may serve as a starting point for the theoretical framework. The in-medium modifications may be expressed allowing the density dependencies of the constants entering into the initial free space Lagrangian. It is necessary to note, that in principle one should be able also to reproduce the medium dependencies in the effective Lagrangian starting from the first principles but it is not known yet the form of general low energy Lagrangian and the peculiarities of its ingredients. For this reason, in the present work we use an in-medium modified chiral soliton model [5,6] which may be considered as a truncated version of the general low energy Lagrangian.

In Refs. [5,6] the medium modifications were achieved putting a bit more phenomenology into the initial model, i.e. putting the density dependent functions into the free space Skyrme Lagrangian according to the pionic-atoms data at low energies [7] and properties of asymmetric nuclear matter at saturation density ρ_0. Although the in-medium modified Skyrme Lagrangians is assumed to be very truncated version of the possible general Lagrangian, it must be applicable to the studies of nuclear many-body problems in the spirit of chiral effective Lagrangians. The pay for the truncation may be the possible deviations from the experimental observables in the sense of quantitative description. Nevertheless, the model has obvious virtues: i) it has the simplest Lagrangian among the same class Lagrangians, and ii) it seams have all necessary ingredients for the qualitative description of the strong interaction

physics. These ideas were the basic ruling philosophy behind of the approach developed in Refs. [5,6] and we continue our model studies in the present work.

The model is phenomenological one and must pass as much as possible tests on its applicability to strong interacting systems comparing with other approaches and the experimental indications. The previous nuclear matter studies [6] showed that the in-medium modified Skyrme term is responsible for preventing the collapse of nuclear matter to the singularity at high densities in analogy to the free space case where the Skyrme term is responsibly for the stabilization of finite size solitons. The modifications showed that at some values of model parameters the properties of infinite and isospin asymmetric nuclear matter can be reproduced well near the saturation point of symmetric nuclear matter ρ_0.

It will be interesting to test the applicability of model to the strong interacting systems under the extreme conditions extrapolating the modified Equations of State to the high density regions. Therefore, in the present work we make further check of the basic philosophy considering an application of the model for the studies of neutron stars structure.

2. Formalism

2.1. The Lagrangian

Our starting point is the in-medium modified Skyrme-model Lagrangian described in Ref. [5]

$$\mathcal{L}^* = \mathcal{L}_2^* + \mathcal{L}_4^* + \mathcal{L}_m^* + \mathcal{L}_e^*,$$

$$\mathcal{L}_2^* = \frac{F_\pi^2}{16}\alpha_\tau \, \mathrm{Tr}\left(\partial_0 U \partial_0 U^\dagger\right) - \frac{F_\pi^2}{16}\alpha_s \, \mathrm{Tr}\left(\partial_i U \partial_i U^\dagger\right),$$

$$\mathcal{L}_4^* = -\frac{1}{16e^2\zeta_\tau} \, \mathrm{Tr}\left[U^\dagger\partial_0 U, U^\dagger\partial_i U\right]^2$$
$$+ \frac{1}{32e^2\zeta_s} \, \mathrm{Tr}\left[U^\dagger\partial_i U, U^\dagger\partial_j U\right]^2,$$

$$\mathcal{L}_m^* = -\frac{F_\pi^2 m_\pi^2}{16}\alpha_m \, \mathrm{Tr}\left(2 - U - U^+\right),$$

$$\mathcal{L}_e^* = -\frac{F_\pi^2}{16} m_\pi \alpha_e \varepsilon_{ab3} \, \mathrm{Tr}\left(\tau_a U\right) \mathrm{Tr}\left(\tau_b \partial_0 U^\dagger\right), \tag{1}$$

where Einstein's summation convention is always assumed (if not specified otherwise). The chiral $SU(2)$ matrix $U = \exp(2i\tau_a\pi_a/F_\pi)$ is defined in terms of the Cartesian isospin-components of the pion field π_a ($a = 1, 2, 3$). The density functionals entering into the Lagrangian (α_τ, α_s, ζ_τ, ζ_s, α_m and α_e) represent the influence of surrounding environment to the single soliton properties.

In the single skyrmionic sector, the input parameters of the model have the following values: $F_\pi = 108.783$ MeV, $e = 4.854$ and $m_\pi = 134.976$ MeV. They correctly reproduce the experimental values of the nucleon mass $m_N = 938$ MeV and Δ-isobar mass $m_\Delta = 1232$ MeV in free space, i.e. if $\alpha_\tau = \alpha_s = \alpha_m = \zeta_\tau = \zeta_s = 1$ and $\alpha_e = 0$.

This simple Lagrangian describes the properties of nucleons in free space, the properties of nucleons in nuclear matter as well as the properties of isospin asymmetric nuclear matter starting at the same footing. The formalism for the classical solitonic solutions, the quantization method and the explanations of applications for symmetric and asymmetric matter properties can be found in Refs. [5,6] and references therein. But for selfconsistency, in the next Subsection 2.2 we briefly outline how an infinite size nuclear systems can be described in the framework of the present approach and represent some final formulas.

2.2. Nuclear matter

In the thermodynamic limit at zero temperature, for a system of an infinite number of baryons uniformly distributed in infinite volume but keeping the density per unit volume constant, the binding energy per nucleon can be represented as

$$\varepsilon(\lambda, \delta) = \varepsilon_V(\lambda) + \varepsilon_A(\lambda, \delta), \tag{2}$$

where ε_V and ε_A are known as volume and asymmetry energies, respectively. For the convenience, here we introduced the isoscalar $\lambda = \rho/\rho_0$ and isovector $\delta = \delta\rho/\rho$ density parameters in terms of the isoscalar $\rho = \rho_{\mathrm{neutron}} + \rho_{\mathrm{proton}}$ and isovector $\delta\rho = \rho_{\mathrm{neutron}} - \rho_{\mathrm{proton}}$ nuclear densities, and the normal nuclear matter density ρ_0.

In the framework of present approach, using the spherically symmetric approximation for a single soliton properties via the hedgehog ansatz $U = \exp\{i\tau_i r_i F(r)/r\}$ and considering an isospin asymmetric and infinite size nuclear environment, one can get the symmetric and asymmetric parts of the binding energy per nucleon [5]

$$\varepsilon_V(\lambda) = f_1 m(f_2) + \frac{3f_3}{8\Lambda} - m_N, \tag{3}$$

$$\varepsilon_A(\lambda, \delta) = \frac{\Lambda_2}{2\Lambda}\left(1 + \frac{\Lambda_2 m_\pi f_4}{f_3}\right)m_\pi f_4 \delta^2. \tag{4}$$

Here the functionals m and $\Lambda = \Lambda_2 + \Lambda_4$ are defined as

$$m(f_2) = \frac{\pi F_\pi}{e}\int_0^\infty dx\, x^2 \left\{ f_2 \frac{2m_\pi^2}{e^2 F_\pi^2}(1 - \cos F) \right.$$
$$\left. + \frac{F_x^2}{2} + \frac{2\sin^2 F}{x^2}\left[\frac{1}{2} + 2F_x^2 + \frac{\sin^2 F}{x^2}\right] \right\}, \tag{5}$$

$$\Lambda_2 = \frac{2\pi}{3e^3 F_\pi}\int_0^\infty x^2 \sin^2 F\, dx, \tag{6}$$

$$\Lambda_4 = \frac{8\pi}{3e^3 F_\pi}\int_0^\infty \left(F_x^2 + \frac{\sin^2 F}{x^2}\right)x^2 \sin^2 F\, dx. \tag{7}$$

In the functionals above, using an infinite nuclear matter approximation, the initial medium functionals in Eq. (1) were rearranged by defining the new medium functions $f_{1,2,3,4}$. The rearrangements are made in the following way

$$1 + C_1\rho = f_1 \equiv \sqrt{\frac{\alpha_s}{\zeta_s}}, \tag{8}$$

$$1 + C_2\rho = f_2 \equiv \frac{\alpha_m}{(\alpha_s)^2\zeta_s}, \tag{9}$$

$$1 + C_3\rho = f_3 \equiv \frac{(\alpha_s\zeta_s)^{3/2}}{\alpha_\tau}, \tag{10}$$

$$\frac{C_4\lambda}{1 + C_5\lambda} = f_4 \equiv \frac{\alpha_e}{\alpha_\tau}\delta^{-1} \tag{11}$$

and the basic principle behind the linear density dependent parametrizations of functions $f_{1,2,3}$ was the simplicity of form.

In the last four equations C_i ($i = \overline{1,5}$) are the model parameters and, therefore, the present approach is a simple 5-parametric solitonic model of nuclear matter.[1] As soon as we define 5-parameters

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[1] We would like to note, although at present work we discuss the properties of an asymmetric and infinite nuclear matter the model can be applied for the studies of finite nuclei properties after fitting the density parameters C_i.

at some specific density (e.g. the normal nuclear matter density, ρ_0) the Equations of State for the symmetric and asymmetric nuclear matter are defined for any values of nuclear matter density if one assumes that the extrapolations by means of the linear on density functions f_i are valid.[2] In the next Subsection 2.3 we concentrate our attention on the properties of baryonic systems at high densities discussing the neutron stars.

2.3. Neutron star

As the first approximation in describing the neutron stars, one can consider a spherically symmetric and static mass distribution. Then any part of neutron star mass $\mathcal{M}(r)$ inside a sphere with radius r is given by the integral

$$\mathcal{M}(r) = 4\pi \int_0^r dr\, r^2 \mathcal{E}(r). \qquad (12)$$

Here $\mathcal{E}(r)$ is the mass-energy density distribution of the neutron star in the radial direction. Consequently, the total gravitational mass M of the neutron star with radius R is defined by the condition

$$M = \mathcal{M}(r \geq R). \qquad (13)$$

Further, in the spherically symmetric approximation, the pressure density change in radial direction inside the neutron star is given by the Tolman–Oppenheimer–Volkoff (TOV) equation [8,9]

$$-\frac{dP(r)}{dr} = \frac{G\mathcal{E}(r)\mathcal{M}(r)}{r^2}\left(1 - \frac{2G\mathcal{M}(r)}{r}\right)^{-1}$$
$$\times \left(1 + \frac{P(r)}{\mathcal{E}(r)}\right)\left(1 + \frac{4\pi r^3 P(r)}{\mathcal{M}(r)}\right). \qquad (14)$$

Here $G = 6.707 \times 10^{-39}\hbar c \left(\frac{\text{GeV}}{c^2}\right)^{-2}$ is gravitational constant and $P(r)$ is pressure density in radial direction. The boundary conditions for the functions entering into the equation are

$$\mathcal{M}(0) = 0 \quad \text{and} \quad \mathcal{E}(0) = \mathcal{E}_{\text{cent}}. \qquad (15)$$

After solving TOV equation one can find the radius R of a star with central energy density $\mathcal{E}_{\text{cent}}$. It is defined by the pressure zero condition at the surface of the neutron star, $P(r = R) = 0$.

To obtain the numerical solution for the profile of a star, one solves Eqs. (12) and (14) using the Equation of State

$$P = P(\mathcal{E}). \qquad (16)$$

To find $P(\mathcal{E})$ dependence in present approach, we note that the pressure and energy dependencies on the density parameter λ for the neutron matter ($\delta = 1$) are given by equations

$$P(\lambda) = \rho_0 \lambda^2 \frac{\partial \varepsilon(\lambda, 1)}{\partial \lambda}, \qquad (17)$$

$$\mathcal{E}(\lambda) = [\varepsilon(\lambda, 1) + m_N]\lambda \rho_0. \qquad (18)$$

where $\varepsilon(\lambda, 1)$ is binding energy per nucleon in neutron matter. The system of parametric equations, Eq. (17) and Eq. (18), gives the desired relation $P(\mathcal{E})$ between pressure and energy densities in a neutron star.

3. Results and discussions

In general, the thermodynamically limiting binding energy per nucleon given in Eq. (2) must be valid at any densities and at any given isospin parameter δ. For example, it is valid also at the extreme conditions which are formed in interior regions of neutron stars. Those extreme conditions described by very high density ($\rho \simeq$ several times of ρ_0) and highly isospin asymmetric ($\delta \simeq 1$) form of nuclear matter.

From other side, it is not clear the direct relation of the liquid-drop formula of Bethe and Weizsäcker [10,11]

$$B(Z, N) = a_V A - a_S A^{2/3} - a_C \frac{Z^2}{A^{1/3}} - a_A \frac{(N - Z)^2}{A} + \dots, \qquad (19)$$

describing the binding energy of nucleus to the binding energy of neutron star. The reason is not only due to the relativistic metric factors coming from the Einstein's equations in the calculations of binding energies of neutron stars. The reason here is rather obvious, the semiempirical liquid-drop formula describes the binding energy per nucleon near the normal nuclear matter densities (which correspond to the density profiles of the existing heavy nuclei) and its validity at high densities is not clear.

Nevertheless, around the normal nuclear matter density and for the small values of asymmetry parameter δ, the thermodynamically limiting binding energy per nucleon given in Eq. (2) is well related to the Bethe and Weizsäcker's formula. Because, from one side, in the limit of small δ the binding energy formula in Eq. (2) can be approximated as

$$\varepsilon(\lambda, \delta) = \varepsilon_V(\lambda) + \varepsilon_S(\lambda)\delta^2 + \mathcal{O}(\delta^4), \qquad (20)$$

where $\varepsilon_S(\lambda)$ is called the symmetry energy. From other side, if one ignores the Coulomb and surface effects, the binding energy per nucleon defined from the liquid-drop formula will take the form

$$A^{-1}B(Z, N) \approx a_V - a_A \delta^2 + \dots. \qquad (21)$$

From the comparisons of the last two equations, it is seen that the density dependencies of volume and symmetry energies can be established well at the densities around ρ_0 from the stability conditions and the density variations of resonating heavy nuclei near the ground state.

The symmetry energy ε_S describes the energy increase in the system if the number of protons and neutrons becomes not equal relatively to the case when the neutron and proton numbers are same. Consequently, the symmetry energy is important factor in describing the properties of neutron-rich stable nuclei existing in nature as well as the properties of exotic nuclei formed under extreme conditions where the neutron-to-proton number N/Z is much smaller or much larger than one [12]. Although the definition of symmetry energy in Eq. (20) is model independent its density dependence is clearly model dependent. Therefore, during the fitting to density region near the normal nuclear matter density ρ_0 the different models are modified taking into account the properties of symmetry energy coming from the phenomenological observations. But the extrapolations of EOS to the high density ($\rho >> \rho_0$) and highly asymmetric ($\delta \simeq 1$) regions remain not clear. In particular, this is due to the reason that in neutron stars the higher order terms in δ in Eq. (20) may be also important. Usually, it is assumed that $\mathcal{O}(\delta^4)$ terms are small and the symmetry energy is mostly responsible in describing the properties of neutron stars when the neutron-to-proton number N/Z becomes infinite.[3] In the

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[2] Other details can be found in Refs. [5,6].

[3] Although the neutron star is initially formed from the ordinary matter which has the finite neutron-to-proton number with well separated electrons, due to the
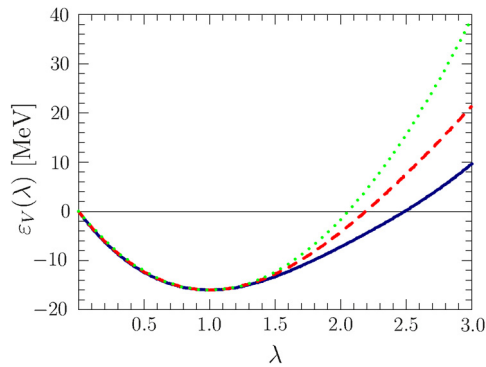
Fig. 1. (Color online.) The volume energy as a function of normalized density $\lambda = \rho/\rho_0$. Solid, dashed and dotted curves correspond to the values of compressibility $K_0 = 230$ MeV, $K_0 = 250$ MeV and $K_0 = 270$ MeV, respectively. Other parameters are defined in Table 1.

present approach, the higher order terms $\mathcal{O}(\delta^4)$ come from the explicit isospin breaking symmetry in the mesonic sector and found to be negligible.[4] Consequently, we ignore them in the present work.

3.1. Nuclear matter and symmetry energy

Let us first discuss the properties of the symmetric nuclear matter. The value of the first experimental parameter in the liquid-drop formula Eq. (19) is well known, $a_V \approx 16$ MeV. It is also well established that the saturation density of symmetric nuclear matter, where the pressure becomes zero $P_{\text{sym}} = 0$, is around 0.16 fm^{-3}. Further, the compressibility of symmetric nuclear matter within the various approaches is found to be $K_0 \sim 290 \pm 70$ MeV at the saturation density ρ_0 [13–18]. Therefore, these three quantities may be used to fix some of the model parameters (e.g. C_1, C_2 and C_3) if one expands the volume energy into the Taylor series

$$\varepsilon_V(\lambda) = \varepsilon_V(1) + \frac{K_0}{18}(\lambda - 1)^2 + \dots \qquad (22)$$

around the saturation density, $\lambda = 1$. After fitting the values of parameters, the EOS of symmetric matter is defined for any values of density.[5] The density dependencies of the binding energy per nucleon in symmetric matter are shown in Fig. 1 for the three possible values of the compressibility K_0. The corresponding parameters and the volume energy coefficients at saturation density ρ_0 are given in Table 1.

It is seen, that our results become consistent with the results from other approaches and the phenomenological indications at some values of density parameters C_i. For example, the density dependence of volume energy from the Set II is close to the APR (Akmal–Pandharipande–Ravenhall) predictions (see the model $A18 + \delta v + UIX^*$ in Ref. [19]) made on the basis of Argonne v_{18} two-nucleon interactions [20]. Similarly, the results from the Set III is very close to the results from the model $A18 + UIX$ [19].

The quantity in the last column, referred as the skewness parameter, is proportional to the third derivative of the volume term at saturation density ρ_0 and defined as

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gravitational collapse and due to further formed electron degenerate states, and due to the following nucleon degenerate states at even higher densities the frequent collisions lead to the intensive nuclear reactions. As a result "an effective" neutron-to-proton number N/Z becomes infinite.

[4] See the discussions in Subsection V.C of Ref. [6].

[5] In this case, the minimization scheme is related to the single in-medium soliton, i.e. the soliton mass is minimized at the given values of density functions f_i.

Table 1
Three sets of parameters which reproduce the symmetric matter properties. The parameters C_1, C_2 and C_3 are chosen in such a way that at saturation point $P_{\text{sym}}(\rho_0) = 0$ the value of volume energy per nucleon equals to its experimental value, $\varepsilon_V(\rho_0) = \varepsilon_V^{\text{exp}} \approx -a_V$, and the compressibility of nuclear matter K_0 is reproduced in a given experimental range.

Set	C_1	C_2	C_3	$\varepsilon_V(\rho_0)$ [MeV]	K_0 [MeV]	Q [MeV]
I	-0.285	0.803	1.753	-16	230	-545
II	-0.273	0.643	1.858	-16	250	-279
III	-0.333	0.281	3.090	-16	270	-133

$$Q = 27\lambda^3 \left. \frac{\partial^3 \varepsilon_V(\lambda)}{\partial \lambda^3} \right|_{\lambda=1}. \qquad (23)$$

Its values presented in Table 1 are outcome results from the present approach. There is a nonlinear correlation between Q and compressibility K_0, Q increases if the value of K_0 increases. Our predictions for Q is qualitatively similar to the results from the Hartree–Fock approach based on Skyrme interactions [21], and to the result from the MDI (isospin and momentum-dependent interaction) model [22]. Another example, the phenomenological momentum-independent model (MID) also predicts the similar results [23]. For comparison, in the present model one has $Q/K_0 \approx -1.71$ at $K_0 = 240$ MeV while MID model gives the result $Q/K_0 \approx -1.6$ at that value of K_0.

Now let us discuss the properties of the asymmetric matter. As we said above, in the approximation that the higher order terms in δ are small (see Eq. (20)), the asymmetric matter properties are completely determined by the density dependence of the symmetry energy. The properties of the symmetry energy can be studied again by expansion into the Taylor series

$$\varepsilon_S(\lambda) = \varepsilon_S(1) + \frac{L_S}{3}(\lambda - 1) + \frac{K_S}{18}(\lambda - 1)^2 + \dots \qquad (24)$$

around the saturation density, $\lambda = 1$. While the first coefficient $\varepsilon_S(1)$ in Eq. (24) is known to be more or less well defined phenomenologically, $\varepsilon_S(1) \sim 29$ to 34 MeV, the values of slop parameter L_S and compressibility of asymmetric matter K_S remain unclear. The reason is that EOS of asymmetric matter is highly sensitive to those parameters and the different models give the different predictions. For example, in relativistic mean field approaches (see Ref. [24], for the recent optimized versions) there are mainly two classes: i) small $\varepsilon_S(\rho_0) \sim 30$ MeV and small $L_S \sim 50$ MeV (BSP, IUFSU*, IUFSU) and ii) large $\varepsilon_S(\rho_0) \sim 37$ MeV and large $L_S \sim 110$ MeV (G1, G2, TM1*, NL3). The recent analyses of the data from heavy-ion collisions [25] and neutron-skin experiments [26] give the prediction $L_S \sim 70 \pm 20$ MeV.

During the modifications of EOS in the present work we choose the value of slop parameter L_S in the interval 30–50 MeV which is in agreement with various empirical constraints [25–27].

The symmetry energies, calculated using two different sets of parameters, are shown in Fig. 2 and the parameters are determined in Table 2. Here we present only two representatives among the many sets producing the symmetry energy parameters in commonly adopted range. Depending on the compressibility K_0 value of the symmetric nuclear matter we classify the results into two models, Model II with relatively smaller value of the compressibility $K_0 = 250$ MeV (more soft EOS) and Model III with relatively bigger value of the compressibility $K_0 = 270$ MeV (more stiff EOS).

The results show that the symmetry energy is less sensitive to the different model parameters presented in Table 2. In the Fig. 2 we have shown the density dependencies of symmetry energy corresponding to two boundary regions referred as more soft (II-a) and more stiff (III-f) Equations of State. The other sets reproduce the density dependence of symmetry energy corresponding

Table 2

The different sets of parameters which reproduce the asymmetric matter properties. The parameters C_4 and C_5 are chosen in such a way that, at saturation density ρ_0, the value of symmetry energy ε_S and the coefficient of its first derivative L_S are reproduced in the commonly adopted range. Other parameters are defined in Table 1 (see the parameters values corresponding to the same K_0 values given in this table).

| Set	K_0 [MeV]	C_4	C_5	$\varepsilon_S(\rho_0)$ [MeV]	L_S [MeV]	K_S [MeV]	K_τ [MeV]	$K_{0,2}$ [MeV]	$\varepsilon_S(0.1\ \mathrm{fm}^{-3})$ [MeV]	$\varepsilon_S(0.11\ \mathrm{fm}^{-3})$ [MeV]
II-a	250	2.338	0.878	30	30	−209	−299	−265	24.26	25.54
II-b	250	1.984	0.594	30	40	−209	−329	−285	23.11	24.55
II-c	250	1.723	0.384	30	50	−197	−347	−291	22.06	23.64
II-d	250	2.559	0.946	32	30	−222	−312	−278	26.11	27.44
II-e	250	2.183	0.660	32	40	−226	−346	−301	24.93	26.44
II-f	250	1.904	0.448	32	50	−217	−367	−311	23.86	25.50
III-a	270	2.670	1.498	30	30	−169	−259	−245	24.57	25.76
III-b	270	2.179	1.024	30	40	−177	−297	−278	23.33	24.71
III-c	270	1.831	0.701	30	50	−172	−322	−298	22.22	23.75
III-d	270	2.980	1.622	32	30	−178	−268	−254	26.46	27.69
III-e	270	2.425	1.133	32	40	−189	−309	−290	25.20	26.62
III-f	270	2.044	0.798	32	50	−188	−338	−313	24.05	25.64
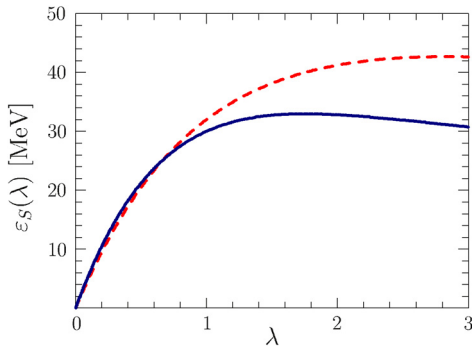
Fig. 2. (Color online.) The symmetry energy as a function of normalized density $\lambda = \rho/\rho_0$. The solid and the dashed curves correspond to the sets II-a and III-f defined in Table 2.

to somewhere in between of these two boundary curves. The relatively more stiff EOS (III-f) in the present approach reproduce the density dependence of symmetry energy which is close to the APR predictions [19], to the result from the MDI model with the parameter $x = 0$ [22] as well as to the results from MID model with the parameter $y = -0.73$ [23].

The quantities in the last five columns in Table 2 are outcome from our model calculations. In particular, the quantities K_τ and $K_{0,2}$ are related to the compressibility of asymmetric matter and defined as

$$K_\tau = K_S - 6L_S, \qquad K_{0,2} = K_\tau - \frac{Q}{K_0}L_S. \qquad (25)$$

They describe the correlations between the symmetry energy coefficients and important for estimating the shift in compressibility value in asymmetric matter

$$K(\rho_0, \delta) = K_0 + K_{0,2}\delta^2 + \mathcal{O}(\delta^4). \qquad (26)$$

The condition for the lowering of saturation density value in asymmetric matter leads to the constraint $K_\tau < 0$. This constraint is fulfilled in the present work.

The calculated values of K_τ and $K_{0,2}$ are consistent with the results from other approaches. For example, the phenomenological momentum-independent model predicts the range for the values of $K_{0,2}$: $-477\ \mathrm{MeV} \leq K_{0,2} \leq -241\ \mathrm{MeV}$ [23]. Our predictions, made using the different sets, are also belong to that range (see Table 2).

It is interesting also to compare the low density behavior of the symmetry energy in the present model with other model predictions. For example, an analysis of the giant dipole resonance (GDR) of ^{208}Pb using a series of microscopic Hartree–Fock plus

Random phase approximation calculations predicts the following values of symmetry energy at subnormal nuclear matter density: $20\ \mathrm{MeV} < \varepsilon_S(\rho = 0.1\ \mathrm{fm}^{-3}) < 25.4\ \mathrm{MeV}$ [28]. The recent analysis of the properties of double magic nuclei [29] puts more constraints, $\varepsilon_S(\rho = 0.1\ \mathrm{fm}^{-3}) = 25.4 \pm 0.8\ \mathrm{MeV}$. An analysis of data on the neutron skin thickness of Sn isotopes and the isotope binding energy difference for heavy nuclei at slightly higher density value gave the result $\varepsilon_S(\rho = 0.11\ \mathrm{fm}^{-3}) = 26.65 \pm 0.2\ \mathrm{MeV}$ [30]. For comparison, our results are presented in the last two columns of Table 2. One can see that they are mainly consistent with the results in Refs. [28–30].

The discussions of density dependencies of the pressure in symmetric and asymmetric matter within the present approach can be found in Ref. [6]. Here we only would like to note, that the modifications are also consistent with the results from other approaches. Moreover, the density dependencies of pressure in symmetric and asymmetric matter are weakly sensitive to the change of model parameters and the reproduced values correspond to the allowed region deduced from the experimental flow data and simulations studies by Danielewicz et al. [31].

In summary, the properties of nuclear matter at saturation density ρ_0 and the extrapolations to the lower than ρ_0 regions are found to be satisfactory and consistent with the results from other approaches. Therefore, we now concentrate on the extrapolations to the high density regions and discuss the properties of neutron stars.

3.2. Properties of neutron stars

Let us start from the mass–radius relations of neutron stars. There are dramatic differences in predictions from the different approaches as concerned the mass–radius relation of neutron stars. But most of the models with Equations of State of normal (nonstrange) nuclear matter predict existence of the regions around the one solar mass where the radius of neutron star is independent from the mass.

In the Fig. 3 we present our results corresponding to the two possible values of the compressibility of symmetric matter $K_0 = 250\ \mathrm{MeV}$ in the left panel and $K_0 = 270\ \mathrm{MeV}$ in the right panel, respectively. One can see that the mass independent regions exist also in the framework of the present approach. Another the common for many models peculiarity, the existence of the maximal mass of the neutron star, is also reproduced well in this model. As we mentioned above, the results are sensitive to the compressibility value, if the compressibility value decreases it leads to decreased values of the maximal mass and radius of neutron star leading in such a way to more compact neutron stars. From other side, the results are not sensitive to the change in the value of
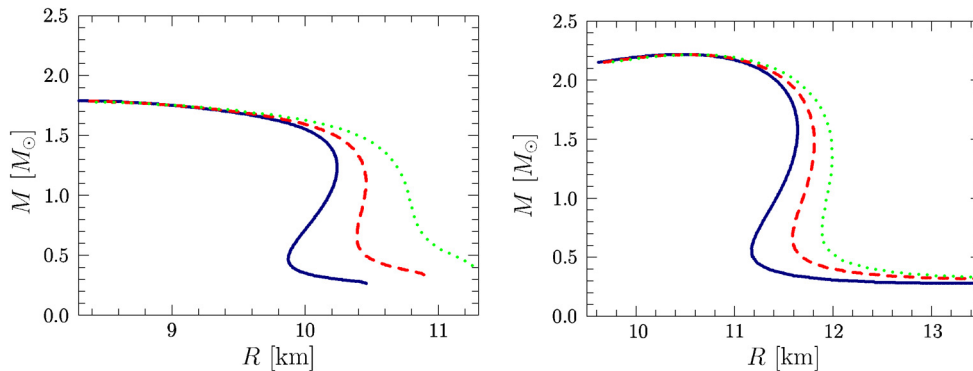
Fig. 3. (Color online.) The mass–radius relations of neutron stars at the values of compressibility $K_0 = 250$ MeV (left panel, Set II) and $K_0 = 270$ MeV (right panel, Set III). The value of symmetry energy at saturation density ρ_0 is chosen as $\varepsilon_S(1) = 32$ MeV. The solid, dashed and dotted curves correspond to the symmetry energy slop parameter L_S values 30, 40 and 50 MeV. They represented by sets d,e and f in the Models II and III, respectively (see Table 2).

Table 3

Properties of the neutron stars from the different sets of parameters (see Tables 1 and 2 for the values of parameters): n_c is central number density, ρ_c is central energy-mass density, R is radius of the neutron star, M_{max} is possible maximal mass, A is number of baryons in the star, E_b is binding energy of the star. In the left panel we represent the neutron star properties corresponding to the maximal mass M_{max} and in the right panel approximately 1.4 solar mass neutron star properties. The last two lines are results from the Ref. [21].

Set	n_c [fm^{-3}]	ρ_c [10^{15}gr/cm^3]	R [km]	M_{max} [$M_\odot$]	A [10^{57}]	E_b [10^{53} erg]	n_c [fm^{-3}]	ρ_c [10^{15}gr/cm^3]	R [km]	M [$M_\odot$]	A [10^{57}]	E_b [10^{53} erg]
III-a	1.046	2.445	10.498	2.226	3.227	8.721	0.479	0.861	11.587	1.402	1.898	3.503
III-b	1.045	2.444	10.547	2.223	3.216	8.557	0.471	0.861	11.772	1.402	1.895	3.453
III-c	1.037	2.424	10.616	2.221	3.200	8.397	0.460	0.832	11.953	1.402	1.887	3.339
III-d	1.047	2.452	10.494	2.221	3.213	8.598	0.481	0.867	11.619	1.402	1.893	3.422
III-e	1.044	2.440	10.554	2.218	3.203	8.495	0.473	0.858	11.809	1.403	1.890	3.384
III-f	1.040	2.433	10.609	2.216	3.189	8.311	0.464	0.842	11.992	1.403	1.887	3.334
SLy230a [21]	1.15	2.69	10.25	2.10	2.99	7.07	0.508	0.925	11.8	1.4	1.85	2.60
SLy230b [21]	1.21	2.85	9.99	2.05	2.91	6.79	0.538	0.985	11.7	1.4	1.85	2.61

$\varepsilon_S(1)$ and, therefore, we presented only the results corresponding to the value of $\varepsilon_S(1) = 32$ MeV. In general, our results are in qualitative agreement with the results from other approaches.

It is also interesting to compare our results in quantitative level too. Measurements of the thermal spectra from the quiescent low-mass X-ray binaries inside globular clusters gave the possibility to fit the data sets with a neutron star radius $R_{NS} = 9.1^{+1.3}_{-1.5}$ km at 90% confidence level [32]. Determinations of the mass–radius relation, based on recent observations of both transiently accreting and bursting sources, gave the radius range between 10.4 and 12.9 km for 1.4 solar mass neutron stars [33].

In the present approach some class of parameters (Model III) defined in Table 2 give very good agreement with the above mentioned estimations in quantitative level. The properties of neutron stars reproduced using some subclasses of Model III is presented in Table 3. One can see that, our results corresponding to more than $2M_\odot$ as well as $\sim 1.4M_\odot$ neutron stars are very similar to the estimations from the Refs. [32,33].

For comparison, in Table 3 we present two of the possible neutron stars parametrizations from the Ref. [21] as the representatives of the Skyrme effective forces used in the density functional approach. One can see that our results in qualitative agreement with the results from the Ref. [21]. It is also necessary to note, that the compressibility value $K_0 = 230$ MeV used in Ref. [21] is smaller in comparison with the compressibility value $K_0 = 270$ MeV in Model III. Therefore, we have more stiff EOS leading to slightly higher maximal mass and larger radius neutron stars. But all of the models presented in Table 3 give more or less similar results as concerned $1.4M_\odot$ mass neutron stars.

The central number densities of maximal mass neutron stars in the present approach are around $6.5\rho_0$ and the corresponding central mass-energy densities are around 2.44×10^{15} gr/cm^3

(≈ 1300 MeV fm^{-3}). Our results are close to the results from non-relativistic potential model approaches discussed in Ref. [34].

It is interesting also to compare the total baryon number of neutron star in the present approach with the results from other approaches. General Relativistic formula for the total baryon number of the neutron star is given by the integral

$$A = 4\pi \int_0^R dr\, r^2 \rho(r) \left(1 - \frac{2G\mathcal{M}(r)}{r}\right)^{-1/2}. \qquad (27)$$

One can find the radial dependence in the number density $\rho(r)$ from the relation $P = P(\rho/\rho_0)$ after finding the radial dependence of the pressure $P = P(r)$. Our results are in qualitative agreement with results from Ref. [21] (see Table 3).

After calculations of the total baryon number, one can also estimate the binding energy E_b of the neutron star. Due to the decrease of the gravitational mass E_b is defined by the formula

$$E_b = Am_N - M, \qquad (28)$$

where we used the mass of nucleon m_N in free space.[6] One can see that the calculated binding energies corresponding to 1.4 solar mass neutron stars are consistent with the estimations made on the analysis of detected neutrinos from SN1987A: $E_b = 3.8 \pm 1.2 \times 10^{53}$ erg [35].

In addition we present the binding energy per unit mass versus GM/Rc^2 relation in Fig. 4 for some representatives of neutron star models given in Table 3. For comparison we represent also the outcome from an approximate formula

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[6] Note, that the authors of the Ref. [21] used 1/56 part of the ^{56}Fe atom mass in calculating the binding energy of the neutron star.
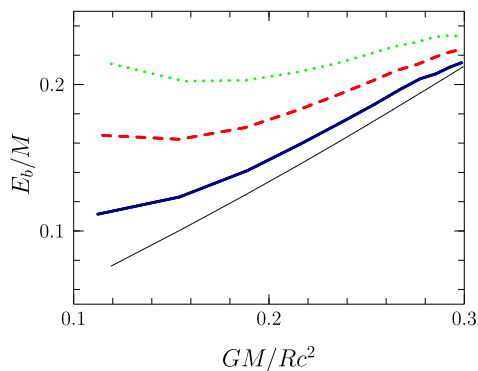
Fig. 4. (Color online.) Binding energy per unit mass E_b/M of the neutron star as a function of GM/Rc^2. The solid, dashed and dotted curves correspond to the sets III-d, III-e and III-f defined in Table 2. Thin and black solid curve represents an approximate relation Eq. (29) presented in Ref. [34].

$$\frac{E_b}{M} \simeq 0.6 \frac{GM}{Rc^2}\left(1 - 0.5\frac{GM}{Rc^2}\right)^{-1} \tag{29}$$

suggested by Lattimer and Prakash [34]. It is seen that in the present approach the small values of slop parameter $L_S \sim 30$ MeV of symmetry energy give the results close to the parametrization in Eq. (29).

4. Summary

In summary, we discussed the application of the in-medium modified chiral soliton model to the studies of asymmetric matter properties and neutron star structure. The symmetric and asymmetric matter equations of state where reproduced by very simple 5-parametric density approach to the single nucleon properties in the nuclear environment introducing the isospin breaking effects in the mesonic sector of the model. After reproducing the asymmetric matter properties near the saturation density of symmetric matter ρ_0 we extrapolated the Equations of State to the high density and highly isospin asymmetric regions. Our primary goal was a crude qualitative analysis of neutron star properties. Nevertheless, at some given set of parameter values, our results are very close to the predictions from analysis of the data compiled during the observation of neutron stars [32,33]. In particular, the calculations showed that the properties of $\sim 1.4M_\odot$ and $\sim 2M_\odot$ neutron stars can be well reproduced within the present approach.

As an outlook for further studies we note that the in-medium chiral soliton model presented here can easily be extended to the studies of finite nuclei properties. This task can be realized in the local density approach for the density of environment, surrounding the soliton under the consideration, taking into account the real density functionals α's and ζ's in the Lagrangian in Eq. (1). Moreover, one should note that the parametrizations of the density functionals are already given by the form of the density functions f_i through the parameters C_i. It is necessary to adjust the form of only one of the density functionals as it is done in the Ref. [5]. Some discussions of further applications of the present approach

and comparisons with other alternate approaches can be found also in Ref. [6]. Some of the above mentioned further studies are under the way.

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