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Multi-objective possibilistic model for portfolio selection with transaction cost

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ABSTRACT

In this paper, we introduce the possibilistic mean value and variance of continuous distribution, rather than probability distributions. We propose a multi-objective Portfolio based model and added another entropy objective function to generate a well diversified asset portfolio within optimal asset allocation. For quantifying any potential return and risk, portfolio liquidity is taken into account and a multi-objective non-linear programming model for portfolio rebalancing with transaction cost is proposed. The models are illustrated with numerical examples.

1. Introduction

The theory of mean–variance efficient portfolios was first given in [1,2], who also gave his critical line method for finding these. Markowitz published his work, which paved the foundation of modern portfolio analysis. It combines probability and optimization theory to model the behavior of economic agents under uncertainty. The mean-variance approach has also been subject to a lot of criticism. One of the most important reasons for this, is the computational difficulty associated with solving a large-scale quadratic programming problem. Konno and Yamazaki [3] used absolute deviation risk function to replace the risk function in Markowitz's model. Consequently, other measures of risk, such as Value at (VaR), expected shortfall, semi-variance and so on are used.

Because of the information incompleteness and the complexity of a financial market, it is impossible to precisely predict the future return and the actual risk of a portfolio. In order to represent vagueness in everyday life, Zadeh [4] introduced the concept fuzzy sets in 1965. Based on this concept, Bellman and Zadeh [5] defined decision-making in a fuzzy environment with a decision set which unifies a fuzzy objective and fuzzy constraint. Watada [6] presented another type of portfolio selection model based on the fuzzy principle. In traditional portfolio theory, a distributive investment has been regarded as a good policy to reduce the risk. Therefore, an application of possibilistic programming to portfolio selection can be expected. In possibilistic programming approaches, the expected return rates are not treated as random variables but as possibilistic variables.

Transaction cost is one of the main concerns for portfolio manager. Obviously, transaction costs have a direct impact on one's performance. Arnott and Wagner [7] found that ignoring transaction costs would result in an inefficient portfolio. In some cases, investors may consider other factors such as liquidity besides two fundamental factors $\cdots$return and risk. The level of return that an investor might aspire to, the risk, and the liquidity of portfolio are vague in an uncertain financial environment. Here we propose a multi-objective non-linear programming model for portfolio rebalancing with transaction cost, by considering liquidity.

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This paper organized as follows. Portfolio Selection Problem and basic concept of fuzzy set are given in Sections 2 and 3. A multi-objective possibilistic model is given in Section 4. In Section 5 we introduce the Mathematical analysis of multi-objective non-linear programming model. Numerical examples are given in Section 6. Some conclusions are finally given in Section 7.

2. Portfolio selection problem

Suppose that a prosperous individuals has a opportunity to invest an asset in n different bonds and stocks.
Notations are as follows:

x_i Proportion of the total amount of money devoted to security i, $i = 1, 2, \ldots, n$
l_i, u_i Minimum, maximum proportion invested to security i respectively
R_i Random rate of return on the risky asset i, $i = 1, 2, \ldots, n$
σ_{ij} $\mathrm{Cov}(R_i, R_j)$, covariance between R_i and R_j, $i, j = 1, 2, \ldots, n$
c_i Rate of transaction cost of i risky asset $i = 1, 2, \ldots, n$

$$\sum_{i=1}^{n} x_i = 1, \quad x_i \geq 0, i = 1, 2, \ldots, n.$$

As in [8] and other, it is assumed that the transaction cost is a V-shaped function of the difference between a new portfolio $x = (x_1, x_2, \ldots, x_n)$, and the existing portfolio $x^o = (x_1^o, x_2^o, \ldots, x_n^o)$. Thus the total transaction cost of the portfolio is $\sum_{i=1}^{n} c_i|x_i - x_i^o|$
For new investors, we can set $x_i^o = 0, i = 1, 2, \ldots, n$.

Model-1: The mean–variance (MV) bi-objective model with transaction costs can be stated as:

$$\begin{aligned}
&\text{Maximize} \quad M(x) = E(R) - \sum_{i=1}^{n} c_i|x_i - x_i^o|, \quad R = \sum_{i=1}^{n} R_i x_i \\
&\text{Minimize} \quad V(x) = \sum_{i=1}^{n}\sum_{j=1}^{n} \mathrm{Cov}(R_i, R_j) x_i x_j \\
&\quad \text{subject to} \quad \sum_{i=1}^{n} x_i = 1 \\
&l_i \leq x_i \leq u_i, \quad x_i \geq 0, \quad i = 1, 2, \ldots, n.
\end{aligned} \tag{1}$$

The Markowitz mean variance (MV) criterion simply states that an investor should always choose an efficient portfolio. The main problem in optimal MV portfolio is that the portfolios are often extremely concentrated on a few assets, which is a contradiction to the notion of diversification. Therefore there is scope for introducing another criterion viz one for diversification and the best candidate for this. It is not surprising that entropy is used as the divergence measure of asset portfolio in finance literature. They usually solve quadratic problem for MV portfolio selection and then, apply an entropy measure to infer how much the portfolio is diversified [9,10]. In this paper, we maximize the entropy function

$$S(x) = -\sum_{i=1}^{n} x_i \log x_i.$$

Model-2: So, the real life problem in analogy to problem (1) is a portfolio selection problem with diversification, which can be written as:

$$\begin{aligned}
&\text{Maximize} \quad S(x) = -\sum_{i=1}^{n} x_i \ln(x_i) \\
&\text{Maximize} \quad M(x) = E(R) - \sum_{i=1}^{n} c_i|x_i - x_i^o|, \quad R = \sum_{i=1}^{n} R_i x_i \\
&\text{Minimize} \quad V(x) = \sum_{i=1}^{n}\sum_{j=1}^{n} \mathrm{Cov}(R_i, R_j) x_i x_j \\
&\quad \text{subject to} \quad \sum_{i=1}^{n} x_i = 1 \\
&l_i \leq x_i \leq u_i, \quad x_i \geq 0, \quad i = 1, 2, \ldots, n.
\end{aligned} \tag{2}$$

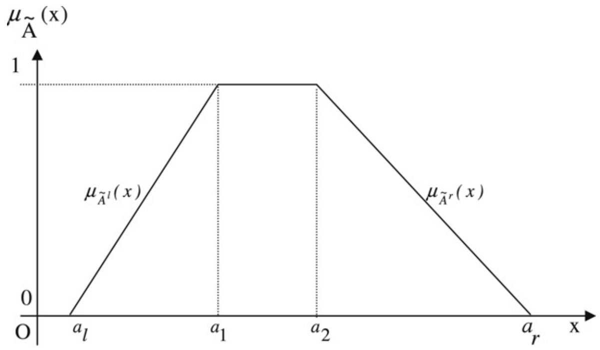

Fig. 1. Trapezoidal fuzzy number.

3. Basic concept in fuzzy set theory

Let X be a collection of objects called the universe of discourse. A fuzzy set $\widetilde{A}$ of X is defined by membership function $\mu_{\widetilde{A}} : X \longrightarrow [0, 1]$. $\mu_{\widetilde{A}}(x)$ is the degree of membership of x in $\widetilde{A}$. The closer the value of $\mu_{\widetilde{A}}(x)$ is to unity, higher the grade of x in $\widetilde{A}$. Therefore, $\widetilde{A}$ is completely characterized by the set of ordered pairs : $\widetilde{A} = \{(x, \mu_{\widetilde{A}}(x))|x \in X\}$.

1. Normally fuzzy set: A fuzzy set $\widetilde{A}$ of the universe of discourse X is called a normal fuzzy set, which implies that there exists at least one $x \in X$, such that $\mu_{\widetilde{A}}(x) = 1$.

2. Convex fuzzy set: A fuzzy set $\widetilde{A}$ of the universe of discourse X is convex iff for all x_1, x_2 in X, $\mu_{\widetilde{A}}(\lambda x_1 + (1 - \lambda)x_2) \geq \min(\mu_{\widetilde{A}}(x_1), \mu_{\widetilde{A}}(x_2)), 0 < \lambda < 1$.

3. Boundary of a fuzzy set: The boundary of a fuzzy set $\widetilde{A}$ on X, is the set of points in X, such that

$$\text{Boundary}(\widetilde{A}) = \{x : 0 < \mu_A(x) < 1, x \in X\}.$$

4. Support of a fuzzy set: The support of a fuzzy set $\widetilde{A}$ on X, is denoted by Supp($\widetilde{A}$), is the set of points in X at which is $\mu_{\widetilde{A}}(x)$ positive, i.e.

$$\text{Supp}(\widetilde{A}) = \{x \in X : \mu_{\widetilde{A}}(x) > 0\}.$$

5. α-cut of a fuzzy set: The α level set of the fuzzy set $\widetilde{A}$ of X is a crisp set A_α, that contains all the elements of X that have membership values in $\widetilde{A}$ greater than or equal to α i.e.

$$A_\alpha = \{x, \mu_{\widetilde{A}}(x)) \geq \alpha, x \in X, 0 \leq \alpha \leq 1\} = [u_1(\alpha), u_2(\alpha)].$$

3.1. Fuzzy number

A fuzzy number $\widetilde{A}$ is a fuzzy set of the real line ($X \equiv \mathbb{R}$, set of real numbers as the universe of discourse) that satisfies the conditions for normality and convexity.

Trapezoidal Fuzzy Number (TrFN): In this paper, we consider a Trapezoidal fuzzy number. A TrFN can be represented completely by a quadruplet $\widetilde{A} = (a_l, a_1, a_2, a_r)$ and its membership function has the following form:

$$\mu_{\widetilde{A}}(x) = \begin{cases} \mu^l_{\widetilde{A}}(x) = \dfrac{x - a_l}{a_1 - a_l}, & \text{if } a_l \leq x \leq a_1 \\ 1, & \text{if } a_1 \leq x \leq a_2 \\ \mu^r_{\widetilde{A}}(x) = \dfrac{a_r - x}{a_r - a_2}, & \text{if } a_2 \leq x \leq a_r \\ 0, & \text{otherwise} \end{cases} \tag{3}$$

where $a_l \leq a \leq a_r$ are real numbers and $\mu^l_{\widetilde{A}}, \mu^r_{\widetilde{A}}$ are left, right membership function of fuzzy number $\widetilde{A}$ respectively. The pictorial representation of the above membership function is Fig. 1. It is noted that a Triangular Fuzzy Number (TFN) is a special type of a TrFN with $a_1 = a_2$.

Consider two trapezoidal fuzzy numbers $\widetilde{A} = (a_l, a_1, a_2, a_r)$ and $\widetilde{B} = (b_l, b_1, b_2, b_r)$ The sum of the two trapezoidal fuzzy numbers is also a trapezoidal fuzzy number and the product with scalar k is also a trapezoidal fuzzy number ie. we have

$$\widetilde{A} + \widetilde{B} = (a_l + b_l, a_1 + b_1, a_2 + b_2, a_r + b_r)$$

$$\begin{aligned} k \times \widetilde{A} &= (ka_l, ka_1, ka_2, ka_r), \quad k \geq 0 \\ &= (ka_r, ka_2, ka_1, ka_l), \quad k < 0 \end{aligned}$$

$$\widetilde{A} - \widetilde{B} = (a_l - b_r, a_1 - b_2, a_2 - b_1, a_r - b_l).$$

3.2. Posssibilistic mean value and variance

In 1987 Dubois and Prade [11] defined an interval valued expectation of fuzzy numbers, viewing them as consonant random sets. In this paper we use crisp possibilistic mean and variance of continuous possibility distributions [12], which are consistent with the extension principle.

Let $A_\alpha = [u_1(\alpha), u_2(\alpha)]$, then the crisp possibilistic mean value of $\widetilde{A} = (a_l, a_1, a_2, a_r)$ is

$$\begin{aligned} E(\widetilde{A}) &= \int_0^1 \alpha[u_1(\alpha) + u_2(\alpha)]\mathrm{d}\alpha \\ &= \int_0^1 \alpha[a_l + \alpha(a_1 - a_l) + a_r - \alpha(a_r - a_2)]\mathrm{d}\alpha \\ &= \frac{a_1 + a_2}{3} + \frac{a_l + a_r}{6}. \end{aligned} \tag{4}$$

Now we introduce the possibilistic variance of $\widetilde{A}$ as

$$\begin{aligned} \mathrm{Var}(\widetilde{A}) &= \frac{1}{2}\int_0^1 \alpha[u_2(\alpha) - u_1(\alpha)]^2\mathrm{d}\alpha \\ &= \frac{1}{2}\int_0^1 \alpha[a_r - \alpha(a_r - a_2) - a_l - \alpha(a_1 - a_l)]^2\mathrm{d}\alpha \\ &= \frac{(a_1 + a_r - a_l - a_2)^2}{8} + \frac{(a_r - a_l)^2}{4} - \frac{(a_1 + a_r - a_l - a_2)(a_r - a_l)}{3}. \end{aligned} \tag{5}$$

Let $A_\alpha = [u_1(\alpha), u_2(\alpha)]$ and $B_\alpha = [v_1(\alpha), v_2(\alpha)]$, then the possibilistic covariance between fuzzy numbers $\widetilde{A} = (a_l, a_1, a_2, a_r)$ and $\widetilde{B} = (b_l, b_1, b_2, b_r)$ is defined as

$$\begin{aligned} \mathrm{Cov}(\widetilde{A}, \widetilde{B}) &= \frac{1}{2}\int_0^1 \alpha[[u_2(\alpha) - u_1(\alpha)][v_2(\alpha) - v_1(\alpha)]]\mathrm{d}\alpha \\ &= \frac{(a_1 + a_r - a_l - a_2)(b_1 + b_r - b_l - b_2)}{8} + \frac{(a_r - a_l)(b_r - b_l)}{4} \\ &\quad - \frac{(a_1 + a_r - a_l - a_2)(b_r - b_l) + (b_1 + b_r - b_l - b_2)(a_r - a_l)}{6}. \end{aligned} \tag{6}$$

4. Multi-objective posibilistic model

Multi-criteria portfolio optimization started with two fundamental factors; expected return and risk. In some cases, investors may consider other factors, such as liquidity and use the turnover rate of securities to measure it. With marketable cash investments, liquidity generally means how easily and quickly one may exchange a security for cash, with little price concession from its going rate, and usually investors prefer greater liquidity. Due to the uncertain financial market it is impossible to predict turnover rates of securities. In many cases, it might be easier to estimate the possibility distribution of turnover rate on securities rather than probability distribution. Here we consider a turnover rate of the i securities by the trapezoidal fuzzy number $\widetilde{T_i} = (T_{il}, T_{i1}, T_{i2}, T_{ir})$, $i = 1, 2, \ldots, n$ and crisp possibilistic mean value of the turnover rate of the portfolio is

$$E(\widetilde{T}) = \sum_{i=1}^{n} \left(\frac{T_{i1} + T_{i2}}{3} + \frac{T_{il} + T_{ir}}{6} \right) x_i, \quad x_i \geq 0 \tag{7}$$

in which the portfolio liquidity is always greater or equal to a given constant λ by the investor, by rebalancing the existing portfolio.

Denote the rate of return on security i, $(i = 1, 2, \ldots, n)$ by the trapezoidal fuzzy numbers $\widetilde{R_i} = (R_{il}, R_{i1}, R_{i2}, R_{ir})$ and we formulate the possibilistic portfolio model as

Model-3

$$\text{Maximize} \quad S(x) = -\sum_{i=1}^{n} x_i \ln(x_i)$$

$$\text{Maximize} \quad M(x) = E\left(\sum_{i=1}^{n} \widetilde{R}_i x_i\right) - \sum_{i=1}^{n} c_i |x_i - x_i^o|$$

$$\text{Minimize} \quad V(x) = \sum_{i=1}^{n}\sum_{j=1}^{n} \text{Cov}(\widetilde{R}_i, \widetilde{R}_j) x_i x_j$$

$$\text{Subject to} \quad E\left(\sum_{i=1}^{n} \widetilde{T}_i x_i\right) \geq \lambda$$

$$\sum_{i=1}^{n} x_i = 1 \quad l_i \leq x_i \leq u_i, \quad l_i, u_i > 0, \quad i = 1, 2, \ldots, n$$

$$\text{where} \quad E\left(\sum_{i=1}^{n} \widetilde{R}_i x_i\right) = \sum_{i=1}^{n} \left(\frac{R_{i1} + R_{i2}}{3} + \frac{R_{il} + R_{ir}}{6}\right) x_i$$

$$\text{Cov}(\widetilde{R}_i, \widetilde{R}_j) = \frac{(R_{i1} + R_{ir} - R_{il} - R_{i2})(R_{j1} + R_{jr} - R_{jl} - R_{j2})}{8} + \frac{(R_{ir} - R_{il})(R_{jr} - R_{jl})}{4} - \frac{(R_{i1} + R_{ir} - R_{il} - R_{i2})(R_{jr} - R_{jl}) + (R_{j1} + R_{jr} - R_{jl} - R_{j2})(R_{ir} - R_{il})}{6}.$$

5. Mathematical Analysis: Multi-objective non-linear programming (MONLP) problem

Here, we discuss the general form of the MONLP problem and technique to solve this type of problem:

5.1. Multi-objective non-linear programming (MONLP) problem

A general MONLP problem may be taken in the following Vector Minimization Problem (VMP):
Minimize k non-linear objective functions

$$\text{Minimize} \quad Z(x) = [Z_1(x), Z_2(x), \ldots, Z_k(x)]. \tag{8}$$

Subject to the inequality constraints

$$\text{Subject to} \quad x \in X = \{x : g_j(x) \leq b_j, (j = 1, 2, \ldots, m), l_i \leq x_i \leq u_i \ (i = 1, 2, \ldots, n)\}.$$

A direct application of the optimality for single objective non-linear programming to the MONLP leads us to the following complete optimality concept.

5.2. Fuzzy programming technique to solve MONLP problem

Zimmermann [13] showed that a fuzzy programming technique could be used nicely to solve the multi-objective programming problem. To solve the MONLP (8) problem, the following steps are used:

Step 1: Solve the MONLP (8) as a single objective non-linear programming problem using only one objective at a time and ignoring the others. These solutions are known as ideal solutions.

Step 2: From the results of step 1, determine the corresponding values for every objective at each solution derived. With the values of all objectives at each ideal solution, the pay-off matrix can be formulated as follows:

	$Z_1(x)$	$Z_2(x)$	......	$Z_k(x)$
x^1	$Z_1^*(x^1)$	$Z_2(x^1)$	......	$Z_k(x^1)$
x^2	$Z_1(x^2)$	$Z_2^*(x^2)$	......	$Z_k(x^2)$
	..	...	......	...
x^k	$Z_1(x^k)$	$Z_2(x^k)$	......	$Z_k^*(x^k)$

where $x^1, x^2, \ldots, x^k$ are the ideal solutions of the k objective function.

$$U_r = \max\{Z_r(x^1), Z_r(x^2), \ldots, Z_r(x^k)\}$$
$$L_r = \min\{Z_r(x^1), Z_r(x^2), \ldots, Z_r(x^k)\}.$$

Step 3: Using aspiration levels of objective functions of the VMP (8) written as follows:
Find x such that

$$Z_r(x) \tilde{\leq} L_r, \quad (r = 1, 2, \ldots, k), \quad x \in X. \tag{9}$$

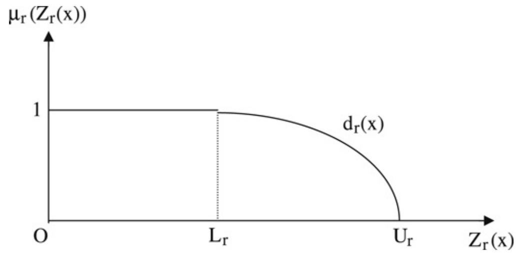

Fig. 2. Membership function.

Here objective functions (8) are considered as fuzzy constraints, which are quantified by the membership function

$$\begin{aligned}\mu_r(Z_r(x)) &= 0 \quad \text{if } Z_r(x) \geq U_r(x)\\ &= d_r(x) \quad \text{if } L_r(x) \leq Z_r(x) \leq U_r(x)\\ &= 1 \quad \text{if } Z_r(x) \leq L_r(x).\end{aligned} \tag{10}$$

Here, $d_r(x)$ is a strictly monotonic decreasing function with respect to $Z_r(x)$ and the Fig. 2 illustrates the graph of $\mu_r(Z_r(x))$.

Having elicited the membership functions (as in (10)) $Z_r(x)$) for $r = 1, 2, \ldots, k$, a general aggregation function which is in the following form.

$$\mu_{\widetilde{D}}(x) = G(\mu_1(Z_1(x)), \mu_2(Z_2(x)), \ldots, \mu_k(Z_k(x))).$$

So a fuzzy multi-objective decision making problem can be defined as

$$\begin{aligned}&\text{Maximize} \quad \mu_{\widetilde{D}}(x)\\ &\text{Subject to} \quad x \in X.\end{aligned} \tag{11}$$

Based on the the convex operator in fuzzy decision making process [5], the problem (11) is reduced to

$$\begin{aligned}&\text{Maximize} \quad \mu_{\widetilde{D}}(x) = \sum_{r=1}^{k} w_r \mu_r(Z_r(x))\\ &\text{Subject to} \quad 0 \leq \mu_r(Z_r(x)) \leq 1, \quad \text{for } r = 1, 2, \ldots, k\\ &x \in X \quad \text{and} \quad w_r \geq 0, \sum_{r=1}^{k} w_r = 1.\end{aligned} \tag{12}$$

Step 4: Solve (12) to get Pareto optimal solution

Some basic definitions on Pareto optimal solutions are introduced below.

Definition 1 (*Complete Optimal Solution*). x^* is said to be a complete optimal solution to the MONLP (2) if and only if, there exists $x^* \in X$ such that $Z_r(x^*) \leq Z_r(x)$, for $r = 1, 2, \ldots, k$ and for all $x \in X$. However, when the objective functions of the MONLP conflict with each other, a complete optimal solution does not always exist, and hence the Pareto Optimality Concept arises and it is defined as follows.

Definition 2 (*Pareto Optimal Solution*). x^* is said to be a Pareto optimal solution to the MONLP (2) if and only if, there does not exist another $x \in X$, such that $Z_r(x^*) \leq Z_r(x)$, for $r = 1, 2, \ldots, k$ and $Z_j(x) \neq Z_j(x^*)$ for at least one $j, j \in \{1, 2, \ldots, k\}$.

5.3. Fuzzy programming technique to solve multi-objective possibilistic portfolio selection model (MOPM)

Model—can be formulated as Vector Minimization problem (VMP)

$$\begin{aligned}&\text{Minimize} \quad [-S(x)] = \sum_{i=1}^{n} x_i \ln(x_i)\\ &\text{Minimize} \quad [-M(x)] = -E\left(\sum_{i=1}^{n} \widetilde{R}_i x_i\right) + \sum_{i=1}^{n} c_i |x_i - x_i^o|\\ &\text{Minimize} \quad V(x) = \sum_{i=1}^{n}\sum_{j=1}^{n} \text{Cov}(\widetilde{R}_i, \widetilde{R}_j) x_i x_j\\ &\text{Subject to} \quad \sum_{i=1}^{n}\left(\frac{T_{i1}+T_{i2}}{3} + \frac{T_{il}+T_{ir}}{6}\right) x_i \geq \lambda\end{aligned} \tag{13}$$

$$\sum_{i=1}^{n} x_i = 1 \quad l_i \le x_i \le u_i, \quad x_i \ge 0, \quad i = 1, 2, \ldots, n$$

$$\text{where} \quad E\left(\sum_{i=1}^{n} \widetilde{R}_i x_i\right) = \sum_{i=1}^{n}\left(\frac{R_{i1}+R_{i2}}{3}+\frac{R_{il}+R_{ir}}{6}\right)x_i$$

$$\mathrm{Cov}(\widetilde{R}_i,\widetilde{R}_j) = \frac{(R_{i1}+R_{ir}-R_{il}-R_{i2})(R_{j1}+R_{jr}-R_{jl}-R_{j2})}{8}+\frac{(R_{ir}-R_{il})(R_{jr}-R_{jl})}{4} - \frac{(R_{i1}+R_{ir}-R_{il}-R_{i2})(R_{jr}-R_{jl})+(R_{j1}+R_{jr}-R_{jl}-R_{j2})(R_{ir}-R_{il})}{6}.$$

To solve VMP form (13), step-1 of (5.2) is used. After that, the pay-off matrix is formulated as follows :

	$S(x)$	$M(x)$	$V(x)$
x^1	$S(x^1)$	$M(x^1)$	$V(x^1)$
x^2	$S(x^2)$	$M(x^2)$	$V(x^2)$
x^3	$S(x^3)$	$M(x^3)$	$V(x^3)$

Now we find the upper bounds U_S, U_M, U_V and lower bounds L_S, L_M, L_V

where $U_S = \max\{S(x^1), S(x^2), S(x^3)\}, \qquad L_S = \min\{S(x^1), S(x^2), S(x^3)\}$

$U_M = \max\{M(x^1), M(x^2), M(x^3)\}, \qquad L_M = \min\{M(x^1), M(x^2), M(x^3)\}$

$U_V = \max\{V(x^1), V(x^2), V(x^3)\}, \qquad L_V = \min\{V(x^1), V(x^2), V(x^3)\}.$

For simplicity, the membership functions are $\mu_{-S}(-S(x))$, $\mu_{-M}(-M(x))$ and $\mu_V(V(x))$ for the objective functions $S(x)$, $M(x)$ and $V(x)$ are defined as follows:

$$\mu_{-S}(-S(x)) = \begin{cases} 0, & \text{if } -S(x) \ge -L_S \\ \left(\dfrac{S(x)-L_S}{U_S-L_S}\right), & \text{if } -U_S < -S(x) < -L_S \\ 1, & \text{if } -S(x) \le -U_S \end{cases}$$

$$\mu_{-M}(-M(x)) = \begin{cases} 0, & \text{if } -M(x) \ge -L_M \\ \left(\dfrac{M(x)-L_M}{U_M-L_M}\right), & \text{if } -U_M < -M(x) < -L_M \\ 1, & \text{if } -M(x) \le -U_M \end{cases}$$

$$\mu_V(V(x)) = \begin{cases} 0, & \text{if } V(x) \ge U_V \\ \left(\dfrac{U_V-V(x)}{U_V-L_V}\right), & \text{if } L_V < V(x) < U_V \\ 1, & \text{if } D_k(v) \ge U_{D_k}. \end{cases}$$

According to step 3, with the above membership functions crisp non-linear programming problem of (13) is formulated as follows:

$$\text{Maximize} \quad F = w_1\mu_{-S}(-S(x)) + w_2\mu_{-M}(-M(x)) + w_3\mu_V(V(x)) \tag{14}$$

$$\text{Subject to} \quad \sum_{i=1}^{n}\left(\frac{T_{i1}+T_{i2}}{3}+\frac{T_{il}+T_{ir}}{6}\right)x_i \ge \lambda$$

$$\sum_{i=1}^{n} x_i = 1 \quad l_i \le x_i \le u_i, \quad x_i \ge 0, \quad i = 1, 2, \ldots, n.$$

$$0 \le \mu_{-S}(-S(x)), \quad \mu_{-M}(-M(x)), \quad \mu_V(V(x)) \le 1, \quad \sum_{i=1}^{3} w_i = 1.$$

Problem (14) can also be written as

$$\text{Maximize} \quad F = w_1\left(\frac{S(x)-L_S}{U_S-L_S}\right) + w_2\left(\frac{M(x)-L_M}{U_M-L_M}\right) + w_3\left(\frac{U_V-V(x)}{U_V-L_V}\right) \tag{15}$$

$$\text{Subject to} \quad \sum_{i=1}^{n}\left(\frac{T_{i1}+T_{i2}}{3}+\frac{T_{il}+T_{ir}}{6}\right)x_i \ge \lambda$$

$$\sum_{i=1}^{n} x_i = 1 \quad l_i \le x_i \le u_i, \quad x_i \ge 0, \quad i = 1, 2, \ldots, n.$$

$$0 \le \left(\frac{S(x)-L_S}{U_S-L_S}\right), \quad \left(\frac{M(x)-L_M}{U_M-L_M}\right), \quad \left(\frac{U_V-V(x)}{U_V-L_V}\right) \le 1$$
$$\sum_{i=1}^{3} w_i = 1.$$

The problem is equivalent to

$$\text{Maximize} \quad F_1 = -K_1 \sum_{i=1}^{n} x_i \ln(x_i) + K_2 \left[E\left(\sum_{i=1}^{n} \widetilde{R}_i x_i\right) - \sum_{i=1}^{n} c_i |x_i - x_i^o| \right] - K_3 \sum_{i=1}^{n}\sum_{j=1}^{n} \mathrm{Cov}(\widetilde{R}_i, \widetilde{R}_j) x_i x_j$$
$$\text{Subject to} \quad \sum_{i=1}^{n} \left(\frac{T_{i1}+T_{i2}}{3} + \frac{T_{il}+T_{ir}}{6}\right) x_i \ge \lambda$$
$$\sum_{i=1}^{n} x_i = 1 \quad l_i \le x_i \le u_i, \quad x_i \ge 0, \quad i = 1, 2, \ldots, n.$$
$$K_1 = \frac{w_1}{U_S - L_S}, \quad K_2 = \frac{w_2}{U_M - L_M}, \quad K_3 = \frac{w_3}{U_V - L_V}$$
$$F(x) = F_1(x) - \frac{w_1 L_S}{U_S - L_S} - \frac{w_2 L_M}{U_M - L_M} + \frac{w_3 U_V}{U_V - L_V}$$
$$\sum_{i=1}^{3} w_i = 1$$
$$\text{where} \quad E\left(\sum_{i=1}^{n} \widetilde{R}_i x_i\right) = \sum_{i=1}^{n} \left(\frac{R_{i1}+R_{i2}}{3} + \frac{R_{il}+R_{ir}}{6}\right) x_i \tag{16}$$
$$\mathrm{Cov}(\widetilde{R}_i, \widetilde{R}_j) = \frac{(R_{i1}+R_{ir}-R_{il}-R_{i2})(R_{j1}+R_{jr}-R_{jl}-R_{j2})}{8} + \frac{(R_{ir}-R_{il})(R_{jr}-R_{jl})}{4} - \frac{(R_{i1}+R_{ir}-R_{il}-R_{i2})(R_{jr}-R_{jl}) + (R_{j1}+R_{jr}-R_{jl}-R_{j2})(R_{ir}-R_{il})}{6}.$$

6. Numerical examples

In this section, we will give a numerical example to illustrate the proposed multi-objective possibilistic portfolio selection model. Consider six securities problems with the following possibility distribution.

$0 \le x_i \le 0.5$

i	$(R_{il}, R_{i1}, R_{i2}, R_{ir})$	c_i	x_i^o	$(T_{il}, \lambda T_{i1}, \lambda T_{i2}, T_{ir}), \lambda = 0.3$
1	(0.046, 0.069, 0.074, 0.081)	0.003	0.15	(0.002, 0.012, 0.024, 0.042)
2	(0.048, 0.070, 0.076, 0.084)	0.001	0.20	(0.003, 0.015, 0.027, 0.045)
3	(0.048, 0.072, 0.078, 0.088)	0.005	0.15	(0.001, 0.012, 0.015, 0.028)
4	(0.050, 0.076, 0.082, 0.090)	0.004	0.15	(0.002, 0.012, 0.042, 0.072)
5	(0.060, 0.078, 0.086, 0.095)	0.003	0.20	(0.001, 0.009, 0.024, 0.039)
6	(0.062, 0.088, 0.098, 0.100)	0.001	0.15	(0.0015, 0.006, 0.030, 0.045)

In Table 1 results have been presented for different weights to the objectives. In type III expected return is higher than that in type-I. but risk also increases.

7. Conclusion

Portfolio selection is a usual multi-objective problem. In this paper, we considered an optimum portfolio for a private investor, taking into account multi-criteria: return, risk, liquidity and added entropy as the objective function to generate well diversified asset. These objectives, in general, are not crisp from the point of view of the investor, so we will deal with them in fuzzy terms. we considered a trapezoidal possibility distribution as the possibility distribution of the rates of returns on the securities. In this study, we proposed a multi-objective non-linear programming model with transaction cost, and a fuzzy non-linear programming technique is used to solve the problem.

Table 1
Pareto optimal solution of Model – IV using different weights

Weights	x_1^*	x_2^*	x_3^*	x_4^*	x_5^*	x_6^*	$S(x^*)$	$M(x^*)$	$V(x^*)$	Type
$w_1 = 1/3$ $w_2 = 1/3$ $w_3 = 1/3$	0.083	0.086	0.115	0.097	0.202	0.417	0.524	0.072	0.041	I
$w_1 = 0.45$ $w_2 = 0.45$ $w_3 = 0.10$	0.217	0.057	0.133	0.167	0.171	0.255	0.409	0.047	0.029	II
$w_1 = 0.45$ $w_2 = 0.15$ $w_3 = 0.40$	0.157	0.129	0.089	0.121	0.150	0.354	0.217	0.089	0.051	III

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